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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

Tystysgrif Addysg Gyffredinol
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MARKING SCHEMES

SUMMER 2006

MATHEMATICS
C1-C4 and FP1-FP3

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1.	(*a*)	Gradient of $AC(BD) = \dfrac{\text{increase in } y}{\text{increase in } x}$	M1
		Gradient $AC = 2$	A1
		Gradient $BD = -½$	A1
		Gradient $AC \times$ Gradient $BD = -1$	M1
		$\therefore$ AC and BD are perpendicular	A1
	(*b*)	A correct method for finding the equation of $AC(BD)$	M1
		Equation of AC : $y - 2 = 2(x - 3)$ (or equivalent)	
		(f.t. candidate's gradient of AC)	A1
		Equation of AC : $2x - y - 4 = 0$ (convincing)	A1
		Equation of BD : $y - 3 = -½(x + 4)$ (or equivalent)	
		(f.t. candidate's gradient of BD)	A1
	(*c*)	An attempt to solve equations of AC and BD simultaneously	M1
		$x = 2, y = 0$ (c.a.o.)	A1
	(*d*)	A correct method for finding the length of AE	M1
		$AE = \sqrt{5}$	A1
2.	(*a*)	$\dfrac{5-\sqrt{3}}{\sqrt{3}+1} = \dfrac{(5-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$	M1
		Numerator: $5\sqrt{3} - 5 - 3 + \sqrt{3}$	A1
		Denominator: $3 - 1$	A1
		$\dfrac{5-\sqrt{3}}{\sqrt{3}+1} = 3\sqrt{3} - 4$	A1
		Special case If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $a + \sqrt{3}b$	
	(*b*)	Removing brackets	M1
		$\sqrt{12} = 2 \times \sqrt{3}$	B1
		$\sqrt{12} \times \sqrt{3} = 6$	B1
		$(2 + \sqrt{3})(4 - \sqrt{12}) = 2$ (c.a.o.)	A1

3.	(a)	An attempt to find $\dfrac{dy}{dx}$		M1
		$\dfrac{dy}{dx} = 2x - 4$		A1
		Value of $\dfrac{dy}{dx}$ at $A = -2$	(f.t. candidate's $\dfrac{dy}{dx}$)	A1
		Equation of tangent at A: $y - 4 = -2(x - 1)$	(or equivalent) (f.t. one error)	A1
	(b)	Gradient of normal × Gradient of tangent = –1		M1
		Equation of normal at A: $y - 4 = ½(x - 1)$	(or equivalent) (f.t. candidate's numerical value for $\dfrac{dy}{dx}$)	A1
4.	(a)	An expression for $b^2 - 4ac$, with $b = \pm4$, and at least one of a or c correct		M1
		$b^2 - 4ac = 4^2 - 4k(k - 3)$		A1
		$b^2 - 4ac = 4(k - 4)(k + 1)$		A1
		Putting $b^2 - 4ac = 0$		m1
		$k = -1, 4$	(f.t. one slip)	A1
	(b)	$a = 4$		B1
		$b = -14$		B1
		Least value = –14	(f.t. candidate's b)	B1
5.	(a)	Use of $f(2) = -20$		M1
		$8p - 4 + 2q - 6 = -20$		A1
		Use of $f(3) = 0$		M1
		$27p - 9 + 3q - 6 = 0$		A1
		Solving simultaneous equations for p and q		M1
		$p = 2, q = -13$	(c.a.o.)	A1
		Special case assuming $p = 2$		
		Use of one of the above equations to find q		M1
		$q = -13$		A1
		Use of other equation to verify $q = -13$		A1
	(b)	Dividing $f(x)$ by $(x - 3)$ and getting coefficient of x^2 to be 2		M1
		Remaining factor = $2x^2 + ax + b$ with one of a, b correct		A1
		$f(x) = (x - 3)(2x + 1)(x + 2)$	(c.a.o.)	A1

6.	(*a*)	$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	B1
		An attempt to substitute $3x$ for a and $\pm\ \frac{1}{3}$ for b in r. h. s. of above expansion	M1
		Required expression = $81x^4 - 36x^2 + 6 - \frac{4}{9x^2} + \frac{1}{81x^4}$	
		(3 terms correct)	A1
		(all terms correct)	A2
		(f.t. one slip in coefficients of $(a + b)^4$)	
	(*b*)	Either: $\frac{n(n-1)}{2} \times 2^k = 40 (k = 1,2)$	
		Or: ${}^nC_2 \times 2^2 = 40$	M1
		$n = 5$	A1
7.	(*a*)	$y + \delta y = (x + \delta x)^2 - 3(x + \delta x) + 4$	B1
		Subtracting y from above to find δy	M1
		$\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$	A1
		Dividing by δx, letting $\delta x \to 0$ and referring to limiting value of $\frac{\delta y}{\delta x}$	M1
		$\frac{dy}{dx} = 2x - 3$	A1
	(*b*)	Required derivative $= -4x^{-3} + \frac{7x^{-\frac{1}{2}}}{2}$	B1, B1
8.	(*a*)	An attempt to collect like terms across the inequality	M1
		$x > -\frac{7}{6}$	A1
	(*b*)	An attempt to remove brackets	M1
		$x^2 + 6x + 8 < 0$	A1
		Graph crosses x-axis at $x = -4$, $x = -2$	B1
		Either: $-4 < x < -2$	
		Or: $-4 < x$ and $x < -2$	
		Or: $(-4, -2)$	B1

9.	(a)	Translation along y-axis so that stationary point is $(0, a)$, $a = 0, -8$	M1
		Correct translation and stationary point at (0, 0)	A1
	(b)	Translation of 2 units to left along x-axis	M1
		Stationary point is $(-2, -4)$	A1
		Points of intersection with x-axis are $(-4, 0)$ and $(0, 0)$	A1
		Special case	
		Translation of 2 units to right along x-axis with correct labelling	B1

10.	$\frac{dy}{dx} = 3x^2 - 6x - 9$			B1
	Putting derived $\frac{dy}{dx} = 0$			M1
	$x = 3, -1$	(both correct)	(f.t. candidate's $\underline{dy}$)	A1
	Stationary points are $(-1, 7)$ and $(3, -25)$	(both correct)	(c.a.o)	A1
	A correct method for finding nature of stationary points			M1
	$(-1, 7)$ is a maximum point		(f.t. candidate's derived values)	A1
	$(3, -25)$ is a minimum point		(f.t. candidate's derived values)	A1

MATHEMATICS C2

1.	$h = 0{\cdot}1$	
		M1 (correct formula $h = 0{\cdot}1$)
	Integral $\approx \frac{0\cdot 1}{2}$ [1 + 1·012719 + 2(1·0000500 + 1·0007997	B1 (3 values)
	+ 1·0040418)]	B1 (2 values)
	$\approx 0{\cdot}401$	A1 (F.T. one slip)
	<u>S. Case</u> $h = 0{\cdot}08$	
	Integral $\approx \frac{0\cdot 08}{2}$ [1 + 1·012719 + 2 (1·0000205 + 1·0003276 + 1·0016575 + 1·0052292)]	M1 (correct formula $h = 0{\cdot}08$)
	$\approx 0{\cdot}401$	B1 (all values) A1 (F.T. one slip)
		4
2.	*(a)* $x = 158{\cdot}2^\circ, 338{\cdot}2^\circ$	B1, B1
	(b) $3x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$	B1 (any value)
	$x = 20^\circ, 100^\circ, 140^\circ$	B1, B1, B1
	(c) $2(1 - \sin^2\theta) + 3\sin\theta = 0$	M1 (correct use of $\cos^2\theta = 1 - \sin^2\theta$)
	$2\sin^2\theta - 3\sin\theta - 1 = 0$ $(2\sin\theta + 1)(\sin\theta - 2) = 0$	M1 (attempt to solve quad in $\sin\theta$ correct formula or $(a\cos\theta + b)(c\sin\theta + d)$ with ac = coefft. of $\sin^2\theta$ bd = constant term)
	$\sin\theta = -\frac{1}{2}, 2$	A1
	$\theta = 210^\circ, 330^\circ$	B1 (210º) B1 (330º)
		11
3.	*(a)* Area $= \frac{1}{2}x \times 8\sin 150^\circ$	B1
	$\frac{1}{2}x \times 8 \times \sin 150^\circ = 10$	B1 (correct equation)
	$x = \frac{10}{4\sin 150^\circ} = 5$	B1 (C.A.O.)

	(b)	$BC^2 = 5^2 + 8^2 + 2.5.8 \cos 30°$ (o.e.)	B1	
		$= 25 + 64 + 68{\cdot}29$	B1	
		$BC \approx 12{\cdot}58$	B1	
				6
4.	*(a)*	$S_n = a + a + d + ... + a + (n-2)d + a(n-1)d$	B1 (at least 3 terms one at each end)	
		$S_n = a + (n-1)d + a + (n-2)d + ... + a + d + a$		
		$2S_n = 2a + (n-1)d + 2a + (n-1)d + ...$	M1	
		$+ 2a + (n-1)d + 2a + (n-1)d$		
		$= n[2a + (n-1)d]$		
		$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$	A1 (convincing)	
	(b)	(i) $\frac{20}{2}[2a + 19d] = 540$	B1	
		$\frac{30}{2}[2a + 29d] = 1260$	B1	
		$2a + 19d = 54$ (1)		
		$2a + 29d = 84$ (2)		
		Solve (1), (2), $d = 3$	M1 (reasonable attempt to solve equations)	
		$a = -\frac{3}{2}$	A1 (both) C.A.O.	
		(ii) 50^{th} term $= -\frac{3}{2} + (n-1)3$ ($n = 50$)	M1 (correct)	
		$= 145{\cdot}5$	A1 (F.T. derived values)	
				9
5.	*(a)*	$ar = 9\,ar^3$	M1 ($ar = kar^3$, $k = 9, \frac{1}{9}$) A1 (correct)	
		$1 = 9r^2$	A1 (F.T. value of k)	
		$r = \pm\frac{1}{3}$	A1 (F.T. value of k, $r = \pm 3$)	

(b) $\dfrac{a}{1-\dfrac{1}{3}} = 12$ M1 (use of correct formula)

$a = 8$ A1 (F.T. derived r)

Third term = $8 \times \left(\dfrac{1}{3}\right)^2 = \dfrac{8}{9}$ (F.T. r) Al

7

6. $3x^{\frac{4}{3}} + \dfrac{3}{2}x^{-2} + 5x(+C)$ B1, B1, B1

3

7. (a) $7 + 2x - x^2 = x + 1$ M1 (equating ys)

$x^2 - x - 6 = 0$ M1 (correct attempt to solve quad)

$(x - 3)(x + 2) = 0$

$x = 3, - 2$ A1

$B\ (3, 4)$ A1

(b) Area $= \displaystyle\int_0^3 (7 + 2x - x^2)dx$ M1 (use of integration to find areas)

$- \displaystyle\int_0^3 (x + 1)dx$ m1

$= \displaystyle\int_0^3 (6 + x - x^2)dx$ B1 (simplified)

$= \left[6x + \dfrac{x^2}{2} - \dfrac{x^3}{3}\right]_0^3$ B3 (3 correct integrations)

$= 18 + \dfrac{9}{2} - 9 - (0 + 0 - 0)$ M1 (use of limits)

$= \dfrac{27}{2}$ A1 (C.A.O.)

12

8.	*(a)*	Let $\log_a x = p$	B1 (props of logs)
		$\therefore x = a^p$	
		$x^n = (a^p)^n = a^{pn}$	B1 (laws of indices)
		$\log_a x^n = pn = n \log_a x$	B1 (convincing)
	(b)	$\ln 5^{3x+1} = ln6$	M1 (taking logs)
		$(3x + 1)\ ln\ 5 = ln6$	A1 (correct)
		$3x\ ln\ 5 = ln\ 6 - ln\ 5$	
		$\therefore\ x = \dfrac{ln6 - ln5}{3ln5}$ (o.e.)	m1 (reasonable attempt to isolate x)
		$\approx 0{\cdot}0378$	A1 (C.A.O.)
			7
9.	*(a)*	Centre $(-1, 4)$	B1
		Radius = $\sqrt{1^2 + 4^2 - 8} = 3$	B1 (use of formula or std form)
			B1 (answer)
	(b)		
		$DP^2 = 29$ (o.e.)	B1 (F.T. coords of centre)
		$PT^2 = DP^2 - (\text{radius})^2$	M1 (use of Pythagoras)
		$= 29 - 9$	
		$= 20$	
		$PT = \sqrt{20}$	A1 (convincing)
	(c)	Equation of circle is	M1 (use of $x^2 + y^2 + 2gx + 2fy + c = 0$
		$(x - 4)^2 + (y - 6)^2 = 20$	or $(x - 4)^2 + (y - 6)^2$ = any +ve no)
		or $x^2 + y^2 - 8x - 12y + 32 = 0$	A1 (either)
			8
10.	*(a)*	$x = 2 \times 4 + 4\theta = 8 + 4\theta$	B1
		$A = \dfrac{1}{2} \times 4^2\theta = 8\theta$	B1
		$8 + 4\theta = 3 \times 8\theta$	B1 (correct equation)
		$20\theta = 8,\ \theta = 0{\cdot}4$	B1 (convincing)
	(b)	Area = $\dfrac{1}{2} \times 4^2 \times 0{\cdot}4 - \dfrac{1}{2} \times 4^2 \times \sin 0{\cdot}4$	B1 (sector)
			B1 (Δ)
			M1 (sector – Δ)
		$\approx 0{\cdot}085$	A1 (C.A.O.)
			8

MATHEMATICS C3

1.	$h = 0{\cdot}25$		M1 ($h = 0{\cdot}25$ correct formula)
	Integral $\approx \dfrac{0.25}{3}$ [0 + 0·8325546 + 4(0·4723807 + 0·7480747)		B1 (3 values)
	+ 2(0·6367614)]		B1 (2 values)
	$\approx 0{\cdot}582$		A1 (F.T. one slip)
			4
2.	*(a)*	$a = b = 45^{\circ}$, for example	B1 (choice of values)
		$\cos(a + b) = 0$	
		$\cos a + \cos b = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2} \approx 1 \cdot 41$	B1 (for correct demonstration)
		$(\therefore \cos(a + b) \neq \cos a + \cos b)$	
		[other cases, $\cos 1^{\circ} + \cos 2^{\circ} = 0{\cdot}999848 + 0{\cdot}999391$ $\approx 1{\cdot}999$ $\cos 3^{\circ} \approx 0{\cdot}999$ $\cos 2^{\circ} + \cos 3^{\circ} \approx 0{\cdot}9994 + 0{\cdot}9986 = 1{\cdot}998$ $\cos 5^{\circ} = 0{\cdot}9962$]	
	(b)	$7 - (1 + \tan^2\theta) = \tan^2\theta + \tan\theta$	M1 (substitution of $\sec^2\theta = 1 + \tan^2\theta$)
		$2\tan^2\theta + \tan\theta \;\; 6 = 0$	M1 (attempt to solve quad ($a \tan\theta + b$)($c \tan\theta + d$)
		$(2\tan\theta - 3)(\tan\theta + 2) = 0$	with ac = coefficient of $\tan^2\theta$ bd = constant term,
		$\tan\theta = \dfrac{3}{2}, -2$	or formula)
			A1
		$\theta = 56{\cdot}3^{\circ}, 236{\cdot}3^{\circ}, 116{\cdot}6^{\circ}, 296{\cdot}6^{\circ}$	B1 ($56{\cdot}3^{\circ}, 236{\cdot}3^{\circ}$) B1 ($116{\cdot}6^{\circ}, 296{\cdot}6^{\circ}$)
		Full F.T. for $\tan\theta = t$, 2 marks for $\tan\theta = -, -$ 1 mark for $\tan\theta = +, +$	
			8

3.	*(a)*	$\dfrac{dy}{dx} = \dfrac{2\cos 2t}{-\sin t}$	M1 (attempt to use $\dfrac{dy}{dx} = \dfrac{\dot{y}}{\dot{x}}$),
			B1 ($-\sin t$)
			B1 ($k\cos 2t$, $k = 1, 2, -2, \dfrac{1}{2}$)
			A1 $\left(\dfrac{2\cos 2t}{-\sin t}, \text{C.A.O.}\right)$
	(b)	$4x^3 + 2x^2\dfrac{dy}{dx} + 4xy + 2y\dfrac{dy}{dx} = 0$	B1 ($2x^2 + 4xy$)
			B1 ($2y\dfrac{dy}{dx}$)
			B1 ($4x^3$, 0)
		$\dfrac{dy}{dx} = \dfrac{(4x^3 + 4xy)}{2x^2 + 2y}$	B1 (C.A.O.)

8

4.	*(a)*	(i) $\left[\dfrac{e^{2x}}{2} - x\right]_0^a = \dfrac{e^{2a}}{2} - a - \dfrac{1}{2}$	M1 (ke^{2x}, $k = \dfrac{1}{2}, 1$)
			A1 $\left(\dfrac{e^{2x}}{2} - x\right)$
			A1 (F.T. one slip (in k))
		(ii) $\dfrac{e^{2a}}{2} - a - \dfrac{1}{2} = \dfrac{1}{2}(9 - a)$	
		$e^{2a} - 2a - 1 = 9 - a$	
		$e^{2a} - a - 10 = 0$	B1 (convincing)

(b)

$\underline{a}$	$\underline{f(a)}$		
1	–3·61	change of sign	M1 (attempt to find values or signs)
2	42·6	indicates presence	
		of root (between 1 and 2)	A1 (correct values or signs and conclusion)

$a_0 = 1{\cdot}2$, $a_1 = 1{\cdot}2079569$, $a_2 = 1{\cdot}20831198$, $a_3 = 1{\cdot}2083278$	B1 (a_1)
$a_4 \approx 1{\cdot}20833$ (1·2083285)	B1 (a_4 to 5 places, C.A.O.)

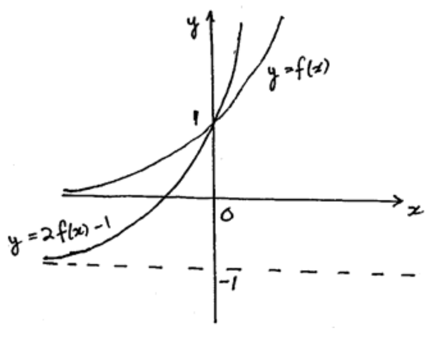
Try 1·208324, 1·208335

$\underline{a}$	$\underline{f(a)}$	
1·208325	–0·00008	M1 (attempt to find values or signs)
1·208335	0·00014	A1 (correct values or signs)

Change of sign indicates root is 1·20833
(correct to 5 decimal places) A1

11

5.	*(a)*	(i)	$\frac{1}{1+(4x)^2}\times 4\left(=\frac{4}{1+16x^2}\right)$	M1 $\left(\frac{k}{1+(4x)^2}\; k = 1,4\right)$
			(Allow M1 for $\frac{4}{1+4x^2}$)	A1 ($k = 4$)
		(ii)	$\frac{1}{1+x^2}\times 2x = \frac{2x}{1+x^2}$	M1 $\left(\frac{f(x)}{1+x^2},\; f(x)\neq 1\right)$ A1 ($f(x) = 2x$)
		(iii)	$3x^2\, e^{3x} + 2x\, e^{3x}$	M1 ($x^2 f(x) + e^{3x}\, g(x)$) A1 ($f(x) = ke^{3x}$, $g(x) = 2x$) A1 (all correct)
	(b)		$\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x}$	M1 $\left(\frac{\sin x f(x) - \cos x g(x)}{\sin^2 x}\right)$ A1 ($f(x) = -\sin x$, $g(x) = \cos x$)
		=	$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$	
		=	$\frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$	
		=	$-\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$	A1 (convincing)
				10
6.	*(a)*	$5\,\lvert x \rvert = 2$		B1
		$x = \pm\,\frac{2}{5}$		B1 (both) (F.T. $a\,\lvert x \rvert = b$)
	(b)	$7x - 5 \geq 3$		
		$x \geq \frac{8}{7}$		B1
		$7x - 5 \leq -3$		M1 ($7x - 5 \leq -3$)
		$x \leq \frac{2}{7}$		A1
				5
7.	*(a)*	(i)	$-\frac{7}{15(5x+2)^3}$ (+C) (o.e.)	M1 $\left(\frac{k}{(5x+2)^3}\right)$ A1 $\left(k = \frac{7}{15}\right)$
		(ii)	$\frac{1}{4}\, ln\;\lvert 8x+7 \rvert$ (+C) (o.e.)	M1 ($k \ln \lvert 8x+7 \rvert$) A1 ($k = \frac{1}{4}$ (o.e.))

	(b)	$\left[\frac{1}{3}\sin 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$	M1 ($k\sin 3x$, $k = \frac{1}{3}, -\frac{1}{3}, 3$)
			A1 ($k = \frac{1}{3}$)
		$= \frac{1}{3}\sin\left(\frac{\pi\times 3}{3}\right) - \frac{1}{3}\sin\left(\frac{\pi}{6}\times 3\right)$	M1 (use of limits, F.T. allowable k)
		$= -\frac{1}{3}$	A1 (C.A.O.)
			8
8.	*(a)*	$f'(x) = 1 + \frac{1}{x^2}$	B1
		$f'(x) > 0$ since ($1 > 0$ and) $\frac{1}{x^2} > 0$	B1
		Least value when $x = 1$ and is 0	B1
	(b)	Range of f is $[o, \infty)$	B1
	(c)	$3(x - \frac{1}{x})^2 + 2 = \frac{3}{x^2} + 8$	B1 (correct composition)
			M1 (writing equations and correct expansion of binomial)
		$3(x^2 - 2 + \frac{1}{x^2}) = \frac{3}{x^2} + 8$	
		$3x^2 = 12$	
		$x = \pm 2$ (accept 2)	A1 (C.A.O.)
		$x = 2$ since domain of f is $x \geq 1$	B1 (F.T. removal of –ve root)
			8
9.			$y = f(x)$ B1 (0,1) B1 (correct behaviour for large +ve, –ve x) $y = 2f(x) - 1$ B1 (0, 1) B1 (behaviour for –ve x, must approach –ve value of y) B1 (greater slope even only if in 1st quadrant)
			5

10. *(a)* Let $y = \sqrt{x+1}$ M1 (attempt to isolate x, $y^2 = x + 1$)

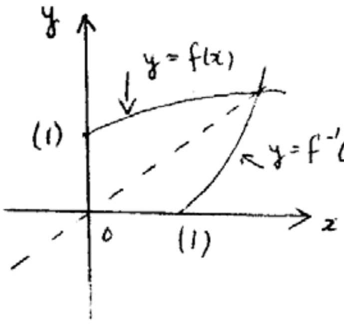
$y^2 = x + 1$

$x = y^2 - 1$ A1

$f^{-1}(x) = x^2 - 1$ A1 (F.T. one slip)

(b) domain $[1, \infty)$, range $[0, \infty)$ B1, B1

(c)



B1 (parabola $y = x^2 - 1$)

B1 (relevant part of parabola)

B1 ($y = f(x)$, F.T. graph of $y = f^{-1}(x)$)

8

MATHEMATICS C4

1.	*(a)* Let $\dfrac{2x^2+4}{(x-2)^2(x+4)} \equiv \dfrac{A}{x+4} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$	M1 (Correct form)
	$2x^2 + 4 \equiv A(x-2)^2 + B(x-2) + C(x+4)$	M1 (correct clearing of fractions and attempt to substitute)
	$\underline{x = 2}$ $\quad 12 = C(6), \quad C = 2$	
	$\underline{x = -4}$ $\quad 36 = A(36), \quad A = 1$	A1 (two constants, C.A.O.)
	$\underline{\text{Coefft of } x^2}$ $\quad 2 = A + B, \quad B = 1$	A1 (third constant, F.T. one slip)
	(b) $f'(x) = \dfrac{-1}{(x+4)^2} - \dfrac{1}{(x-2)^2}$	B1 (first two terms)
	$-\dfrac{4}{(x-2)^3}$	B1 (third term)
	$f'(0) = -\dfrac{1}{16} - \dfrac{1}{4} + \dfrac{4}{8} = \dfrac{3}{16}$ (o.e.)	B1 (C.A.O.)
		7
2.	$6x^2 + 6y^2 + 12xy\dfrac{dy}{dx} - 4y^2\dfrac{dy}{dx} = 0$	B1 ($4y^3\dfrac{dy}{dx}$)
		B1 ($6y^2 + 12xy\dfrac{dy}{dx}$)
	$\dfrac{dy}{dx} = -\dfrac{3}{2}$	B1 (C.A.O.)
	Gradient of normal = $\dfrac{3}{2}$	M1 (F.T. candidate's $\dfrac{dy}{dx}$)
	Equation is $y - 1 = \dfrac{2}{3}(x-2)$	A1 (F.T. candidate's gradient of normal)
		5

3. $2 + 3\,(2\cos^2\theta - 1) = \cos\theta$ — M1 (correct substitution for cos 2θ)

$6\cos^2\theta - \cos\theta - 1 = 0$ — M1 (correct method of solution, $(a\cos\theta + b)(\cos\theta + d)$ with ac = coefft of $\cos^2\theta$, bd = constant term, or correct formula)

$(3\cos\theta + 1)(2\cos\theta - 1) = 0 \quad \cos\theta = -\frac{1}{3}, \frac{1}{2}$ — A1

$\theta = 109{\cdot}5^\circ, 250{\cdot}5^\circ, 60^\circ, 300^\circ$ — B1 ($109{\cdot}5^\circ$)
B1 ($250{\cdot}5^\circ$)
B1 (60°, 300°)

6

Full F.T. for cos θ = +, –
2 marks for cos θ = –, –
1 mark for cos θ = +, +
Subtract 1 mark for each additional value in range, for each branch.

4. *(a)* $R\cos\alpha = 4,\ R\sin\alpha = 3$

$R = 5,\ \alpha = \tan^{-1}\left(\frac{3}{4}\right) = 36{\cdot}87^\circ$ — M1 (reasonable approach)

(or $36{\cdot}9^\circ$, $37{\cdot}0^\circ$) — A1 (α)
B1 ($R = 5$)

(b) Write as $\dfrac{1}{5\sin(x^\circ + 36\cdot 87^\circ) + 7}$ — M1 (attempt to use $\sin(x + \alpha) = \pm 1$)

Greatest value = $\dfrac{1}{-5+7} = \dfrac{1}{2}$ — A1 (F.T. one slip)

5

5. Volume $= \pi\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ — B1

$= (\pi)\displaystyle\int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2}dx$ — M1 ($a + b\cos 2x$)

A1 ($a = \frac{1}{2}$, $b = \frac{1}{2}$)

$= (\pi)\left[\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right]$ — A1 $\left(\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right)$ (F.T. a, b))

$= \dfrac{\pi^2}{4}$ (≈ 2·467, accept 2·47, 3 sig. figures) — A1 (C.A.O.)

5

6.	*(a)*	$\dfrac{dy}{dx} = \dfrac{2t}{-\dfrac{1}{t^2}} - 2t^3$	M1 $\left(\dfrac{\dot{y}}{\dot{x}}\right)$
			A1 (simplified)
		Equation of tangent is	
		$y - p^2 = -2p^3\left(x - \dfrac{1}{p}\right)$	M1 ($y - y_1 = m(x - x_1)$) o.e.
		$y - p^2 = -2p^3x + 2p^2$	
		$2p^3x + y - 3p^2 = 0$	A1 (convincing)
	(b)	$y = 0,\ x = \dfrac{3}{2p}$ (o.e.)	B1
		$x = 0,\ y = 3p^2$	B1
		$PA^2 = \left(\dfrac{3}{2p} - \dfrac{1}{p}\right)^2 + \left(p^2 - 0\right)^2 = \dfrac{1}{4p^2} + p^4$ (o.e.)	M1 (correct use of distance formula in context)
			A1 (one correct simplified distance C.A.O.)
		$PB^2 = 4PA^2,\ PB = 2PA$	A1 (convincing)
			9
7.	*(a)*	$\displaystyle\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\frac{1}{x}\,dx$	M1 $\left(f(x)\ln x - \displaystyle\int g(x).\frac{1}{x}dx\right)$
			A1($f(x)$=$g(x)$) A1 ($f(x) = g(x) = \dfrac{x^2}{2}$)
		$= \dfrac{x^2}{2}\ln x - \displaystyle\int \frac{x}{2}dx$	A1
		$= \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4}(+C)$	A1 (C.A.O.)
	(b)	$u = 2\sin x + 3,\ du = 2\cos x dx$ (limits are 3, 4)	
		$\displaystyle\int_3^4 \frac{1}{2}\frac{du}{u^2}$	M1 $\left(\displaystyle\int \frac{a}{u^2}du \text{ with } a = \pm\frac{1}{2}, 1, 2\right)$
			A1 ($a = \dfrac{1}{2}$)
		$= \left[-\dfrac{1}{2u}\right]_3^4$	A1 ($-\dfrac{a}{u}$, allowable a)
		$= \dfrac{1}{24}$ (o.e.)	A1 (F.T. allowable a <u>or</u> one slip)
			9

8.	*(a)*	$\frac{dx}{dt} = -k\sqrt{x}$	B1
	(b)	$\int \frac{dx}{\sqrt{x}} = \int -k\,dt$	M1 (attempt to separate variables, allow similar work)
		$2x^{\frac{1}{2}} = -kt + C$	A1 (unsimplified version, allow absence of C)
		$t = 0,\ x = 9, \qquad 2\sqrt{9} = C$	M1 (correct attempt to find C)
		$C = 6$	
		$kt = 6 - 2\sqrt{x}$	A1 (convincing)
	(c)	$t = 20,\ x = 4$ gives $k = \frac{1}{10}$ (o.e.)	M1 (attempt to find k)
			A1
		Tank is empty when $6 = \frac{1}{10}t$, $t = 60$ (mins.)	A1
			8
9.	*(a)*	**OP** = **OA** + λ **AB**	M1 (correct formula for r.h.s. and method for finding **AB**)
		= **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**)	B1 (**AB**)
		r = **i** + 3**j** + **k** + λ (**i** + 5**j** – 3**k**)	A1 (must contain **r**, F.T. **AB**)
	(b)	(Point of intersection is on both lines) Equate coefficients of **i** and **j**	
		$1 + \lambda = 2 + \mu$	M1 (attempt to write equations using candidate's equations, one correct)
		$3 + 5\lambda = -1 + 2\mu$	A1 (two correct, using candidate's equations)
		$\lambda = -2,\ \mu = -3$	M1 (attempt to solve equations) A1 (C.A.O.)
		(Consider coefficients in **k**)	
		$p - \mu = 1 - 3\lambda$	M1 (use of equation in **k** to find p)
		$p = \mu + 1 - 3\lambda = 4$	A1 (F.T. candidate's λ, μ)
	(c)	**b** . **c** = (2**i** + 8**j** – 2**k**).(3**i** – **j** – **k**)	M1 (correct method)
		= 6 – 8 + 2 = 0	A1 (correct)
		b and **c** are $\perp^r$ vectors	A1 (C.A.O.)
			12

10. $\left(1+\frac{x}{8}\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(\frac{x}{8}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1.2}\left(\frac{x}{8}\right)^2+...$

$= 1+\frac{x}{16}-\frac{x^2}{512}+\ldots$ B1 $\left(1+\frac{x}{16}\right)$

B1 $\left(-\frac{x^2}{512}\right)$

Expansion is valid for $|x| < 8$ B1 (only for conditions on $|x|$)

$\left(1+\frac{1}{8}\right)^{\frac{1}{2}} = 1+\frac{1}{6}-\frac{1}{512}$

$\frac{3}{2\sqrt{2}} = \frac{543}{512}$ (o.e.) B1 (expression must involve $\sqrt{2}$)

$\sqrt{2} = \frac{3}{2}\times\frac{512}{543} = \frac{256}{181}$ B1 (convincing)

5

11. $4k^2 = 2b^2$ B1

$b^2 = 2k^2$
b^2 has a factor 2 } one or other of these statements B1

$\therefore b$ has a factor 2 B1 (if and only if previous B gained)

a and *b* have a common factor
Contradiction B1 (depends upon previous B being gained)

($\therefore$ $\sqrt{2}$ is irrational)

4

MATHEMATICS FP1

1. Mod = $\sqrt{3+1} = 2$ B1

Arg = $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ (30°) B1

$\sqrt{3} + \mathrm{i} = 2(\cos\left(\frac{\pi}{6}\right) + \mathrm{i}\sin\left(\frac{\pi}{6}\right))$ B1

Arg of $(\sqrt{3} + \mathrm{i})^n = \frac{n\pi}{6}$ M1A1

Power is real when

$\frac{n\pi}{6}$ is an integer multiple of π A1

Smallest $n = 6$. A1

2. Det = 1.2 + 2λ + λ(3λ - 4) M1m1A1

$= 3\lambda^2 - 2\lambda + 2$ A1

EITHER $= 3\left((\lambda - 1/3)^2 + 5/9\right)$ M1

> 0 for all λ A1

OR $'b^2 - 4ac' = -20$ M1

So determinant not equal to zero for any real λ. A1

3. $f(x+h) - f(x) = \frac{1}{1-(x+h)^2} - \frac{1}{1-x^2}$ M1A1

$= \frac{1 - x^2 - [1-(x+h)^2]}{[1-(x+h)^2](1-x^2)}$ m1

$= \frac{h(2x+h)}{[1-(x+h)^2](1-x^2)}$ A1

$f'(x) = \lim_{h \to 0} \frac{(2x+h)}{[1-(x+h)^2](1-x^2)}$ M1

$= \frac{2x}{(1-x^2)^2}$ A1

4. $\frac{11+7i}{1+i} = \frac{(11+7i)(1-i)}{(1+i)(1-i)}$ M1A1

$= \frac{18-4i}{2} = 9 - 2i$ A1

$2(x + \mathrm{i}y) + (x - \mathrm{i}y) = 9 - 2\mathrm{i}$ M1

Equating real and imaginary parts m1

$x = 3$ and $y = -2$ A1

5. (a) $\mathbf{T_1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T_2} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ M1

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ A2

Note: Multiplying the wrong way gives

$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ Award M1A1

(b) Fixed points satisfy

$\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ M1

giving $-y-2 = x$ and $x+1 = y$ A1

Fixed point is (–3/2, –1/2). M1A1

FT from wrong answer to (a) – the incorrect matrix above leads to (–1/2,3/2).

6. (a) Assume the proposition is true for $n = k$, that is

$\sum_{r=1}^{k}(2r+1) = (k+1)^2$ B1

Consider

$\sum_{r=1}^{k+1}(2r+1) = (k+1)^2 + 2k + 3$ M1A1

$= (k+2)^2$ A1

So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. A1

(b) Since $3 \neq 4$, P is false for $n = 1$ is therefore false. B2

7. *(a)* Using reduction to echelon form,

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & -1 & \lambda-2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu-4 \end{bmatrix}$$ M1A1A1

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & \lambda-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ \mu-10 \end{bmatrix}$$ A1

The solution will not be unique if $\lambda = 3$. A1

(b) The equations are consistent if $\mu = 10$. B1

Put $z = \alpha$ M1
$y = \alpha - 6$ A1
$x = 16 - 4\alpha$ M1A1

8. *(a)* Let the roots be $\alpha - 1, \alpha, \alpha + 1$. M1

Then $\alpha(\alpha - 1) + \alpha(\alpha + 1) + (\alpha - 1)(\alpha + 1) = 47$ m1
$\alpha^2 = 16$ A1
$\alpha = -4$ A1
The roots are -3, -4 and -5. A1

(b) $p = 12$, $q = 60$ B1B1

9. *(a)* $\ln f(x) = \frac{1}{x}\ln(x)$ M1

$$\frac{f'(x)}{f(x)} = -\frac{1}{x^2}\ln(x) + \frac{1}{x^2}$$ m1A1

$$= \frac{1}{x^2}\left(1 - \ln(x)\right)$$ A1

At the stationary point,
$\ln(x) = 1$ M1

so $x = e$ (2.72) and $y = e^{\frac{1}{e}}$ (1·44) A1A1

(b) We now need to determine its nature.
We see from above that
For $x < e$, $f'(x) > 0$ and for $x > e$, $f'(x) < 0$ M1
Showing it to be a maximum. A1

10. $w = \dfrac{z+3}{z+1}$

$wz + w = z + 3$ M1

$z(w-1) = 3 - w$ A1

$z = \dfrac{3-w}{w-1}$ A1

Since $|z| = 1$, it follows that

$|3 - w| = |w - 1|$ M1

$\sqrt{(3-u)^2 + v^2} = \sqrt{(u-1)^2 + v^2}$ A1

$9 - 6u + u^2 + v^2 = u^2 - 2u + 1 + v^2$ A1

leading to $u = 2$. A1

Straight line parallel to the v-axis (passing through (2,0)). B1

ALTERNATIVE METHOD

$u + \mathrm{i}v = \dfrac{x+3+\mathrm{i}y}{x+1+\mathrm{i}y}.\dfrac{x+1-\mathrm{i}y}{x+1-\mathrm{i}y}$ M1

$= \dfrac{(x+1)(x+3) + y^2 + \mathrm{i}y(x+1-x-3)}{(x+1)^2 + y^2}$ A1

Equating real and imaginary parts, M1

$u = \dfrac{x^2 + y^2 + 4x + 3}{x^2 + y^2 + 2x + 1}$ A1

$v = \dfrac{-2y}{x^2 + y^2 + 2x + 1}$ A1

Putting $x^2 + y^2 = 1$, M1

$u = 2$ A1

which is a straight line parallel to the v-axis (passing through (2,0)).

B1

MATHEMATICS FP2

1. (i) f is continuous because $f(x)$ passes through zero from both sides around $x = 0$. B1B1

(ii) For $x \geq 0$, $f'(x) = \cos x = 1$ when $x = 0$ and for $x < 0$, $f'(x) = 1$. M1A1

So f' is continuous. A1

2. Converting to trigonometric form,

$i = \cos(\pi/2) + i\sin(\pi/2)$ B2

Cube roots $= \cos(\pi/6 + 2n\pi/3) + i\sin(\pi/6 + 2n\pi/3)$ (si) M1A1

$n = 0$ gives $\cos(\pi/6) + i\sin(\pi/6) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ M1A1

$n = 1$ gives $\cos(5\pi/6) + i\sin(5\pi/6)$ M1

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$ A1

$n = 2$ gives $\cos(3\pi/2) + i\sin(3\pi/2) = -i$ A1

[FT on candidate's first line but award a max of 4 marks for the cube roots of 1]

3. *(a)* $f'(x) = -\frac{1}{x^2(1+x^2)^2} \times (3x^2 + 1)$ M1A1

< 0 for all $x > 0$ (cso) A1

(b) f is odd because $f(-x) = -f(x)$ B2

(c) The asymptotes are $x = 0$ and $y = 0$. B1B1

(d) Graph G2

4. *(a)* Completing the square,

$2\{(x-1)^2-1\}-(y+2)^2+4=4$ M1A1

$\dfrac{(x-1)^2}{1}-\dfrac{(y+2)^2}{2}=1$ A1

The centre is (1,–2) A1

(b) In the usual notation, $a = 1$, $b = \sqrt{2}$. M1

$2 = 1(e^2-1)$ A1

$e=\sqrt{3}$ A1

Foci are $(1 \pm \sqrt{3}, -2)$, Directrices are $x = 1 \pm \dfrac{1}{\sqrt{3}}$ (FT) B1B1

5. Putting $t = \tan(\theta/2)$ and substituting, M1

$\dfrac{3(1-t^2)}{1+t^2}+\dfrac{4.2t}{1+t^2}=3-t$ A1

$3-3t^2+8t=3+3t^2-t-t^3$ A1

$t^3-6t^2+9t=0$ A1

Either $t = 0$ B1

giving $\theta/2 = n\pi$ or $\theta = 2n\pi$ B1

Or $t = 3$ B1

giving $\theta/2 = 1.25 + n\pi$ B1

$\theta = 2.50 + 2n\pi$ (Accept degrees) B1

6. *(a)* Putting $n = 1$,

LHS = $\cos\theta + \mathrm{i}\sin\theta$ = RHS so true for $n = 1$. B1

Let the result be true for $n = k$, ie

$(\cos\theta+\mathrm{i}\sin\theta)^k=\cos k\theta+\mathrm{i}\sin k\theta$ M1

Consider

$(\cos\theta+\mathrm{i}\sin\theta)^{k+1}=(\cos k\theta+\mathrm{i}\sin k\theta)(\cos\theta+\mathrm{i}\sin\theta)$ M1

$=\cos k\theta\cos\theta-\sin k\theta\sin\theta+\mathrm{i}(\sin k\theta\cos\theta+\sin\theta\cos k\theta)$ A1

$=\cos(k+1)\theta+\mathrm{i}\sin(k+1)\theta$ A1

Thus, true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$,
proof by induction follows. A1

(b) $\cos 5\theta+\mathrm{i}\sin 5\theta=(\cos\theta+\mathrm{i}\sin\theta)^5$ M1

$=\cos^5\theta+5\cos^4\theta(\mathrm{i}\sin\theta)+10\cos^3\theta(\mathrm{i}\sin\theta)^2$
$+10\cos^2\theta(\mathrm{i}\sin\theta)^3+5\cos\theta(\mathrm{i}\sin\theta)^4+(\mathrm{i}\sin\theta)^5$ A1

$=\cos^5\theta+5\mathrm{i}\cos^4\theta\sin\theta-10\cos^3\theta\sin^2\theta$
$-10\mathrm{i}\cos^2\theta\sin^3\theta+5\cos\theta\sin^4\theta+\mathrm{i}\sin^5\theta$ A1

Equating imaginary parts,

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta \qquad \text{M1}$$
$$= 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta \qquad \text{A1}$$
$$= 5\sin\theta - 10\sin^3\theta + 5\sin^5\theta - 10\sin^3\theta + 10\sin^5\theta + \sin^5\theta \qquad \text{A1}$$
$$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \qquad \text{A1}$$

7. *(a)* Let $\dfrac{x}{(x+2)(x^2+4)} \equiv \dfrac{A}{x+2} + \dfrac{Bx+C}{(x^2+4)}$ M1

$x \equiv A(x^2+4) + (x+2)(Bx+C)$

$x = -2$ gives $A = -1/4$ A1

Coeff of x^2 gives $B = 1/4$ A1

Constant term gives $C = 1/2$ A1

(b) Integral $= \displaystyle\int_2^3 \left(-\frac{1}{4(x+2)} + \frac{x}{4(x^2+4)} + \frac{1}{2(x^2+4)}\right)\mathrm{d}x$ M1

$= \left[-\dfrac{1}{4}\ln(x+2) + \dfrac{1}{8}\ln(x^2+4) + \dfrac{1}{4}\arctan\left(\dfrac{x}{2}\right)\right]_2^3$ A1A1A1

$= -\dfrac{1}{4}\ln 5 + \dfrac{1}{8}\ln 13 + \dfrac{1}{4}\arctan\left(\dfrac{3}{2}\right) + \dfrac{1}{4}\ln 4 - \dfrac{1}{8}\ln 8 - \dfrac{1}{4}\arctan 1$ A1

$= 0.054$ cao A1

8. *(a)* The line meets the circle where

$x^2 + m^2(x-2)^2 = 1$ M1

$(1+m^2)x^2 - 4m^2x + 4m^2 - 1 = 0$ A1

x coordinate of M $= \dfrac{\text{Sum of roots}}{2}$ M1

$= \dfrac{2m^2}{1+m^2}$ AG

Substitute in the equation of the line.

$y = m\left(\dfrac{2m^2}{1+m^2} - 2\right)$ M1A1

$= -\dfrac{2m}{1+m^2}$ AG

(b) Dividing,

$\dfrac{x}{y} = -m$ or $m = -\dfrac{x}{y}$ M1A1

Substituting,

$y = \dfrac{2x/y}{1+x^2/y^2}$ M1A1

$= \dfrac{2xy}{x^2+y^2}$ A1

whence $x^2 + y^2 - 2x = 0$ A1

[Accept alternative forms, e.g.

$y = \sqrt{x(2-x)}$]

MATHEMATICS FP3

1.	*(a)*	$2\sinh^2 x + 1 = 2\left(\dfrac{e^x - e^{-x}}{2}\right)^2 + 1$	M1
		$= \dfrac{(e^{2x} - 2 + e^{-2x} + 2)}{2}$	A1
		$= \dfrac{(e^{2x} + e^{-2x})}{2} = \cosh 2x$	A1
	(b)	Substitute to obtain the quadratic equation	M1
		$2\sinh^2 x - 3\sinh x + 1 = 0$	A1
		$\sinh x = 1, 1/2$	M1A1
		$x = 0{\cdot}881,\ 0{\cdot}481$ cao	A1A1
2.		$t = \tan\left(\dfrac{x}{2}\right) \Rightarrow dx = \dfrac{2dt}{1+t^2}$	B1
		$(0,\pi/2) \to (0,1)$	B1
		$I = \displaystyle\int_0^1 \frac{2dt/(1+t^2)}{1+3(1-t^2)/(1+t^2)}$	M1A1
		$= \displaystyle\int_0^1 \frac{dt}{2-t^2}$	A1
		$= \dfrac{1}{2\sqrt{2}}\left[\ln\left(\dfrac{\sqrt{2}+t}{\sqrt{2}-t}\right)\right]_0^1$ or $\left[\dfrac{1}{\sqrt{2}}\tanh^{-1}\left(\dfrac{t}{\sqrt{2}}\right)\right]_0^1$	M1A1
		$= \dfrac{1}{2\sqrt{2}}\ln\left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ or $\dfrac{1}{\sqrt{2}}\tanh^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ (0.623) cao	A1
3.	*(a)*	$f(0) = 0$ si	B1
		$f'(x) = \dfrac{1}{\sec x} \times \sec x \tan x = \tan x,\quad f'(0) = 0$ si	B1B1
		$f''(x) = \sec^2 x,\ f''(0) = 1$ si	B1
		$f'''(x) = 2\sec^2 x \tan x,\ f'''(0) = 0$ si	B1B1
		$f''''(x) = 4\sec^2 x \tan^2 x + 2\sec^4 x,\ f''''(0) = 2$ si	B1B1

[This expression has several similar looking forms, eg $6\sec^2 x \tan^2 x + 2\sec^2 x$]

The series is

$$f(x) = \frac{x^2}{2} + \frac{x^4}{12} + \ldots \qquad \text{B1}$$

(b) Substituting the series gives

$$x^4 + 126x^2 - 12 = 0 \qquad \text{M1}$$

Solving,

$$x^2 = 0.0952, \qquad \text{M1A1}$$

$$x = 0.3085 \qquad \text{A1}$$

4. *(a)* $\frac{dx}{d\theta} = 1 + \cos\theta, \frac{dy}{d\theta} = -\sin\theta$ B1B1

$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 1 + 2\cos\theta + \cos^2\theta + \sin^2\theta$ M1A1

$= 2(1 + \cos$ A1

$= 4\cos^2\left(\frac{\theta}{2}\right)$ AG

(b) Arc length $= \int_0^\pi 2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1

$= 4\left[\sin\left(\frac{\theta}{2}\right)\right]_0^\pi$ A1

$= 4$ A1

(c) CSA $= 2\pi\int_0^\pi (1+\cos\theta).2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1

$= 2\pi\int_0^\pi 2\cos^2\left(\frac{\theta}{2}\right).2\cos\left(\frac{\theta}{2}\right)d\theta$ or $4\pi\int_0^\pi \cos\left(\frac{\theta}{2}\right)d\theta + 4\pi\int_0^\pi \cos\theta\cos\left(\frac{\theta}{2}\right)d\theta$ A1

$= 8\pi\int_0^\pi \cos^3\left(\frac{\theta}{2}\right)d\theta$ or $4\pi\int_0^\pi \cos\left(\frac{\theta}{2}\right) + 2\pi\int_0^\pi (\cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right))d\theta$ A1

$= 16\pi\int_0^\pi \left\{1 - \sin^2\left(\frac{\theta}{2}\right)\right\}d\sin\left(\frac{\theta}{2}\right)$ or $6\pi\int_0^\pi \cos\left(\frac{\theta}{2}\right)d\theta + 2\pi\int_0^\pi \cos\left(\frac{3\theta}{2}\right)d\theta$ M1A1

$= 16\pi\left[\sin\left(\frac{\theta}{2}\right) - \frac{1}{3}\sin^3\left(\frac{\theta}{2}\right)\right]_0^\pi$ or $12\pi\left[\sin\left(\frac{\theta}{2}\right)d\theta\right]_0^\pi + \frac{4\pi}{3}\left[\sin\left(\frac{3\theta}{2}\right)\right]_0^\pi$ A1

$= \frac{32}{3}\pi$ (33·5) A1

5. *(a)* $I_n = -\int_0^\pi \theta^n d\cos\theta$ M1

$= \left[-\theta^n\cos\theta\right]_0^\pi + \int_0^\pi \cos\theta.n\theta^{n-1}d\theta$ A1A1

$= \pi^n + n\int_0^\pi \theta^{n-1}\cos\theta d\theta$ A1

$= \pi^n + n\int_0^\pi \theta^{n-1}d\sin\theta$ M1

$= \pi^n + n\left[\theta^{n-1}\sin\theta\right]_0^\pi - n(n-1)\int_0^\pi \theta^{n-2}\sin\theta d\theta$ A1A1

$= \pi^n - n(n-1)I_{n-2}$ A1

(b) $I_4 = \pi^4 - 12I_2$ B1

$= \pi^4 - 12(\pi^2 - 2I_0)$ B1

$= \pi^4 - 12\pi^2 + 24\int_0^{\pi} \sin\theta \mathrm{d}\theta$ M1

$= \pi^4 - 12\pi^2 + 24[-\cos\theta]_0^{\pi}$ A1

$= \pi^4 - 12\pi^2 + 48$ A1

6. (a) Area $= \frac{1}{2}\int_0^{\pi/2} \sinh^2\theta \mathrm{d}\theta$ M1A1

$= \frac{1}{4}\int_0^{\pi/2} (\cosh 2\theta - 1)\mathrm{d}\theta$ A1

$= \frac{1}{4}\left[\frac{\sinh 2\theta}{2} - \theta\right]_0^{\pi/2}$ A1

$= \frac{1}{4}\left(\frac{\sinh\pi}{2} - \frac{\pi}{2}\right)$ (1·05) A1

(b) (i) Consider

$x = r\cos\theta = \sinh\theta\cos\theta$ M1A1

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \cosh\theta\cos\theta - \sinh\theta\sin\theta$ M1A1

At P,

$\cosh\theta\cos\theta = \sinh\theta\sin\theta$ M1

so $\tanh\theta = \cot\theta$ A1

(ii) The Newton-Raphson iteration is

$x \to x - \frac{(\tanh\theta - \cot\theta)}{(\mathrm{sech}^2\theta + \mathrm{cosec}^2\theta)}$ M1A1

Starting with $x_0 = 1$, we obtain M1

$x_1 = 0 \cdot 9348$ A1

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