C1 May 2012

1. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5\right) dx$$

giving each term in its simplest form.

(4)

$\int 6x^2 + 2x^{-2} + 5\, dx = \frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x + C = 2x^3 - 2x^{-1} + 5x + C$

2. (a) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

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a) $(\sqrt[5]{32})^3 = 2^3 = 8$

b) $\left(\frac{4}{25x^4}\right)^{\frac{1}{2}} = \frac{\sqrt{4}}{\sqrt{25x^4}} = \frac{2}{5x^2}$

3. Show that $\frac{2}{\sqrt{(12)} - \sqrt{(8)}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers.

(5)

$\frac{2(\sqrt{12} + \sqrt{8})}{(\sqrt{12} - \sqrt{8})(\sqrt{12} + \sqrt{8})} = \frac{2 \times 2\sqrt{3} + 2 \times 2\sqrt{2}}{12 - 8}$

$= \frac{4\sqrt{3}}{4} + \frac{4\sqrt{2}}{4} = \sqrt{3} + \sqrt{2}$

4. $$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{dy}{dx}$ giving each term in its simplest form. **(4)**

a) $y' = 15x^2 - 8x^{\frac{1}{3}} + 2$

(b) Find $\frac{d^2y}{dx^2}$ **(2)**

b) $y'' = 30x - \frac{8}{3}x^{-\frac{2}{3}}$

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5. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2 **(1)**

(b) Show that $a_3 = 12 - 3c$ **(2)**

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Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c. **(4)**

a) $a_1 = 3$

$a_2 = 2(3) - c = 6 - c$

b) $a_3 = 2(6-c) - c = 12 - 3c$ #

c) $a_4 = 2(12-3c) - c = 24 - 7c$

$\sum_1^4 a_i = 3 + (6-c) + (12-3c) + (24-7c) = 45 - 11c$

$\therefore 45 - 11c \geqslant 23 \Rightarrow 11c \leqslant 22 \Rightarrow c \leqslant 2$

6. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

(2)

(b) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m.

(1)

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a) $a=10 \quad d=5 \qquad U_{15}=a+14d = 10+14(5) = 80p$

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b) $U_{60} = a+59d = 10+59(5) = 305$

$S_n = \frac{1}{2}n(a+L) \qquad S_{60} = 30(10+305) = 30 \times 315$

$= 9450p = £94.50$

c) $a=10 \quad d=10$

$S_m = \frac{1}{2}m(2a+(m-1)d) = \frac{1}{2}m(20+(m-1)10)$

$= \frac{1}{2}m(10m+10) = 5m(m+1)$

$5m(m+1) = 6300 \Rightarrow m(m+1) = 1260 = 35 \times 36$ #

d) $m^2+m-(35 \times 36)=0 \qquad (m+36)(m-35)=0$

$\cancel{m=-36} \qquad m=35$

7. The point P (4, –1) lies on the curve C with equation $y = \mathrm{f}(x)$, $x > 0$, and

$$\mathrm{f}'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

(a) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

(b) Find f(x).

(4)

b) $f'(x) = \frac{1}{2}x - 6x^{-\frac{1}{2}} + 3$

$f(x) = \dfrac{\frac{1}{2}x^2}{2} - \dfrac{6x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$

$f(x) = \frac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + c$

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$(4,-1) \Rightarrow \quad -1 = \frac{1}{4}(4)^2 - 12(4)^{\frac{1}{2}} + 3(4) + c$

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$-1 = 4 - 24 + 12 + c \Rightarrow c = 6$

$\therefore f(x) = \frac{1}{4}x^2 - 12\sqrt{x} + 3x + 6$.

a) $f'(4) = m_t = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3$

$m_t = 2 - 3 + 3 = 2$

$y - y_1 = m(x - x_1) \qquad y + 1 = 2(x - 4)$

$\Rightarrow y + 1 = 2x - 8$

$\Rightarrow \underline{y = 2x - 9}$

8.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q. **(3)**

(b) Calculate the discriminant of $4x - 5 - x^2$ **(2)**

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes. **(3)**

a) $x^2 - 4x + 5 = (x-2)^2 - 4 + 5$

$= (x-2)^2 + 1$

$\therefore 4x - 5 - x^2 = -1 - (x-2)^2$ $\quad p = -2$, $q = -1$

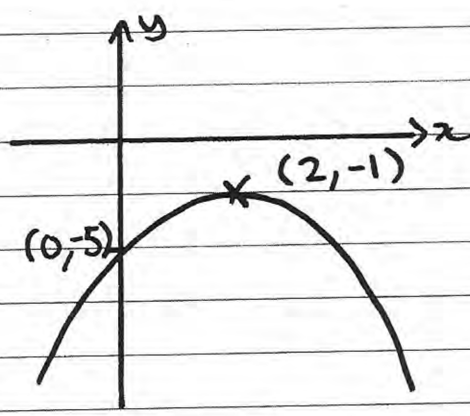
b) $b^2 - 4ac = (4)^2 - 4(-1)(-5)$

$= 16 - 20$

$= -4$

c) $y = -(x-2)^2 - 1$ $\quad \therefore \bigcap$ TP$(2,-1)$

crosses y at -5



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9. The line L_1 has equation $4y + 3 = 2x$

The point A $(p, 4)$ lies on L_1

(a) Find the value of the constant p. **(1)**

The line L_2 passes through the point C $(2, 4)$ and is perpendicular to L_1

(b) Find an equation for L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

The line L_1 and the line L_2 intersect at the point D.

(c) Find the coordinates of the point D. **(3)**

(d) Show that the length of CD is $\frac{3}{2}\sqrt{5}$ **(3)**

A point B lies on L_1 and the length of $AB = \sqrt{(80)}$

The point E lies on L_2 such that the length of the line $CDE = 3$ times the length of CD.

(e) Find the area of the quadrilateral $ACBE$. **(3)**

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a) $y=4 \Rightarrow 4(4)+3=2x \Rightarrow 19=2x$

$\underline{x=9.5}$

b) $4y=2x-3$

$y=\frac{1}{2}x-\frac{3}{4} \Rightarrow m_{L1}=\frac{1}{2} \Rightarrow m_{L2}=-2$

$(2,4)$

$y-y_1=m(x-x_1) \Rightarrow y-4=-2(x-2)$

$y-4=-2x+4$

$\therefore\ 2x+y-8=0$

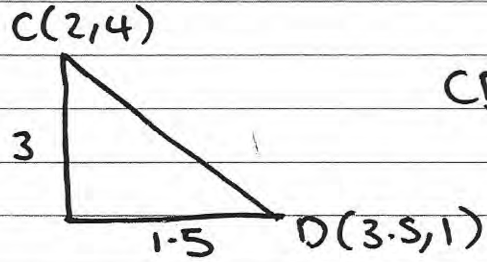
c) $2x = 4y + 3$ $\qquad 2x = 8 - y$

$\therefore \quad 4y + 3 = 8 - y \Rightarrow 5y = 5 \Rightarrow y = 1$

$2x = 4(1) + 3 \Rightarrow 2x = 7 \Rightarrow x = 3.5$

$D(3.5, 1)$

d)



$CD^2 = 3^2 + \left(\frac{3}{2}\right)^2 = 9 + \frac{9}{4}$

$CD^2 = \frac{36+9}{4} = \frac{45}{4}$

$\therefore \quad CD = \frac{\sqrt{45}}{2} = \frac{\sqrt{9}\sqrt{5}}{2}$

$= \frac{3}{2}\sqrt{5}$ #

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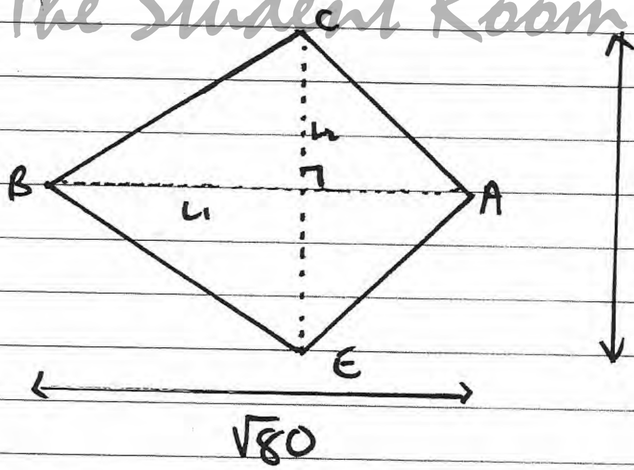
e)



$3 \times \frac{3}{2}\sqrt{5}$

$= \frac{9}{2}\sqrt{5}$

$\text{Area} = \dfrac{\sqrt{80} \times \frac{9}{2}\sqrt{5}}{2} = \dfrac{\sqrt{16}\sqrt{5} \times \frac{9}{2}\sqrt{5}}{2}$

$\text{Area} = \dfrac{4 \times 5 \times \frac{9}{2}}{2} = \underline{45}$

10.

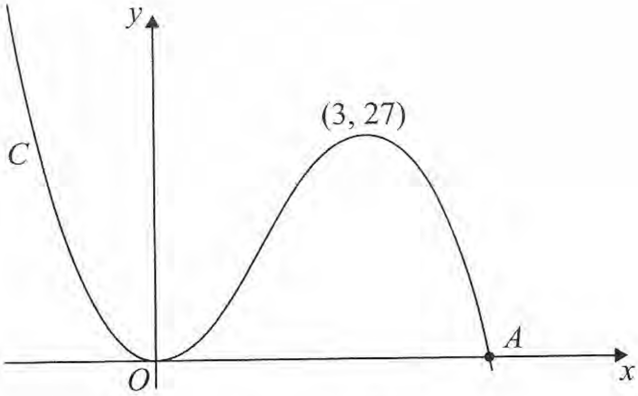


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A. $9 - 2x = 0 \Rightarrow x = 4.5$

(1)

$A(4.5, 0)$

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(b) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x + 3)$

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(ii) $y = \mathrm{f}(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

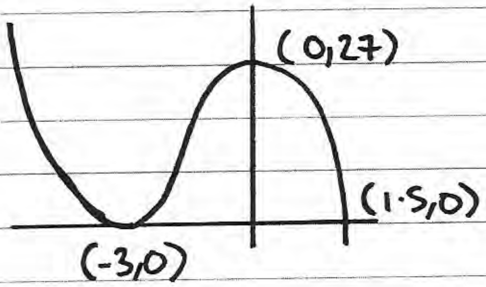
(6)

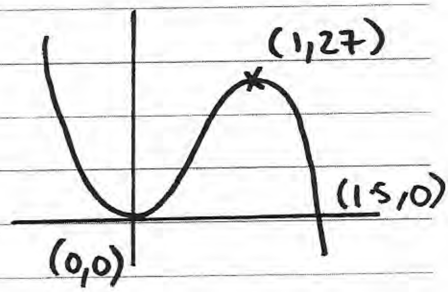
The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

a) $f(x+3) \xleftarrow{3}$ $x-3$ (ii) $f(3x) \rightarrow 3 \leftarrow$ $\div x$ by 3





c) $f(x)+k$ TP(3,10) ↓17 down $k = -17$