

Friday 20 January 2012 – Afternoon

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The complex number $a + 5i$, where a is positive, is denoted by z . Given that $|z| = 13$, find the value of a and hence find $\arg z$. [4]
- 2 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and $\mathbf{I}$ is the 2×2 identity matrix. Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants p and q . [5]
- 3 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})i$. Give your answers in the form $x + iy$, where x and y are exact real numbers. [6]
- 4 Find $\sum_{r=1}^n r(r^2 - 3)$, expressing your answer in a fully factorised form. [6]
- 5 (a) Find the matrix that represents a reflection in the line $y = -x$. [2]
- (b) The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.
- (i) Describe fully the geometrical transformation represented by $\mathbf{C}$. [2]
- (ii) State the value of the determinant of $\mathbf{C}$ and describe briefly how this value relates to the transformation represented by $\mathbf{C}$. [2]
- 6 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - i| = 2$ and $\arg z = \frac{1}{6}\pi$. [6]
- 7 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.
- (i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. [3]
- (ii) Hence suggest a suitable form for the matrix $\mathbf{M}^n$, where n is a positive integer. [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [4]
- 8 (i) Show that $\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$. [2]
- (ii) Hence find an expression, in terms of n , for
- $$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$
- [4]
- (iii) Hence find $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [2]

9 The matrix $\mathbf{X}$ is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

(i) Find the determinant of $\mathbf{X}$ in terms of a . [3]

(ii) Hence find the values of a for which $\mathbf{X}$ is singular. [3]

(iii) Given that $\mathbf{X}$ is non-singular, find $\mathbf{X}^{-1}$ in terms of a . [4]

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α , β and γ .

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2 , β^2 and γ^2 .

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b . [9]

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE



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