MECHANICS UNIT

KINEMATIC RELATIONSHIPS AND RELATIVISTIC MOTION

Calculus notation $v = \dfrac{ds}{dt}$; $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

derive $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$

Rest Mass (m_o) The mass of an object which is at rest relative to an observer. (The mass of an object increases with its velocity).

Relativistic Mass (m) The mass of an object which is travelling at a velocity comparable to the velocity of light.

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$ [*equation will be given*]

Relativistic Energy $E = mc^2$

ANGULAR MOTION

Angular Displacement (θ) measured in radians. (2π radians = $360°$)

Angular Velocity (ω) $\omega = \dfrac{d\theta}{dt}$ (rad s^{-1})

Angular Acceleration (α) $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$ (rad s^{-2})

Equations of Motion

CIRCULAR MOTION [**no** derivations required]	***LINEAR MOTION*** [derivations required]
$\omega = \omega_o + \alpha t$	$v = u + at$
$\theta = \omega_o t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at^2$
$\omega^2 = \omega_o^2 + 2\alpha\theta$	$v^2 = u^2 + 2as$

$v = r\omega$ [*derivation required*]

$a = r\alpha$ [***no*** *derivation required*]

Central Force The force required to maintain a particle in circular motion.

Central acceleration $a = \dfrac{v^2}{r}$ and $a = r\omega^2$

[*derivation required*]

Central Force equations $F = \dfrac{mv^2}{r}$ and $F = m\omega^2 r$

ROTATIONAL DYNAMICS

Moment of a Force The magnitude of the moment of a force (or the turning effect) is force x perpendicular distance

Torque (T) $T = F \times r$ where r is the perpendicular distance from the force to the axis of rotation

Moment of Inertia (I) The moment of inertia depends on the mass and the distribution of the mass about a fixed axis.

$I = m r^2$ mass m at distance r from axis of rotation

[$I = \Sigma m r^2$ (Σ is the 'sum of') *equation not required*]

Torque (T) $T = I \alpha$

Angular Momentum(L) $L = I \omega$ (for a rigid body)

$L = mr^2\omega = mrv$ (for a particle)

Rotational Kinetic Energy $E_{rot} = \frac{1}{2} I \omega^2$ (for a rigid body)

GRAVITATION

Law of Gravitation $F = \dfrac{G m_1 m_2}{r^2}$

Gravitational Potential $V = -\dfrac{Gm}{r}$ (zero of V is at infinity)

Conservative Field The gravitational field is an example of a conservative field where the total work done moving a mass around any closed path is zero.

Equipotentials Lines joining points of equal gravitational potential.

Escape Velocity The velocity a projectile must have in order to escape from a planet's gravitational field.

$v_{esc} = \sqrt{\dfrac{2G M}{r}}$ [*derivation required*]

Black Hole A body with a sufficiently high density to make the escape velocity greater than c, the speed of light.

SIMPLE HARMONIC MOTION

SHM	The unbalanced force, or acceleration, is proportional to the displacement of the object and acts in the opposite direction.
SHM Equation	$\frac{d^2y}{dt^2} = -\omega^2 y$ and $\omega = \frac{2\pi}{T}$
SHM Solutions	$y = a \sin \omega t$ if $y = 0$ at $t = 0$ $y = a \cos \omega t$ if $y = a$ at $t = 0$
Velocity $(\frac{dy}{dt})$	$v = \pm \omega\sqrt{a^2 - y^2}$ a = amplitude of motion. $v_{max} = \pm \omega a$ and occurs at the centre of the motion, $v_{min} = 0$ at extremes.
Acceleration $(\frac{d^2y}{dt^2})$	$acc = -\omega^2 y$ $acc_{max} = -\omega^2 a$ and occurs at $y = a$. $acc_{min} = 0$ at centre.
Energy	$E_k = \frac{1}{2} m \omega^2 (a^2 - y^2)$ [*derivation required*] $E_p = \frac{1}{2} m \omega^2 y^2$ [*derivation required*] $E_{tot} = E_k + E_p = \frac{1}{2} m \omega^2 a^2$
Damping	Damping causes the amplitude of the oscillation to decay.

WAVE PARTICLE DUALITY

Particles as Waves	Particles such as electrons can exhibit wave properties, such as diffraction.
de Broglie Wavelength	$\lambda = \frac{h}{p}$ (h is the Planck constant and p is momentum)
The Bohr Model of the Atom	The electrons occupy only certain allowed orbits. Angular momentum is quantised. Radiation is emitted when electrons move from higher energy levels to lower energy levels.
Quantisation of Angular Momentum	$mvr = \frac{nh}{2\pi}$
Quantum Mechanics and Probability	Quantum mechanics provides methods to determine probabilities.

ELECTRICAL PHENOMENA UNIT

ELECTRIC FIELDS

Coulomb's Inverse Square Law

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2} \quad \text{or} \quad \left[\frac{1}{4\pi\varepsilon_o}\right]\cdot\frac{Q_1 Q_2}{r^2}$$

(ε_o is the permittivity of free space)

Electric Field Strength (E)

Force on one coulomb of positive charge at that point.

$$E = \frac{F}{Q}$$

Electric Field Strength for a uniform electric field

$$E = \frac{V}{d}$$ [*derivation required*]

Electric Field Strength for a point charge

$$E = \frac{Q}{4\pi\varepsilon_o r^2} \quad \text{or} \quad \left[\frac{1}{4\pi\varepsilon_o}\right]\cdot\frac{Q}{r^2}$$

[*no derivation required*]

Charging by Induction

Conducting objects can be charged by separating the positive and negative charges on the objects and then removing one set of charges by earthing.

Conducting Shapes

When a conducting shape is in an electric field the induced charge stays on its surface and the electric field inside the conducting shape is zero.

Electrostatic Potential

Work done by an external force to bring one coulomb of positive charge from infinity to that point.

$$V = \frac{Q}{4\pi\varepsilon_o r} \quad \text{or} \quad \left[\frac{1}{4\pi\varepsilon_o}\right]\cdot\frac{Q}{r}$$

[*no derivation required*]

Charged Particles in uniform electric fields - non relativistic

$\frac{1}{2}mv^2 = QV$ (kinetic energy to electrical energy)

Charged Particles in uniform electric fields - relativistic case

Relativistic effects must be considered when the velocity of the charged particle is more than 10% of the velocity of light.

[*no relativistic calculations required*]

Particle head-on collisions

Change in E_k = change in E_p

$$\frac{1}{2}mv^2 = \frac{qQ}{4\pi\varepsilon_o}\cdot\frac{1}{r}$$

where r is closest distance of approach

Millikan's Experiment

Quantisation of charge.

$E q = mg$ (neglecting upthrust)

ELECTROMAGNETISM

Tesla	The tesla is the magnetic induction of a magnetic field in which a conductor of length one metre, carrying a current of one ampere perpendicular to the field is acted on by a force of one newton
Magnetic Induction (B)	$F = IlB \sin\theta$ (θ is the angle between B and l) The direction of F is perpendicular to the plane containing B and I.
The Magnetic Induction around an 'infinite', straight conductor	$B = \frac{\mu_o I}{2\pi r}$ (μ_o is the permeability of free space) (r is the perpendicular distance from conductor)
Force between parallel conductors	$\frac{F}{l} = \mu_o \frac{I_1 I_2}{2 \pi r}$ [*derivation required*]

MOTION IN A MAGNETIC FIELD

Force on charge q, speed v, in field B:	$F = q v B \sin\theta$ (θ is the angle between v and B) **The direction of F is perpendicular to the plane containing v and B.**
Helical path	This is the spiral path followed by a charge when its velocity makes an angle θ with the direction of B. $v \sin\theta$ is the component perpendicular to the direction B, while $v \cos\theta$ is the component parallel to the direction of B.
J.J. Thomson	Measured the charge to mass ratio of the electron by using electric and magnetic deflection of an electron beam.
'Crossed' fields	Electric and magnetic fields are applied at right angles to each other. Charged particles of certain speeds will pass through undeviated - velocity selector: $v = \frac{E}{B}$

SELF-INDUCTANCE

Growth and Decay of current	The current takes time to grow and decay in a d.c. circuit containing an inductor
Self-Induction	An e.m.f. is induced across a coil when the current in the coil changes.
Self Inductance (L)	$e = - L \frac{dI}{dt}$ (L is the self inductance of the coil)
Henry	The inductance of an inductor is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.
Direction of induced e.m.f.	The direction of the induced e.m.f. is such that it opposes the change of current. This is known as Lenz's Law. The negative sign in the above equation indicates this opposing direction.
Energy stored	The work done in building up the current in an inductor is stored in the magnetic field of the inductor. The magnetic field can be a source of energy when the magnetic field is allowed to collapse.
Energy equation	$E = \frac{1}{2} L I^2$ (energy E stored in inductor L)
Current and frequency in an inductive circuit	Current is inversely proportional to the frequency in an inductive circuit.
Reactance	The opposition to flow of an alternating current is called reactance.
C and L in a.c. circuits	For an inductor the reactance increases as the frequency of the a.c. increases. Conversely the reactance of a capacitor decreases as the frequency of the a.c. increases.
Uses	Inductors can be used to block a.c. signals while allowing d.c signals to pass. Capacitors can block d.c signals, but allow high frequency a.c. signals to pass. Inductors can be used to generate a high voltage when the magnetic field is allowed to collapse suddenly.

FORCES OF NATURE

Strong Force	The force of attraction between nucleons in a nucleus, with a very short range < 1 x 10^{-14} m.
Weak Force	This is the force associated with β-decay.
Quarks	Neutrons and protons are made up of quarks.

WAVE PHENOMENA SUMMARY

WAVES

Wave motion	Energy is transferred with no net mass transport.
Travelling Wave	The displacement, y, of any point on a travelling wave in the positive x direction is given by: $y = a \sin 2\pi (ft - \frac{x}{\lambda})$ [*explain not derive*]
Intensity of a wave	Intensity is directly proportional to $(\text{amplitude})^2$.
Superposition	The displacement at a point, due to two or more waves, is the algebraic sum of the individual displacements.
Phase Difference	For two points separated by distance x, the phase difference is $\phi = 2\pi \frac{x}{\lambda}$ (ϕ is the phase angle)
Stationary Wave	This wave is produced by the interference of two identical waves travelling in opposite directions
Nodes	These are points of zero displacement on a stationary wave separated by a distance of $\frac{\lambda}{2}$.
Antinodes	These are points of maximum displacement on a stationary wave, also separated by $\frac{\lambda}{2}$.
Doppler Effect	This is the change in frequency which is observed when a source of sound waves moves relative to a stationary observer.

Apparent frequency when source of sound moves	$\boxed{f_{obs} = f_s \frac{v}{(v - v_s)}}$	source moving **towards** stationary observer
	$\boxed{f_{obs} = f_s \frac{v}{(v + v_s)}}$	source moving **away from** stationary observer
Apparent frequency when observer moves	$\boxed{f_{obs} = f_s \frac{v + v_o}{v}}$	observer moving **towards** stationary source
	$\boxed{f_{obs} = f_s \frac{v - v_o}{v}}$	observer moving **away from** stationary source

[*derivation of the above expressions for f_{obs} required*]

INTERFERENCE – DIVISION OF AMPLITUDE

Coherent Sources of light	Coherent sources must have a **constant phase difference**.
Optical path length	Optical path length = n x geometrical path length
Optical path difference	For *optical* path lengths S_1P and S_2P: $(S_2P - S_1P) = m\lambda$ for constructive interference $(S_2P - S_1P) = (m + \frac{1}{2})\lambda$ for destructive interference
Phase difference and optical path length	phase difference = $\frac{2\pi}{\lambda}$ x optical path length
Phase change on reflection	When light reflects off an optically more dense medium a phase change of π occurs.
Thin Film	**Destructive** interference: $2nt \cos r = m\lambda$ For viewing at near normal incidence $2nt = m\lambda$ [*derivation required*]
Wedge Fringes	At normal incidence, fringe separation Δx is $\Delta x = \frac{\lambda}{2\tan\theta} = \frac{\lambda L}{2D}$ [*derivation required*] (D is the wedge separation, and L is the wedge length)
Non-Reflective Coatings	Thickness of coating, $d = \frac{\lambda}{4n}$ [*derivation required*]

INTERFERENCE – DIVISION OF WAVEFRONT

Point or line source	Explain why division of wavefront requires a point or line source. Describe why division of amplitude can use an extended source.
Young's Slits	Fringe separation $\Delta x = \frac{\lambda D}{d}$ [*derivation required*]

POLARISATION

Plane Polarised Light	Linearly polarised light waves consist of vibrations of the electric field strength vector in one plane only.
Polarisers and Analysers	A polariser and analyser held so that their planes of polarisation are at right angles can prevent the transmission of light.
Brewster's angle	At the polarising angle i_p, known as Brewster's angle, the refracted and reflected rays are separated by 90°.
Brewster's law	$n = \tan i_p$ [*derivation required*]