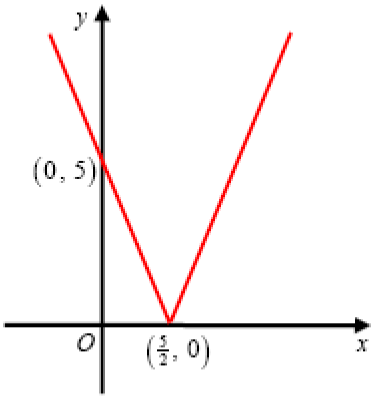
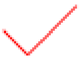
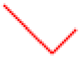
1. (a)



M1 A1 2

<u>Note</u>

M1: V or  or  graph with vertex on the x-axis.
A1: $\left(\frac{5}{2}, \{0\}\right)$ and ({0}, 5) seen and the graph appears in both the first and second quadrants.

(b) $\underline{x = 20}$ B1

$2x - 5 = -(15 + x)$; $\Rightarrow$ $\underline{x = -\frac{10}{3}}$ M1; A1 oe. 3

<u>Note</u>

M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$

(c) $\text{fg}(2) = \text{f}(-3) = |2(-3) - 5|; = |-11| = 11$ M1; A1 2

<u>Note</u>

M1: ***Full method*** of inserting g(2) into f(x) = $|2x - 5|$ or for inserting $x = 2$ into $|2(x^2 - 4x + 1) - 5|$. There must be evidence of the modulus being applied.

(d) $g(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $g_{min} = -3$ M1

Either $g_{min} = -3$ or $g(x) \geq -3$ B1

or $g(5) = 25 - 20 + 1 = 6$

$\underline{-3 \leq g(x) \leq 6}$ or $\underline{-3 \leq y \leq 6}$ A1 3

<u>Note</u>

M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $g_{min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into f(x) in order to find the minimum.

B1: For either finding the correct minimum value of g (can be implied by $g(x) \geq -3$ or $g(x) > -3$) or for stating that $g(5) = 6$

A1: $\underline{-3 \leq g(x) \leq 6}$ or $\underline{-3 \leq y \leq 6}$ or $\underline{-3 \leq g \leq 6}$. **Note that**: $-3 \leq x \leq 6$ is A0.

Note that: $-3 \leq f(x) \leq 6$ is A0. **Note that**: $-3 \geq g(x) \geq 6$ is A0.

Note that: $g(x) \geq -3$ or $g(x) > -3$ or $x \geq -3$ or $x > -3$ with no working gains M1B1A0.

Note that for the final Accuracy Mark:

If a candidate writes down $-3 < g(x) < 6$ or $-3 < y < 6$, then award M1B1A0.

If, however, a candidate writes down $g(x) \geq -3$, $g(x) \leq 6$, then award A0.

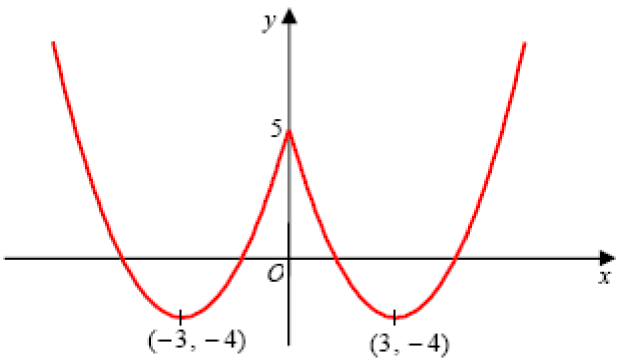
If a candidate writes down $g(x) \geq -3$ or $g(x) \leq 6$, then award A0.

[10]

2. (a) (i) $(3, 4)$ B1 B1

(ii) $(6, -8)$ B1 B1 4

(b)



B1 B1 B1 3

Note

B1: Correct shape for $x \geq 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.).
B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$.
Note: The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0.
B1: Correct turning points of (–3, –4) and (3, –4). Also, ({0}, 5) is marked where the graph cuts through the y-axis. Allow (5, 0) rather than (0, 5) if marked in the "correct" place on the y-axis.

(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1 A1	2

Note

M1: Either states f(x) in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$
Or uses a complete method on $\mathrm{f}(x) = x^2 + ax + b$, with f(0) = 5 and f(3) = – 4 to find both a and b.
A1: Either $(x - 3)^2 - 4$ or $x^2 - 6x + 5$

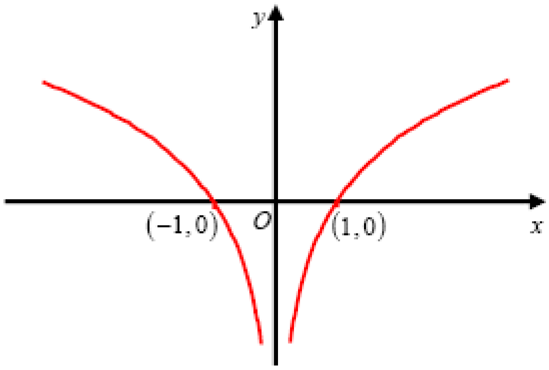
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1	1

Note

B1: Or: The inverse is a one-many {mapping and not a function}.
Or: Because f(0) = 5 and also f(6) = 5.
Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.

[10]

3. $y = \ln|x|$



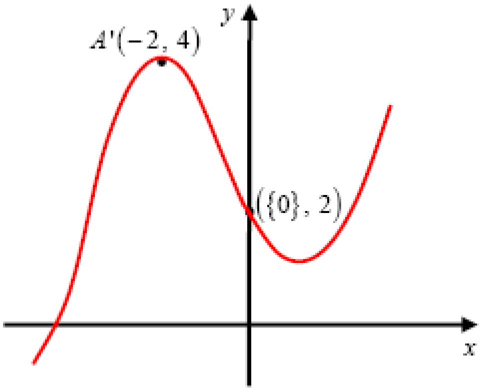
Right-hand branch in quadrants 4 and 1. Correct shape. B1

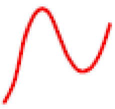
Left-hand branch in quadrants 2 and 3. Correct shape. B1

Completely correct sketch and both (–1,{0}) and (1,{0}) B1 3

[3]

4. (i) $y = \text{f}(-x) + 1$

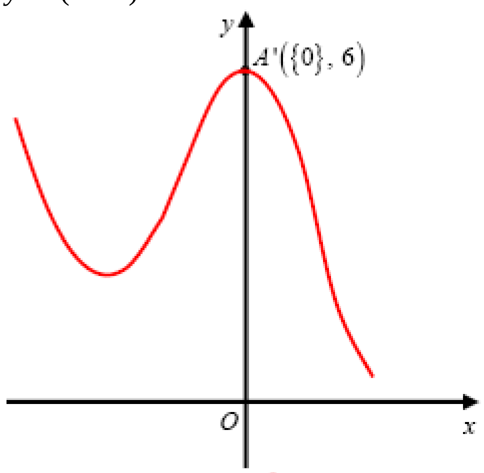


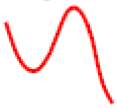
Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive *y*-axis. B1

Either ({0}, 2) or *A*'(–2, 4) B1

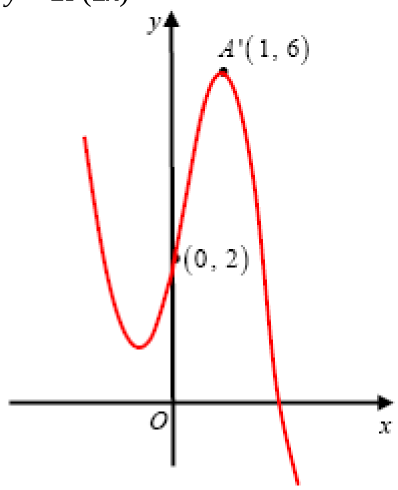
Both ({0}, 2) and *A*'(–2, 4) B1 3

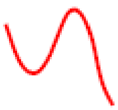
(ii) $y = \mathrm{f}(x + 2) + 3$



Any translation  of the original curve.	B1	
The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.		
The translated curve has maximum ({0}, 6) and is in the correct position on the Cartesian axes.	B1	3

(iii) $y = 2\mathrm{f}\,(2x)$



Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1	
Either ({0}, 2) or A'(1, 6)	B1	
Both ({0}, 2) and A'(1, 6)	B1	3

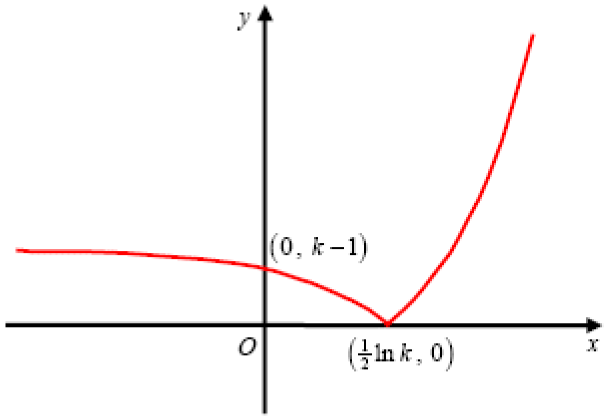
[9]

5.	(i)	(a)	$\ln(3x-7)=5$			
			$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation.		
				This can be implied by $3x-7=e^5$.	M1	
				Then rearranges to make x the subject.	dM1	
			$3x-7=e^5 \Rightarrow$			
			$x=\dfrac{e^5+7}{3}\{=51.804...\}$	***Exact answer*** of $\dfrac{e^5+7}{3}$.	A1	3
		(b)	$3^x e^{7x+2}=15$			
			$\ln(3^x e^{7x+2})=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1	
			$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1	
			$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe	
			$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1	
			$x=\dfrac{-2+\ln 15}{7+\ln 3}\{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe	5

(ii)	(a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$			
		$y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$	Attempt to make x (or swapped y) the subject	M1	
		$\Rightarrow \ln(y-3) = 2x$			
		$\Rightarrow \frac{1}{2}\ln y - 3 = x$	Makes e^{2x} the subject and takes ln of both sides	M1	
		Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x-3)}$	$\frac{1}{2}\ln(x-3)$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(x) = \frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**	
		$f^{-1}(x)$: Domain: $\underline{x > 3}$ or $\underline{(3, \infty)}$	Either $\underline{x > 3}$ or $\underline{(3, \infty)}$ or <u>Domain > 3.</u>	B1	4
	(b)	$g(x) = \ln(x-1), x \in \mathbb{R}, x > 1$			
		$fg(x) = e^{2\ln(x-1)} + 3$ $\{= (x-1)^2 + 3\}$	An attempt to put function g into function f.	M1	
			$e^{2\ln(x-1)} + 3$ or $(x-1)^2 + 3$ or $x^2 - 2x + 4$.	A1 isw	
		$fg(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$	Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or <u>Range>3</u> or <u>fg(x)>3</u>.	B1	3

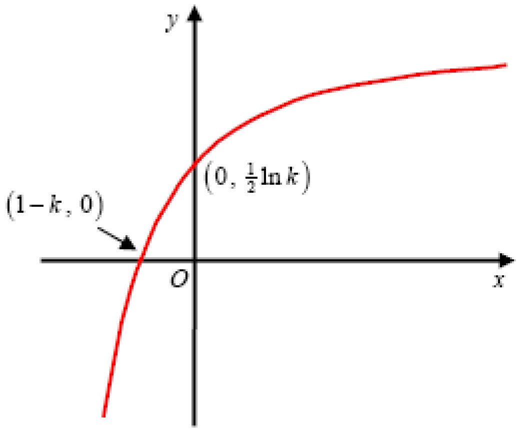
[15]

6. (a)



Curve retains shape when $x > \frac{1}{2}\ln k$	B1	
Curve reflects through the x-axis when $x > \frac{1}{2}\ln k$	B1	
$(0, k-1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1	3

(b)



Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1	
$(1-k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1	2

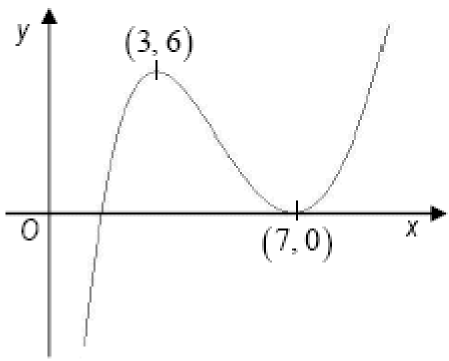
(c) Range of f: <u>f(x) > –k</u> or <u>y > –k</u> or <u>(–k, ∞)</u>

Either <u>f(x) > –k</u> or <u>y> –k</u> or <u>(– k, ∞)</u>or <u>f > – k</u> or <u>Range > – k</u> .	B1	1

(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$	Attempt to make x		
	$\Rightarrow \ln(y + k) = 2x$	(or swapped y) the subject	M1	
	$\Rightarrow \frac{1}{2}\ln(y+k) = x$	Makes e^{2x} the subject and takes ln of both sides	M1	
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	<u>A1</u> cao	3
(e)	$f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\infty)}$	Either $\underline{x > -k}$ or $\underline{(-k,\infty)}$ or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1ft	1

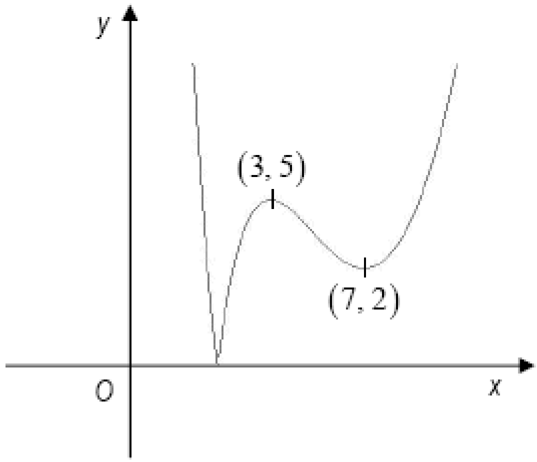
[10]

7. (a)



Shape	B1	
(3, 6)	B1	
(7, 0)	B1	3

(b)



Shape		B1	
(3, 5)		B1	
(7, 2)		B1	3

[6]

8. (a) $g(x) \geq 1$ B1 1

(b) $fg(x) = f(e^{x^2}) = 3e^{x^2} + \ln e^{x^2}$ M1

$= x^2 + 3e^{x^2}$ * A1 2

$(fg : x \mapsto x^2 + 3e^{x^2})$

(c) $fg(x) \geq 3$ B1 1

(d) $\frac{d}{dx}(x^2 + 3e^{x^2}) = 2x + 6xe^{x^2}$ M1 A1

$2x + 6xe^{x^2} = x^2 e^{x^2} + 2x$

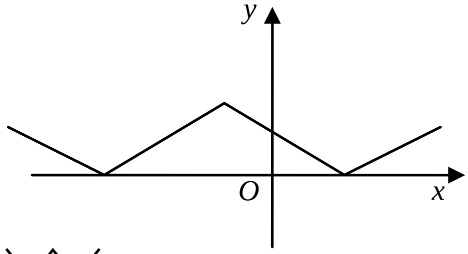
$e^{x^2}(6x - x^2) = 0$ M1

$e^{x^2} \neq 0,\quad 6x - x^2 = 0$ A1

$x = 0, 6$ A1 A1 6

[10]

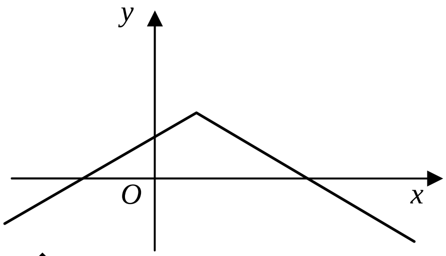
9. (a)



 shape — B1

Vertices correctly placed — B1 — 2

(b)



 shape — B1

Vertex and intersections with axes correctly placed — B1 — 2

(c) P: (–1, 2) — B1

Q: (0, 1) — B1

R: (1, 0) — B1 — 3

(d) $x > -1;\ 2 - x - 1 = \frac{1}{2}x$ — M1A1

Leading to $x = \frac{2}{3}$ — A1

$x < -1;\ 2 + x + 1 = \frac{1}{2}x$ — M1

Leading to $x = -6$ — A1 — 5

[12]

10. (a) $x^2 - 2x - 3 = (x - 3)(x + 1)$ — B1

$\mathrm{f}(x) = \frac{2(x-1)-(x+1)}{(x-3)(x+1)} \left(\text{or } \frac{2(x-1)}{(x-3)(x+1)} - \frac{x+1}{(x-3)(x+1)} \right)$ — M1A1

$= \frac{x-3}{(x-3)(x+1)} = \frac{1}{x+1}$ * — cso — A1 — 4

(b)	$\left(0, \frac{1}{4}\right)$	Accept $0 < y < \frac{1}{4}$, $0 < f(x) < \frac{1}{4}$ etc.	B1B1	2

(c) Let $y = f(x)$

$y = \frac{1}{x+1}$

$x = \frac{1}{y+1}$

$yx + x = 1$

$y = \frac{1-x}{x}$ or $\frac{1}{x} - 1$ M1A1

$f^{-1}(x) = \frac{1-x}{x}$

Domain of f^{-1} is $\left(0, \frac{1}{4}\right)$ ft their part (b)   B1ft   3

(d) $fg(x) = \frac{1}{2x^2 - 3 + 1}$

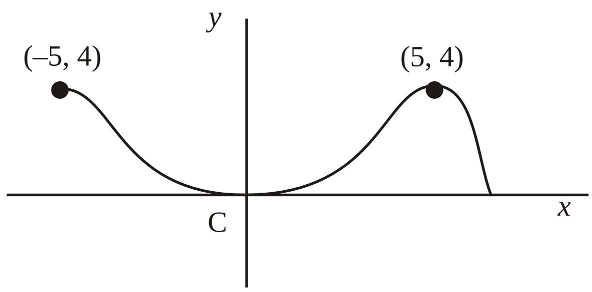
$\frac{1}{2x^2 - 2} = \frac{1}{8}$ M1

$x^2 = 5$ A1

$x = \pm\sqrt{5}$ both   A1   3

[12]

11. (a)

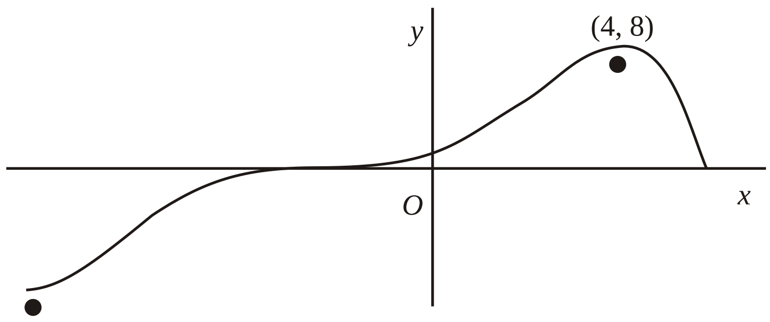


Shape	B1	
(5, 4)	B1	
(–5, 4)	B1	3

(b) For the purpose of marking this paper, the graph is identical to (a)

Shape	B1	
(5, 4)	B1	
(–5, 4)	B1	3

(c)



General shape – unchanged		B1	
Translation to left		B1	
(4, 8)		B1	
(–6, –8)		B1	4

In all parts of this question ignore any drawing outside the domains shown in the diagrams above.

[10]

12.	(a)	$x = 1 - 2y^3 \Rightarrow\ y = \left(\frac{1-x}{2}\right)^{1/3}$ or $\sqrt[3]{\frac{1-x}{2}}$		M1A1	2
		$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{1/3}$	Ignore domain		
	(b)	$gf(x) = \frac{3}{1-2x^3} - 4$		M1A1	
		$= \frac{3-4(1-2x^3)}{1-2x^3}$		M1	
		$= \frac{8x^3-1}{1-2x^3}$ (*)	cso	A1	4
		$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$	Ignore domain		
	(c)	$8x^3 - 1 = 0$	Attempting solution of numerator = 0	M1	
		$x = \frac{1}{2}$	Correct answer and no additional answers	A1	2

(d) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1-2x^3)\times 24x^2 + (8x^3-1)\times 6x^2}{(1-2x^3)^2}$ M1A1

$= \frac{18x^2}{(1-2x^3)^2}$ A1

Solving their numerator = 0 and substituting to find y. M1

$x = 0$, $y = -1$ A1 5

[13]

13. (a) Finding g(4) = k and f(k) = ... or fg(x) = $\ln\left(\frac{4}{x-3}-1\right)$ M1

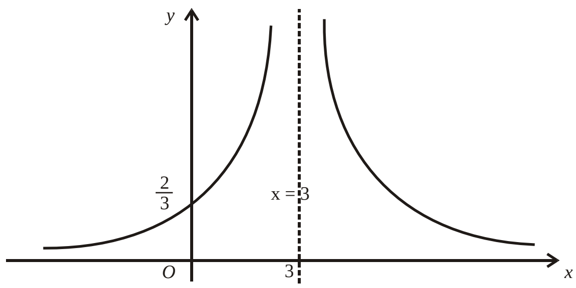
[f(2) = ln $(2x^2 - 1)$ fg(4) = ln (4 – 1)] = ln 3 A1 2

(b) $y = \ln(2x-1) \Rightarrow e^y = 2x - 1$ or $e^x = 2y - 1$ M1, A1

$f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$ A1

Domain $x \in \mathbb{R}$ [Allow $\mathbb{R}$ all reals, $(-\infty, \infty)$] independent B1 4

(c)



Shape, and x-axis should appear to be asymptote B1

Equation x = 3 needed, may see in diagram (ignore others) B1 ind.

Intercept (0, $\frac{2}{3}$) no other; accept $y = \frac{2}{3}$ (0.67) or on graph B1 ind 3

(d) $\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv. B1

$\frac{2}{x-3} = -3, \Rightarrow x = 2\frac{1}{3}$ or exact equiv. M1, A1 3

Note: $2 = 3(x + 3)$ or $2 = 3(-x - 3)$ o.e. is M0A0

Alt: Squaring to quadratic ($9x^2 - 54x + 77 = 0$) **and** solving M1; B1A1

[12]

14. (a) $y = \ln(4 - 2x)$

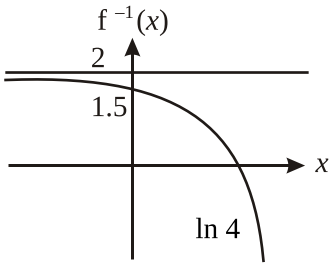
$e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln M1 A1

$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ * cso A1

Domain of f^{-1} is □ B1 4

(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$ □) B1 1

(c)



Shape B1
1.5 B1
ln 4 B1
$y = 2$ B1 4

(d) $x_1 \approx -0.3704$, $x_2 \approx -0.3452$ cao B1, B1 2

If more than 4 dp given in this part a maximum on one mark is lost.
Penalise on the first occasion.

(e) $x_3 = -0.35403019...$
$x_4 = -0.350\ 92688...$
$x_5 = -0.352\ 01761...$
$x_6 = -0.35163386...$ Calculating to at least x_6 to at least four dp M1
$k \approx -0.352$ cao A1 2

Alternative

$k \approx -0.352$ Found in any way

Let $g(x) = x + \frac{1}{2}e^x$

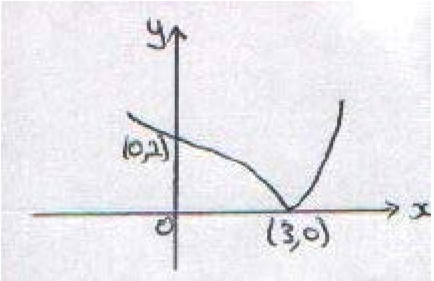
$g(-0.3515) \approx +0.0003$, $g(-0.3525) \approx -0.001$ M1

Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$

$\Rightarrow k = -0.352$ (to 3 dp) A1 2

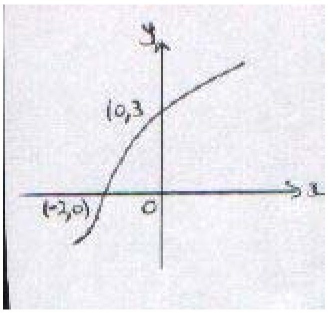
[13]

15. (a)



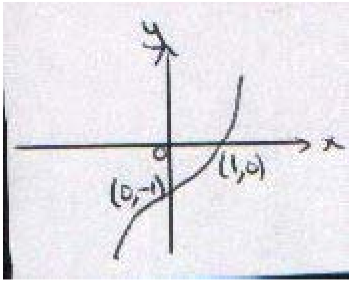
Mod graph, reflect for y < 0	M1	
(0, 2), (3, 0) or marked on axes	A1	
Correct shape, including cusp	A1	3

(b)



Attempt at reflection *in* $y = x$	M1	
Curvature correct	A1	
(–2, 0), (0, 3) or equiv.	B1	3

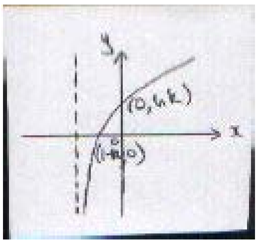
(c)



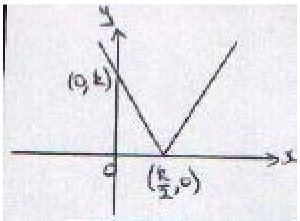
Attempt at ‘stretches’	M1	
(0, −1) or equiv.	B1	
(1, 0)	B1	3

[9]

16. (a)



Log graph: Shape	B1	
Intersection with -ve x-axis	dB1	
(0, ln$k$), $(1-k, 0)$	B1	

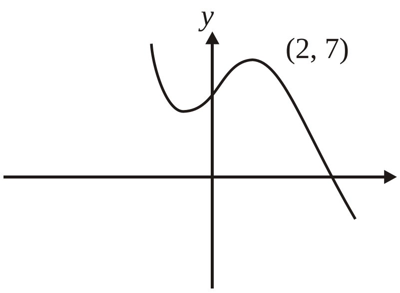


Mod graph :V shape, vertex on +ve x-axis	B1	
$(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1	5

(b)	$f(x) \in \mathbb{R}$, $-\infty < f(x) < \infty$, $\infty < y < \infty$	B1	1
(c)	$\text{fg}\left(\frac{k}{4}\right) = \ln\left\{k + \left\|\frac{24}{4} - k\right\|\right\}$ or $\text{f}\left(\left\|-\frac{k}{2}\right\|\right)$	M1	
	$= \ln\left(\frac{3k}{2}\right)$	A1	2
(d)	$\frac{dy}{dx} = \frac{1}{x+k}$	B1	
	Equating (with $x = 3$) to grad. of line; $\frac{1}{3+k} = \frac{2}{9}$	M1: A1	
	$k = 1\frac{1}{2}$	A1ft	4

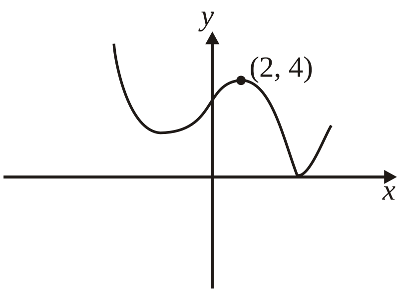
[12]

17. (a)



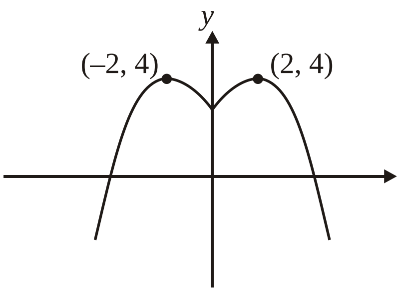
B1 Shape		
B1 Point	2	

(b)



B1 Shape		
B1 Point	2	

(c)

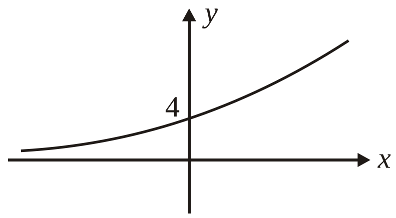


B1 Shape > 0		
B1 Point $x < 0$		
B1 Point		
(–2, 4)	3	
		[7]

18. (a)

$gf(x) = e^{2(2x + \ln 2)}$	M1	
$= e^{4x}e^{2 \ln 2}$	M1	
$= e^{4x}e^{\ln 4}$	M1	
$= 4e^{4x}$ **AG**	A1	4

(b)



B1 shape & (0, 4) 1

(c) $gf(x) > 0$ B1 1

(d) $\frac{d}{dx} gf(x) = 16e^{4x}$ M1

$e^{4x} = \frac{3}{16}$ M1 attempt to solve

$4x = \ln\frac{3}{16}$ A1

$x = -0.418$ A1 4

[10]

19. (a) $f(x) = \frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$ B1

factors of quadratic denominator

$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$

common denominator M1

simplifying to linear numerator M1

$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ AG A1cso 4

(b) $y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow$ M1

$xy = 2 + y$ or $x - 1 = \frac{2}{y}$ A1

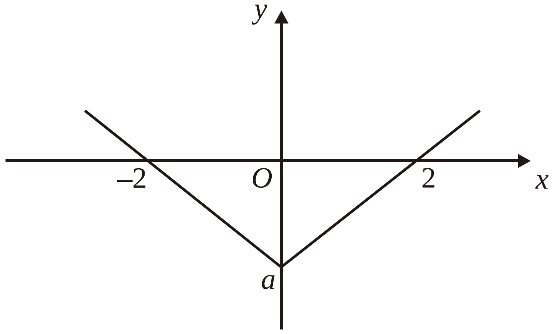
$f^{-1}(x) = \frac{2+x}{x}$ or equiv. A1 3

(c) fg(x) = $\dfrac{2}{x^2+4}$ (attempt) [$\dfrac{2}{"g"-1}$] M1

Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$ DM1; A1 3

[10]

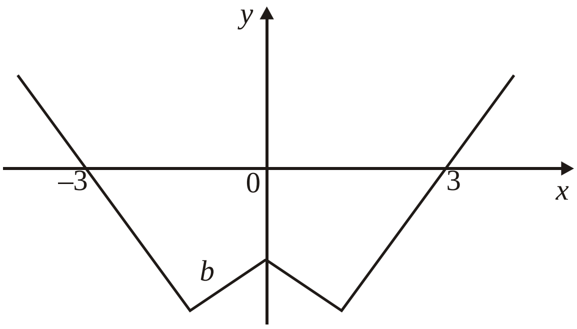
20. (a)



Translation ← by 1 M1

Intercepts correct A1 2

(b)



$x \geq 0$, correct "shape" B1

[provided not just original]

Reflection in y-axis B1ft

Intercepts correct B1 3

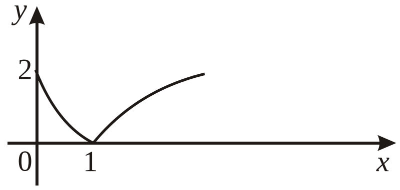
(c) $a = –2$, $b = –1$ B1B1 2

(d) Intersection of $y = 5x$ with $y = – x – 1$ M1A1

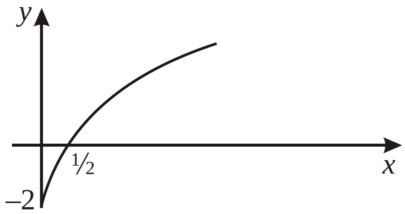
Solving to give $x = -\dfrac{1}{6}$ M1A1 4

[11]

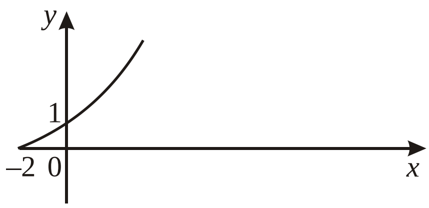
21. (a)



Reflected in x-axis 0 < x < 1	M1	
Cusp + coords	A1	2
Clear curve going correct way		
Ignore curve x < 0		



General shaped and -2	B1	
(1/2, 0)	B1	2
Ignore curve x< 0		



Rough reflection in y = x	B1	
(0,1) or 1 on y-axis	B1	
(-2, 0) or -2 on x-axis and no curve x < -2	B1	3

[7]

22.	(a)	I = $3x + 2e^x$	B1	
		Using limits correctly to give 1 + 2e. (c.a.o.)	M1 A1	3
		must subst 0 and 1 and subtract		
	(b)	$A = (0, 5)$;	B1	
		$y = 5$		
		$\frac{dy}{dx} = 2e^x$	B1	
		Equation of tangent: $y = 2x + 5$; c = –2.5	M1; A1	4
		attempting to find eq. of tangent and subst in $y = 0$, must be linear equation		
	(c)	$y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \Rightarrow 4y - 2 = 5x - xy$	M1; A1	
		putting $y =$ and att. to rearrange to find x .		
		$g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent	A1	3
		must be in terms of x		
	(d)	gf(0) = g(5); =3	M1; A1	2
		att to put 0 into f and then their answer into g		

[12]

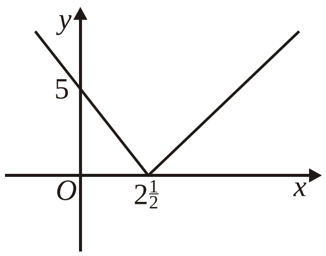
23.	(a)	$\frac{2x+5}{x+3} - \frac{1}{(x+3)(x+2)} = \frac{(2x+5)(x+2)-1}{(x+3)(x+2)}$	M1	
		$= \frac{2x^2+9x+9}{(x+3)(x+2)}$	A1	
		$= \frac{(2x+3)(x+3)}{(x+3)(x+2)}$	M1 A1	
		$= \frac{2x+3}{x+2}$	A1	5
	(b)	$2 - \frac{1}{x+2} = \frac{2(x+2)-1}{x+2} = \frac{2x+3}{x+2}$ or the reverse	M1 A1	2

(c) T_1: Translation of –2 in x direction B1
T_2: Reflection in the x-axis B1
T_3: Translation of (+)2 in y direction B1
All three fully correct B1 4

[11]

One alternative is
T_1: Translation of –2 in x direction
T_2: Rotation of 90° clockwise about O
T_3: Translation of –2 in x direction

24. (a)



Correct shape, vertex on x-axis B1
(0, 5)or 5 on y-axis B1
(2 ½, 0)or 2½ on x-axis B1 3

(b) $2x – 5 = x \Rightarrow x = 5$ M1 A1
accept stated
$2x – 5 = –x$ or equivalent M1
$x = 1^2/_3$ accept exact equivalents A1 4

(c) Method for finding either coordinate of the lowest point (differentiating and equating to zero, completing the square, using symmetry). M1
$x = 3$ or $g(x) = –9$ A1
$g(x) \geq –9$ A1 3

(d) fg(1) = f(–5) M1
= 15 A1 2

[12]

25. (a) $2 + \frac{3}{x+2}\left(=\frac{2(x+2)+3}{x+2}\right) = \underline{\underline{\frac{2x+7}{x+2}}}$ or $\frac{2(x+2)+3}{x+2}$ B1 1

(b)	$y = 2 + \dfrac{3}{x+2}$	<u>OR</u> $y = \dfrac{2x+7}{x+2}$		
	$y - 2 = \dfrac{3}{x+2}$	$y(x+2) = 2x + 7$	M1	
		$yx - 2x = 7 - 2y$		
	$x + 2 = \dfrac{3}{y-2}$	$x(y-2) = 7 - 2y$	M1	
	$x = \dfrac{3}{y-2} - 2$	$x = \dfrac{7-2y}{y-2}$		
	$\therefore\ f^{-1}(x) = \underline{\underline{\dfrac{3}{x-2} - 2}}$	$\underline{\underline{f^{-1}(x) = \dfrac{7-2x}{x-2}}}$	o.e A1	3

Notes

M1 $y = f(x)$ and <u>1st step</u> towards $x =$.

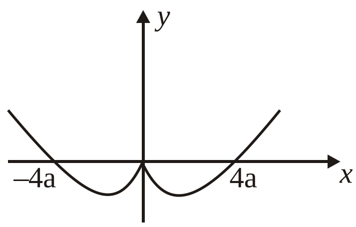
M1 One step from $x =$.

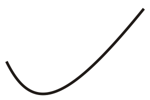
A1 y or $f^{-1}(x) =$ in terms of x.

(c)	Domain of $f^{-1}(x)$ is $x \in \mathbb{R}, x \neq 2$ [NB $x \neq +2$]	B1	1

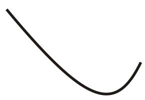
[5]

26. (a)





$x > 0$ B1



$x < 0$ B1 ft

$(4a, 0)$ & $(-4a, 0)$ and shape at $(0, 0)$ B1 3

(b) $f(2a) = (2a)^2 - 4a(2a) = 4a^2 - 8a^2 = \underline{-4a^2}$ B1

$f(-2a)$ [= $f(2a)$ ($\because$ *even function*)] $= \underline{-4a^2}$ B1 ft 2

B1 ft their f(2a)

(c) $a = 3$ and $f(x) = 45 \Rightarrow 45 = x^2 - 12x \quad (x > 0)$ M1

$0 = x^2 - 12x - 45$

$0 = (x - 15)(x + 3)$ M1

$x = 15$ (or -3) A1

$\therefore$ Solutions are $\underline{x = \pm 15}$ $\underline{\text{only}}$ A1 4

M1 Attempt 3TQ in x

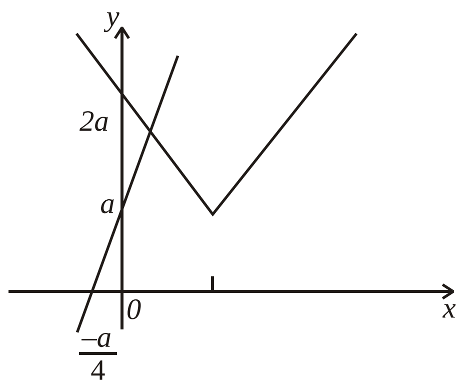
M1 Attempt to solve

A1 At least x = 15 can ignore x = –3

A1 To get final A1 must make clear <u>only</u> answers are ±15.

[9]

27. (a)



V shape right way up B1

vertex in first quadrant B1

g B1

-1 eeoo; $2a$, a, $-\frac{a}{4}$ B2 (1, 0) 5

(b) $4x + a = (a - x) + a$ M1

$5x = a, \quad x = \frac{a}{5}$ M1

$y = \frac{9a}{5}$ A1 3

both correct

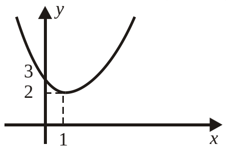
(c) $\text{fg}(x) = |4x + a - a| + a = |4x| + a$ M1 A1 2

(d) $|4x| + a = 3a \Rightarrow |4x| = 2a$ M1

$x = \frac{a}{2}, -\frac{a}{2}$ A1, A1 3

[13]

28. (a) $x^2 - 2x + 3 = (x - 1)^2 + 2$ M1



Full method to establish min. f

f(4) = $3^2 + 2 = 11$ f ≥ 2 A1

f ≤ 11 B1 3

penalise once for x or <

(b) f(2) = 3 ; $\therefore 16 = \text{gf}(2) \;\Rightarrow 16 = 3\lambda + 1$ B1; M1

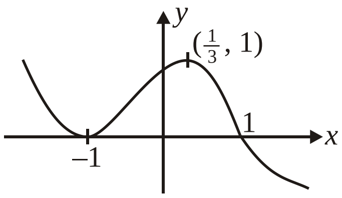
M for using their f(2) for eqn

$\therefore$ $\underline{\lambda = 5}$ A1 ft 3

ft their genuine f(2)

[6]

29. (a)



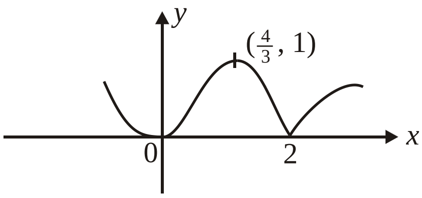
Translation in $\leftarrow$ or $\rightarrow$ B1

Points correct B2/1/0 3

(–1 eeoo)

[Don't insist on graph for $x < -1$ and is more $x > 2$]

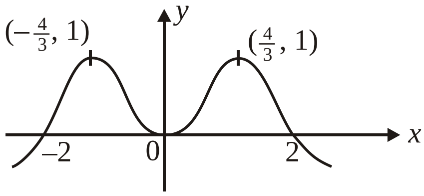
(b)



$x < 2$ including points B1

$x > 2$ correct reflection B1

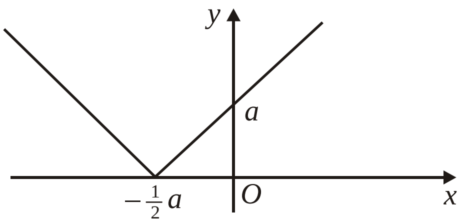
cusp at (2, 0) (not $\cup$) B1 3

(c) 

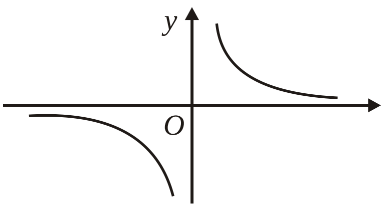
correct shape $x \geq 0$	B1	
symmetry in y-axis	B1	
correct maxima	B1	
correct x intercepts	B1	4

Fully correct (b) and (c) wrong way aroubnd B2

[10]

30. (a) 

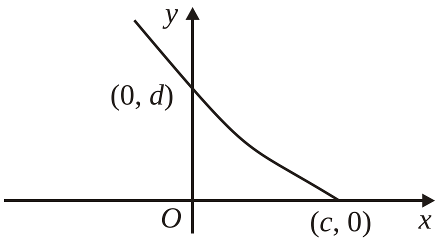
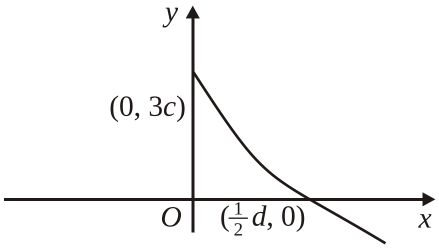
V graph with 'vertex' on x-axis	M1	
$\{-\frac{1}{2}a, (0)\}$ and $\{(0), a\}$ seen	A1	2



Correct graph (could be separate)	B1	1

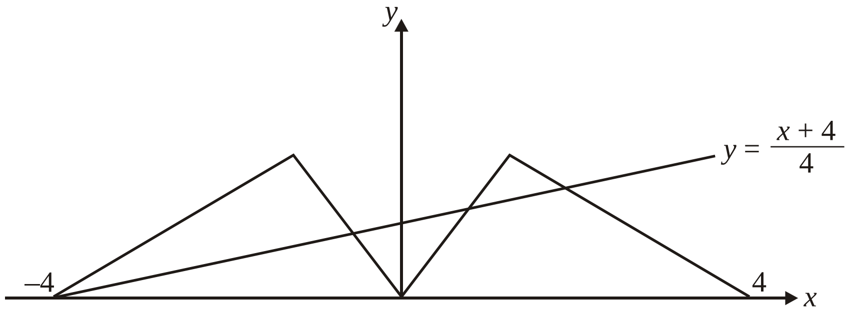
(c)	Meet where $\frac{1}{x} = \lvert 2x + a \rvert \Rightarrow x\lvert 2x + a \rvert - 1 = 0$; only one meet	B1	1
(d)	$2x^2 + x - 1$	B1	
	Attempt to solve; $x = \frac{1}{2}$ (no other value)	M1; A1	3

[7]

31.	(a)			
		shape	B1	
		intersections with axes $(c, 0)$, $(0, d)$	B1	2
	(b)			
		shape	B1	
		x intersection $(\frac{1}{2}d, 0)$	B1	
		y intersection $(0, 3c)$	B1	3
	(c)	(i) $c = 2$	B1	
		(ii) $-1 < f(x) \leq$ (candidate's) c value	B1 B1 ft	3
	(d)	$3(2^{-x}) = 1 \Rightarrow 2^{-x} = \frac{1}{3}$ and take logs; $-x = \dfrac{\ln\frac{1}{3}}{\ln 2}$	M1; A1	
		d (or x) = 1.585 (3 decimal places)	A1	3
	(e)	$fg(x) = f[\log_2 x] = [3(2^{-\log_2 x}) - 1]$; $= [3(2^{\log_2 \frac{1}{x}}) - 1]$ or $\dfrac{3}{2^{\log_2 x}} - 1$	M1; A1	
		$= \dfrac{3}{x} - 1$	A1	3

[14]

32. (a) $y = \text{f}(x)$ G1 G1 G1 G1 4



(b) Drawing line $y = \dfrac{x+4}{4}$ or an analytical complete method for 2 roots (or more) M1 A1

-4; $-\dfrac{4}{5}$, $\dfrac{4}{3}$, $\dfrac{12}{5}$ B1; A2, 1, 0 5

[9]

33. (a) A is (2, 0); B is $(0, e^{-2} - 1)$ B1; B1 2

(b) $y = e^{x-2} - 1$

Change over x and y, $x = e^{y-2} - 1$ M1

$y - 2 = \ln(x + 1)$ M1

$y = 2 + \ln(x + 1)$ A1

$f^{-1} : x / 2 + \ln(x + 1), x > -1$ A1 A1 5

(c) $f(x) - x = 0$ is equivalent to $e^{x-2} - 1 - x = 0$

Let $g(x) = e^{x-2} - 1 - x$

$g(3) = -1.28\ldots$

$g(4) = \quad 2.38\ldots$

Sign change $\Rightarrow$ root α M1 A1 2

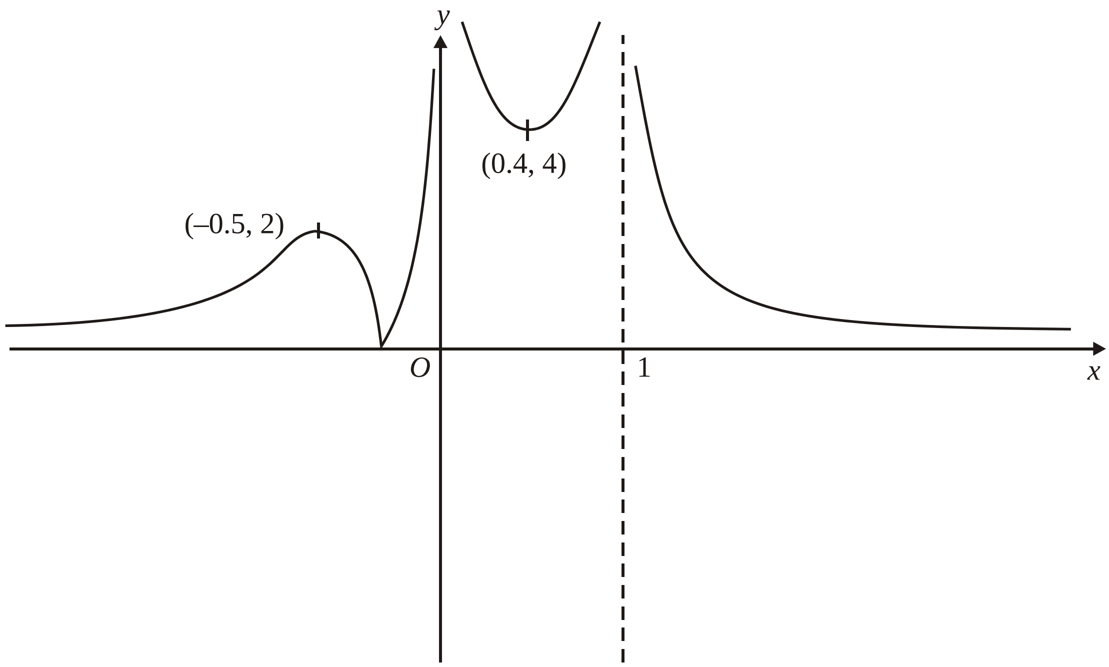
	(d)	$x_{n+1} = 2 + \ln(x_n + 1)$, $x_1 = 3.5$	M1	
		$x_2 = 3.5040774$	A1	
		$x_3 = 3.5049831$	A1	
		$x_4 = 3.5051841$		
		$x_5 = 3.5052288$		
		Needs convincing argument on 3 d.p. accuracy		
		Take 3.5053 and next iteration is reducing 3.50525…	M1	
		Answer: 3.505 (3 d.p.)	A1	5

[14]

34.	(a)	$f^{-1}(x) = \frac{1}{2}x, \; x \in \mathbb{R}$	B1 B1	2
	(b)	$gf^{-1}(x) = g(\frac{1}{2}x) = \frac{3}{4}x^2 + 2$	M1 A1	2
	(c)	Range $gf^{-1}(x) \geq 2$	B1	1

[5]

35. (a)



$x < 0$

B1 shape

$0 < x < 1$

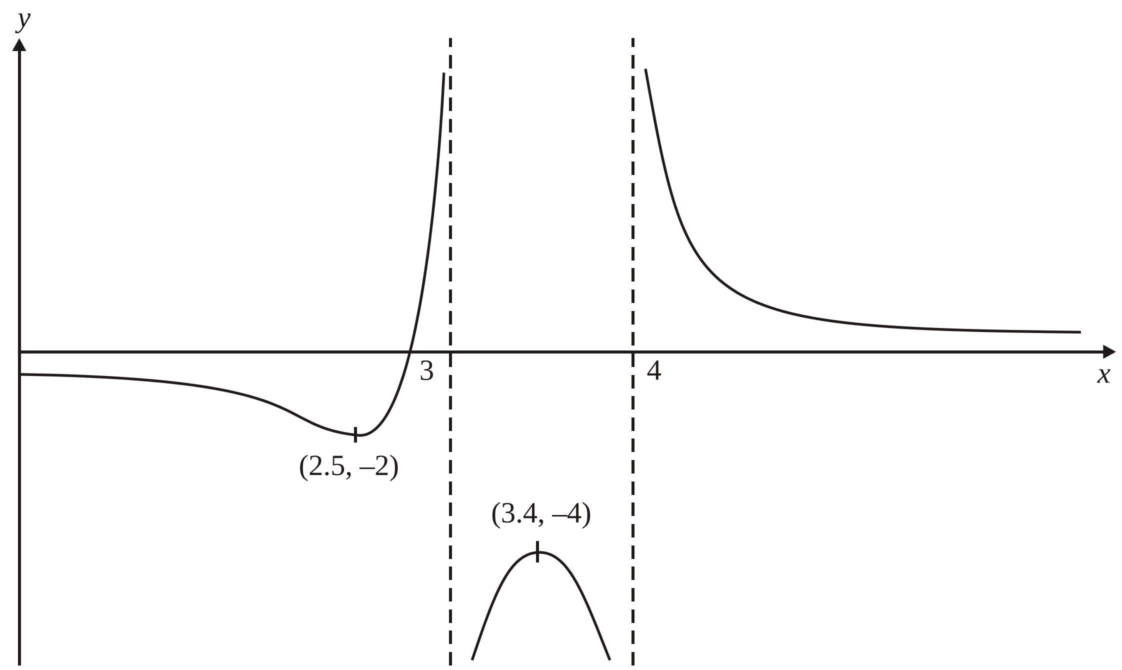
B1 shape

$x > 1$ 4

B1 shape

B1 points

(b)



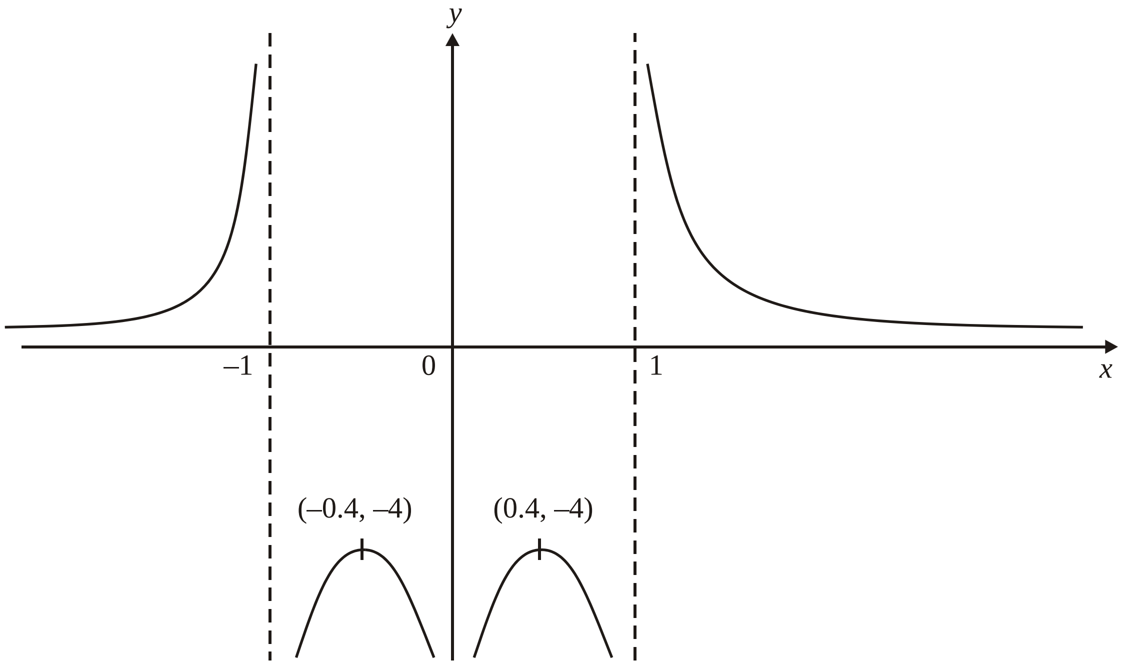
M1any translation

M1 correct direction, translation

B1 points

B1 asymptotes 4

(c)



B1 shape > 0
B1 shape < 0
B1 points
B1 asymptotes 4

[12]