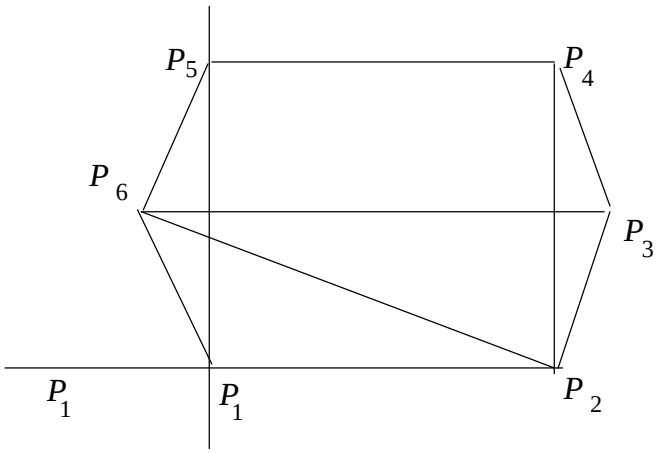
6. (i) $\mathbf{P}_1\mathbf{P}_2 = \mathbf{P}_5\mathbf{P}_4$ and $\mathbf{P}_2\mathbf{P}_3 = \mathbf{P}_6\mathbf{P}_5 \Rightarrow z_2 - z_1 = z_4 - z_5$ and $z_3 - z_2 = z_5 - z_6$
adding we have $z_3 - z_1 = z_4 - z_6 \Rightarrow z_4 - z_3 = z_6 - z_1 \Rightarrow \mathbf{P}_3\mathbf{P}_4 = \mathbf{P}_1\mathbf{P}_6$

$z_1 = 0, z_2 = 1, z_3 = z, z_5 = i$ and $z_6 = w$
From (i) $z_4 = z_2 + z_5 - z_1 = 1 + i$ and $z - 1 = i - w$
from (ii) $\mathrm{P}_2\mathrm{P}_4 \perp \mathrm{P}_3\mathrm{P}_6 \Rightarrow z_4 - z_2 \perp z_6 - z_3 \Rightarrow i \perp w - z \Rightarrow w - z$ is real
From the diagram it is clear that we must have
$\mathrm{Re}(z) > 1,\ \mathrm{Re}(w) < 0$ and conditions (i) and (ii)
$\Rightarrow \mathrm{Re}(z) + \mathrm{Re}(w) = 1$
also $w - z$ real $\Rightarrow \mathrm{Im}(z) = \mathrm{Im}(w) = \frac{1}{2}$
constraints on $\mathrm{Re}(z)$ and $\mathrm{Re}(w)$ are thus
$\mathrm{Re}(z) > 1,\ \mathrm{Re}(w) < 0$ and $\mathrm{Re}(z) + \mathrm{Re}(w) = 1$



let $z = (1 + a) + \frac{1}{2}bi$, so $w = -a + \frac{1}{2}bi$
If $\angle P_4P_2P_6 = \alpha$ and $\angle P_4P_2P_3 = \beta$
then $\tan\alpha = 2(1 + a)$ and $\tan\beta = 2a$
so $\tan\angle P_3P_2P_6 = \tan(\alpha + \beta)$
$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{2 + 4a}{1 - 4a(1 + a)} = -\frac{2}{3} \Rightarrow 6 + 12a = -2 + 8a + 8a^2 \Rightarrow 8a^2 - 4a - 8 = 0$
$\Rightarrow 2a^2 - a - 2 = 0 \Rightarrow a = \frac{1}{4}(1 + \sqrt{17})$ and hence $P_6P_3 = 1 + 2a = \frac{3}{2} + \frac{1}{2}\sqrt{17}$

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