

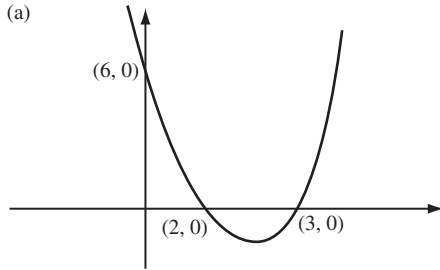
Worked Solutions

AQA C3 Paper A

1. $fg(x) = g^{-1}g(x) = x.$

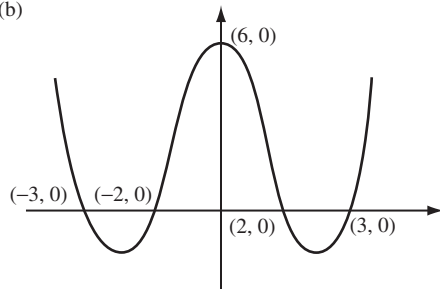
(3 marks)

2. (a)



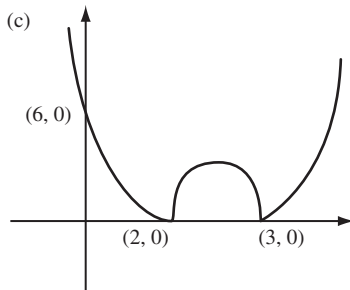
(2 marks)

(b)



(3 marks)

(c)



(3 marks)

3. $e^{2x} - 7e^x + 12 = 0$

$$(e^x - 3)(e^x - 4) = 0$$

$$e^x = 3 \Rightarrow x = \ln 3$$

$$e^x = 4 \Rightarrow x = \ln 4$$

(6 marks)

4. (a) $y = x + 9x^{-1}$

$$\frac{dy}{dx} = 1 - \frac{9}{x^2}$$

$$\text{at min. point } \frac{9}{x^2} = 1, \quad x = 3 \quad (x > 0)$$

$$\text{when } x = 3, \quad y = 3 + \frac{9}{3} = 6$$

$$\text{so } y \geq 6.$$

(4 marks)

$$\begin{aligned} \text{(b) area} &= \int_3^9 \left(x + \frac{9}{x} \right) dx = \left[\frac{x^2}{2} + 9 \ln x \right]_3^9 = \frac{81}{2} + 9 \ln 9 - \left(\frac{9}{2} + 9 \ln 3 \right) \\ &= 36 + 9 \ln 3. \end{aligned}$$

(4 marks)

5. $\frac{dy}{dx} = xe^x + e^x$

$$\text{at turning point, } (x + 1)e^x = 0 \Rightarrow x = -1$$

$$y = -e^{-1} \quad \text{pt } \left(-1, \frac{-1}{e} \right)$$

$$\frac{d^2y}{dx^2} = e^x + xe^x + e^x$$

$$x = -1 \quad \frac{d^2y}{dx^2} = e^{-1} > 0 \quad \therefore \text{minimum}$$

(8 marks)

$$6. \quad \left. \begin{array}{l} \text{(a) } f(1) = 1 - 6 + 7 = 2 \\ f(2) = 4 - 12 + 7 = -1 \end{array} \right\} \text{change of sign}$$

(2 marks)

$$\text{(b) } 7 = 6x - x^2$$

$$7 = x(6 - x)$$

$$x = \frac{7}{6 - x}$$

(2 marks)

$$\text{(c) } x_1 = 1.75, x_2 = 1.6470, x_3 = 1.6081, x_4 = 1.5938, x_5 = 1.5886, x_6 = 1.5868$$

$$\text{Ans. } x = 1.59 \text{ (3 s.f.)}$$

(4 marks)

$$7. \quad \frac{dx}{dy} = -3 \sin 3y,$$

$$\frac{dy}{dx} = -\frac{1}{3 \sin 3y}$$

$$y = \frac{\pi}{6}, \quad \frac{dy}{dx} = -\frac{1}{3}$$

$$y = \frac{\pi}{6}, \quad x = 0.$$

$$y - \frac{\pi}{6} = -\frac{1}{3}(x - 0)$$

$$y - \frac{\pi}{6} = -\frac{1}{3}x$$

$$6y + 2x - \pi = 0$$

(8 marks)

$$8. \quad \text{(a) } \frac{dy}{dx} = e^x - 3$$

$$\text{at } M \quad e^x = 3$$

$$x = \ln 3$$

(2 marks)

$$\text{(b) area} = \int_0^{\ln 3} (e^x - 3x) dx = \left[e^x - \frac{3}{2}x^2 \right]_0^{\ln 3}$$

$$= e^{\ln 3} - \frac{3}{2}(\ln 3)^2 - (1 - 0)$$

$$= 3 - \frac{3}{2}(\ln 3)^2 - 1$$

$$= 2 - \frac{3}{2}(\ln 3)^2$$

(5 marks)

$$9. \quad \text{(a) } f(1) = 1 - 1 - 3 + 3 = 0$$

$$(x - 1)(x^2 - 3) = 0 \quad x = \pm\sqrt{3}$$

(4 marks)

$$\text{(b) } \tan^3 \theta - 3 \tan \theta + 4 = 1 + \tan^2 \theta$$

$$\tan^3 \theta - \tan^2 \theta - 3 \tan \theta + 3 = 0$$

$$\text{same as (i) with } x = \tan \theta$$

(2 marks)

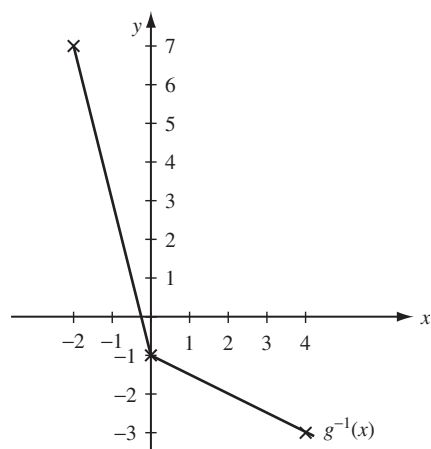
$$\text{(c) } \tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\tan \theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$$

$$\tan \theta = \sqrt{3} \Rightarrow \theta = \frac{2\pi}{3}$$

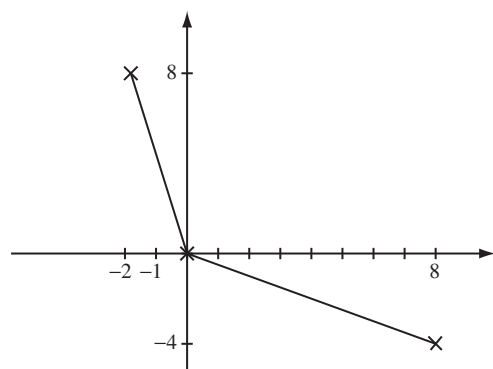
(4 marks)

10. (a)



(3 marks)

(b)



(3 marks)

(c) $g(-3) = 4$

$$hg(-3) = h(4)$$

$$= -2$$

(3 marks)