

Core Mathematics C4 Advanced Level

For AQA

Paper C

Time: 1 hour 30 minutes

Instructions and Information

- Full marks may be obtained for answers to ALL questions.
- The formulae booklet, available from AQA, may be used.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You may use a graphical calculator in this paper.
- The total number of marks for this paper is 75.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. (a) Express y , where $y = \frac{5x+7}{(x+1)(x+2)}$, in partial fractions (3 marks)

(b) Hence find the value of $\frac{d^2y}{dx^2}$ when $x = 1$. (3 marks)

2. Given that A and B are both obtuse angles and that $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, find the exact values of $\sin(A - B)$ and $\tan(A + B)$. (6 marks)

3. (a) Expand $(1 + 2x)^{-\frac{1}{2}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. (4 marks)

(b) State the set of values of x for which the expansion is valid. (1 mark)

(c) In the expansion of

$$(1 + ax)(1 + 2x)^{-\frac{1}{2}},$$

the coefficient of x is 3. Find the value of the constant a and find the coefficient of x^3 . (4 marks)

4. The parametric equations of a curve are

$$x = 2\theta + \sin \theta, \quad y = \cos \theta, \quad 0 \leq \theta \leq 2\pi.$$

(a) Show that the equation of the tangent to the curve, where $\theta = \frac{\pi}{2}$, is $2y + x = \pi + 1$. (4 marks)

(b) Find the coordinates of the stationary points on the curve. (4 marks)

5. (a) Prove that

$$\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1. \quad (4 \text{ marks})$$

(b) Using part (a) solve the equation

$$8 \cos^2 x - 8 \cos^4 x = 1, \quad \text{for } 0 < x < 180^\circ. \quad (3 \text{ marks})$$

6. The number of fish N in a pond is given by the formula

$$N = A e^{-kt},$$

where t is the time in days measured from a time when $N = 5000$.

- (a) Write down the value of A . (2 marks)

- (b) Given that $N = 4000$ when $t = 4$, show that $k = \frac{1}{4} \ln \frac{5}{4}$. (4 marks)

- (c) Find the value of N when $t = 8$. (3 marks)

7. (a) Factorise $(x^2 - 4x + 3)$ and hence express $\frac{2x}{x^2 - 4x + 3}$ in partial fractions. (3 marks)

- (b) Solve the differential equation

$$\frac{dy}{dx} = \frac{2xy}{x^2 - 4x + 3},$$

given that $y = \frac{1}{3}$ when $x = 4$. Give your answer in the form $y = f(x)$.

(6 marks)

8. (a) The line l passes through the points with coordinates $(1, 6, 1)$ and $(4, 0, -8)$. Find a vector equation for the line l . (2 marks)

- (b) The line m has equation $r = \begin{pmatrix} 4 \\ 8 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and intersects the line l .

Find the coordinates of the point of intersection of l and m . (4 marks)

- (c) The line n has direction $\begin{pmatrix} 5 \\ k \\ 5 \end{pmatrix}$, where k is a constant. The angle between m and n is 60° . Find the positive value of k . (3 marks)

9. (a) (i) Express

$40 \cos \theta - 9 \sin \theta$ in the form

$R \cos(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$. (4 marks)

(ii) Hence solve the equation

$$40 \cos \theta - 9 \sin \theta = 4$$

for $0 < \theta < 90^\circ$, giving your answer to 1 decimal place. (3 marks)

(b) Solve the equation

$$6 \sin \theta = 2 \operatorname{cosec} \theta + 1$$

for $0 < \theta < 180$. (5 marks)