Worked Solutions

AQA C4 Paper D

1. $\dfrac{\cancel{(2x-3)}\ \cancel{(x+2)}}{\cancel{(2x-3)}(2x+3)} \times \dfrac{(x-1)}{(x+2)} = \dfrac{x-1}{(2x+3)(x+2)}$ *(4 marks)*

2. (a) $\cos^3 x = \cos x \cdot \cos^2 x$

$$= \cos x(1 - \sin^2 x)$$

$$= \cos x - \cos x \sin^2 x$$ *(1 mark)*

(b) $\dfrac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \cdot \cos x$ *(1 mark)*

(c) $\displaystyle\int \left(\cos x - \cos x \sin^2 x\right)\ dx = \sin x - \frac{1}{3}\sin^3 x + c$ *(3 marks)*

3. Given $\dfrac{dS}{dt} = 640\ \text{cm}^2\ \text{s}^{-1}$. To find $\dfrac{dr}{dt}$.

$S = 4\pi r^2$

$\dfrac{dS}{dr} = 8\pi r$

$\dfrac{dS}{dt} = \dfrac{dS}{dr} \times \dfrac{dr}{dt}$

when $r = 5$,

$640 = 8\pi \times 5 \times \dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{640}{40\pi}$

$= \dfrac{16}{\pi}\ \text{cm s}^{-1}$ *(4 marks)*

4. (a) $2x + 2y\dfrac{dy}{dx} - 2 + 4\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(2y+4) = 2 - 2x$

$\dfrac{dy}{dx} = \dfrac{2-2x}{2y+4} = \dfrac{1-x}{y+2}$ *(2 marks)*

(b) at $(4, 2)$ gradient of tangent $= \dfrac{1-4}{2+2} = -\dfrac{3}{4}$

equation of tangent is $y - 2 = -\dfrac{3}{4}(x-4)$

$4y + 3x = 20$ *(4 marks)*

5. (a) $x = \dfrac{\ln 11}{\ln 5}\left(\text{or } \dfrac{\log 11}{\log 5}\right) = 1.49$ *(2 marks)*

(b) $y = 3^x$

$\ln y = \ln 3^x$

$\ln y = x \ln 3$

$\dfrac{1}{y}\dfrac{dy}{dx} = \ln 3$

$\dfrac{dy}{dx} = y \ln 3 = 3^x \ln 3$ *(3 marks)*

(c) $e^{-0.4x} = 5$

$\ln e^{-0.4x} = \ln 5$

$-0.4x = \ln 5$

$x = -\dfrac{\ln 5}{0.4} = -4.02$ *(2 marks)*

6. (a) $$\text{L.H.S.} = \frac{\cos A\cos B + \sin A\sin B - (\cos A\cos B - \sin A\sin B)}{\sin A\cos B + \cos A\sin B - (\sin A\cos B - \cos A\sin B)}$$

$$= \frac{2\sin A\sin B}{2\cos A\sin B} = \tan A$$ *(6 marks)*

(b) $$\tan 75° = \frac{\cos(75° - 15°) - \cos(75° + 15°)}{\sin(75° + 15°) - \sin(75° - 15°)}$$

$$= \frac{\cos 60° - \cos 90°}{\sin 90° - \sin 60°}$$

$$= \frac{\frac{1}{2} - 0}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= 2 + \sqrt{3}$$ *(4 marks)*

7. (a) $$\frac{dy}{dt} = 4e^{4t} - 3, \qquad \frac{dx}{dt} = 4e^{2t} - 1$$

$$\frac{dy}{dx} = \frac{4e^{4t} - 3}{4e^{2t} - 1}$$ *(3 marks)*

(b) gradient $= 3$, $$\frac{4e^{4t} - 3}{4e^{2t} - 1} = 3$$

$$4e^{4t} - 3 = 12e^{2t} - 3$$

$$e^{4t} = 3e^{2t}$$

$$e^{2t} = 3$$

$$2t = \ln 3$$

$$t = \frac{1}{2}\ln 3$$ *(5 marks)*

8. (a) P: $2\mathbf{i} + \mathbf{k}$, Q: $\mathbf{j} + 2\mathbf{k}$ *(2 marks)*

(b) $\overrightarrow{QP} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ equation of line QP is $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ *(2 marks)*

(c) let $\angle OPQ = \theta$

$$\overrightarrow{PO} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}, \overrightarrow{PQ} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix},$$

$$\left|\overrightarrow{PO}\right| = \sqrt{5}, \left|\overrightarrow{PQ}\right| = \sqrt{6}$$

$$\begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \sqrt{5}\sqrt{6}\cos\theta$$

$$3 = \sqrt{5}\sqrt{6}\cos\theta$$

$$\theta = 56.8°$$ *(3 marks)*

9. (a) (i) $\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1 + \sin 2\theta$ *(2 marks)*

(ii) $$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + \sin 2\theta)\ d\theta = \left[\theta - \frac{1}{2}\cos 2\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - \frac{1}{2}\cos\pi - \left(\frac{\pi}{4} - \frac{1}{2}\cos\frac{\pi}{2}\right)$$

$$= \frac{\pi}{2} - \frac{1}{2}(-1) - \frac{\pi}{4} + 0 = \frac{\pi}{4} - \frac{1}{2}$$ *(3 marks)*

(b) $$\int_0^1 (2x + 1)^4 dx = \left[\frac{1}{5} \cdot \frac{1}{2}(2x + 1)^5\right]_0^1 = \frac{1}{10}(3^5 - 1) = 24.2$$ *(4 marks)*

10. (a) $\dfrac{3+5x-x^2}{(2-x)(1+x)^2}$

$$\equiv \frac{A(1+x)^2 + B(2-x)(1+x) + C(2-x)}{(2-x)(1+x)^2}$$

$3+5x-x^2 \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$

let $x=-1$, $3-5-1 = C(2-(-1)), C=-1$

let $x=2$, $3+10-4 = A(1+2)^2, A=1$

constants, $3 = A+2B+2C, B=2$ *(4 marks)*

(b) $\displaystyle\int_0^1 \left(\frac{1}{2-x} + \frac{2}{1+x} - \frac{1}{(1+x)^2}\right) \mathrm{d}x$

$$= \left[-\ln(2-x) + 2\ln(1+x) + \frac{1}{(1+x)}\right]_0^1$$

$$= -\ln 1 + 2\ln 2 + \frac{1}{2} - (-\ln 2 + 2\ln 1 + 1)$$

$$= 3\ln 2 - \frac{1}{2}$$ *(6 marks)*

(c) $\dfrac{1}{2-x} = \dfrac{1}{2\left(1-\frac{x}{2}\right)} = \dfrac{1}{2}\left(1-\dfrac{x}{2}\right)^{-1}$

$$= \frac{1}{2}\left[1 + (-1)\left(-\frac{x}{2}\right) + \frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{3.2}\left(-\frac{x}{2}\right)^3 + \dots\right]$$

$$= \frac{1}{2}\left[1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}\right] = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

$$\mathrm{f}(x) = \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16} + 2(1+x)^{-1} - (1+x)^{-2}$$

$$= \frac{1}{2} + \frac{x}{4} + \frac{x^2}{8} + \frac{x^3}{16}$$

$$+2\left[1 + (-1)x + \frac{(-1)(-2)}{2}x^2 + \frac{(-1)(-2)(-3)}{3.2}x^3 + \dots\right]$$

$$-\left[1 + (-2)x + \frac{(-2)(-3)}{2}x^2 + \frac{(-2)(-3)(-4)}{3.2}x^3 + \dots\right]$$

$$= \left(\frac{1}{2}+2-1\right) + x\left(\frac{1}{4}-2+2\right) + x^2\left(\frac{1}{8}+2-3\right) + x^3\left(\frac{1}{16}-2+4\right)$$

$$= \frac{3}{2} + \frac{1}{4}x - \frac{7}{8}x^2 + \frac{33}{16}x^3$$ *(4 marks)*

(d) valid for $-1 < x < 1$ *(1 mark)*