M1 June 2016

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1. (a) Q's velocity vector is $20\mathbf{i} - 5\mathbf{j}$. Let θ be the angle between the horizontal and Q's velocity, i.e $\tan\theta = \frac{5}{20}$ So $\theta \approx 14$ to the nearest degree. Hence the bearing is simply $\theta + 90° = 104°$

(3)

(b) $\mathbf{p} = 400\mathbf{i} + t(15\mathbf{i} + 20\mathbf{j})$ which simplifies to $\mathbf{p} = (400 + 15t)\mathbf{i} + 20t\mathbf{j}$. Likewise with Q $\mathbf{q} = 800\mathbf{j} + t(20\mathbf{i} - 5\mathbf{j})$ which simplifies to $\mathbf{q} = 20t\mathbf{i} + (800 - 5t)\mathbf{j}$ (3)

(c) If Q is west of P, then the $\mathbf{j}$ components for $\mathbf{p}$ and $\mathbf{q}$ are the same. Hence $20t = 800 - 5t$ so $t = 32$.
By subbing $t = 32$ into $\mathbf{q}$ then we will get that the position vector of Q. Hence

$$\mathbf{q} = 640\mathbf{i} + 640\mathbf{j}$$

(4)

2. (a) Consider the system together, meaning that the three forces are the weight of the scale pan, the weight of the brick and the tension. Hence

$$\Re(\uparrow) : T - 2g = 2(0.5)$$

Rearranging gives

$$T = 20.6N \approx 21N$$

(3)

(b) Now consider simply the brick. The brick has a mass of its own, but also has a reactional force due to being in contact with the scale pan, let that force be called R. Hence

$$\Re(\uparrow) : R - 1.5g = 1.5(0.5)$$

Rearranging gives

$$R = 15.45N \approx 15.5N$$

(3)

3. (a) To find the Impulse we only simply need the rebound speed of the particle as we already know its mass and the speed it hit the wall. To find the speed we will first need to consider its motion after rebounding. If we consider the forces, friction will act opposite to the motion of the particle. The reactional force of the particle to the ground will be $0.4g$ and so the Frictional Force will be $0.49N$ Hence

$$\Re(\rightarrow) : 0.49 = 0.4a$$

Hence

$$a = 1.225ms^{-2}$$

Remember this actually a deceleration. Now using the suvat equation $v^2 = u^2 + 2as$ and using $v = 0, a = -1.225, s = 5$. Solving this gives

$$0 = u^2 - 12.25$$

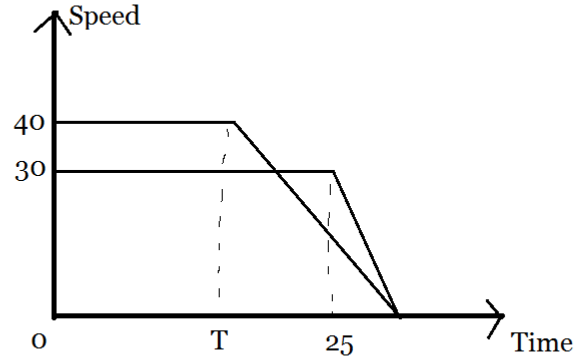
$$u = 3.5ms^{-1}$$

Now knowing the speed we can simply use the equation of impulse or the change in momentum, $I = m(v - u)$where $m = 0.4, v = 3.5andu = -4$ Notice -4, this is because their directions are opposite. Hence

$$I = 0.4(3.5 + 4) = 3Ns$$

(7)

4. (a) The graph looks as follows



(4)

(b) We know they travel the same distance which is given to be 975m, they also take the same time to get to Y from X. We can find this time required for train N. By considering the area and let T_n be the whole time of the journey. Then

$$975 = \frac{1}{2}(25 + T_n)(30)$$

Rearranging gives

$$65 = 25 + T_n$$

$$T_n = 40$$

Now that we know it took 40 seconds for N, it must also take 40 seconds for M, so using the same idea again

$$975 = \frac{1}{2}(T + 40)(40)$$

Rearranging gives

$$48.75 = 40 + T$$

Hence

$$T = 8.75$$

(8)

5. (a) We need to find the normal reaction to the plane so if we resolve perpendicular to the plane we get

$$\Re(\nwarrow) : R = 2g\cos 20^\circ + 40\sin 30^\circ \approx 38.4N$$

Hence we now know that $F \approx 38.4\mu$.
Now let's resolve along the plane as such

$$\Re(\nearrow) : 40\cos 30^\circ - F - 2gsin20^\circ = 0$$

Rearranging and writing $F = \mu R$ we get

$$\mu R = 40\cos 30^\circ - 2g\sin 20^\circ$$

Hence by dividing by R and substituting our above expression of R we get

$$\mu = \frac{40\cos 30^\circ - 2g\sin 20^\circ}{2g\cos 20^\circ + 40\sin 30^\circ} \approx 0.727$$

(10)

6. (a) Let us imagine the first scenario, the normal reaction at T just be zero and hence by resolving perpendicular to the rod, we get the

$$R_s = (30 + M)g$$

Now take moments about A

$$M(A) : 0.5(30 + M)g = (30g)d$$

making M the subject, we get

$$M = 60d - 30$$

Now consider the second scenario. This time the normal reaction at S will be zero and so

$$R_t = (30 + M)g$$

now taking moments about A again

$$M(A) : (30g)d + 6(Mg) = 4(30 + M)g$$

Rearranging gives

$$M = 60 - 15d$$

Now we equate both equations gives

$$75d = 90$$

Hence

$$d = 1.2$$

Putting d back into any of the equations gives

$$M = 60(1.2) - 30 = 42$$

(7)

7. (a) We can let $\mathbf{F_2} = \lambda(\mathbf{i} + \mathbf{j})$ and also our resultant force, $\mathbf{R} = \mu(\mathbf{i} + 3\mathbf{j})$
By resolving forces both horizontal and vertically we get two equations

$$\lambda - 1 = \mu$$

and

$$2 + \lambda = 3\mu$$

Solving for λ gives

$$2 + \lambda = 3(\lambda - 1)$$

hence

$$2\lambda = 5$$
$$\lambda = \frac{5}{2}$$

As we don't need to find μ we shall then just put λ back into our force equation, hence

$$\mathbf{F_2} = \frac{5}{2}\mathbf{i} + \frac{5}{2}\mathbf{j}$$

(7)

(b) We have all the information we need, we now just use the equation $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ where $\mathbf{u} = 3\mathbf{i} - 22\mathbf{j}$, $\mathbf{a} = 3\mathbf{i} + 9\mathbf{j}$ and $t = 3$. Hence

$$\mathbf{v} = 3\mathbf{i} - 22\mathbf{j} + 3(3\mathbf{i} + 9\mathbf{j}) = 12\mathbf{i}5\mathbf{j}$$

so

$$|\mathbf{v}| = \sqrt{12^2 + 5^2} = 13$$

(4)

8. (a) First we find the normal reaction to the plane. i.e $N = 1.5g$ and hence the frictional force becomes $F = 2.94N$ Now let us obtain an equation of motion for the particle on the plane,

$$\Re(\rightarrow) : T - 2.94 = 1.5a$$

and now consider the equation of motion for the particle B attached to the rope

$$\Re(\downarrow) : 3g - T = 3a$$

If we add both equations we obtain

$$3g - 2.94 = 4.5a$$

solving for a gives

$$a = 5.88ms^{-2}$$

Now sub a back into any of the equations of motion, Hence

$$T = 1.5(5.88) + 2.94 = 11.76 \approx 11.8N$$

(8)

(b) You either use pythagoras or resolve along the resultant force produced, if we resolve then we must consider that there is an angle of 45° between the resultant force and **both** tensions. By resolving

$$\Re(\swarrow) : R = 2T\cos(45^\circ) \approx 16.6N$$

or by pythagoras

$$R = \sqrt{T^2 + T^2} \approx 16.6N$$

For the direction, the resultant force acts south west from the pulley and making an angle of 45° to both the plane and the string attached to B.

(4)