① a. $f(x) = 7x - 1$

$g(x) = \frac{4}{x-2}$

$f(g(x)) = \frac{28}{x-2} - 1 = x$

$(x-2)(x+1) = 28$

$x^2 - x - 2 = 28$

$x^2 - x - 30 = 0$

$x = -5, \ 6$

b. highest $(-5, 6) = 6$

② $\frac{d}{dx}\left(4x(x^2+5)^{-1}\right)$

$$= \frac{4}{(x^2+5)^1} - \frac{8x^2}{(x^2+5)^2}$$

$$= \frac{4(x^2+5) - 8x^2}{(x^2+5)^2} = \frac{20-4x^2}{(x+5)^2}$$

b. $20 - 4x^2 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

$x > \sqrt{5}$ $\quad$ $x < -\sqrt{5}$

test e.g. $x = 0$,

$\frac{dy}{dx} > 0$

③ a. $2\cos\theta \quad -\sin\theta$

$$R = \sqrt{2^2+1^2} = \sqrt{5}$$

e.g. $\theta = 0$

$$\sqrt{5}\cos\alpha = 2$$

$$\cos\alpha = \frac{2}{\sqrt{5}}$$

$$\alpha = 26.57^\circ$$

b. $$\frac{2}{\sqrt{5}\cos(\theta + 26.57^\circ) - 1} = 15$$

$$15\sqrt{5}\cos(\theta + 26.57^\circ) = 17$$

$$\cos(\theta + 26.57^\circ) = \frac{17}{15\sqrt{5}}$$

$$\theta + 26.57^\circ = 59.546^\circ,\ 300.453^\circ\ldots$$

$$\theta = 33.0^\circ,\ 273.9^\circ$$

4 a i. $y = |4e^{2x} - 25|$

$x = 0 \quad y = |4 - 25| = 21$

ii. $y = 0$

$4e^{2x} = 25$

$e^{2x} = \frac{25}{4}$

$2x = \ln\left(\frac{25}{4}\right)$

$x = \ln\sqrt{\frac{25}{4}} = \ln\frac{5}{2}$

iii. Asymptote:

As $4e^{2x} \longmapsto 0$

$y \longmapsto |-25| = 25$

$k = 25$

b. $|4e^{2x} - 25| = 2x + 43$

$4e^{2x} = 2x + 68$

$e^{2x} = \frac{x}{2} + 17$

$2x = \ln\left(\frac{x}{2} + 17\right)$

$x = \frac{1}{2}\ln\left(\frac{x}{2} + 17\right)$

c. $x_0 = 1.4 \qquad x_{n+1} = \frac{1}{2}\ln\left(\frac{x_n}{2} + 17\right)$

$x_1 = 1.4368$

$x_2 = 1.4373$

d. let $f(x) = |4e^{2x} - 25| - 2$

$f(1.4365)$ -ive

$f(1.4375)$ +ive

change of sign

$\Rightarrow$ root

(5) i. $y = e^{3x} \cos 4x$

$$\frac{dy}{dx} = 3e^{3x}\cos 4x - 4e^{3x}\sin 4x = 0$$

$$4 \sin 4x = 3\cos 4x$$

$$\tan 4x = \frac{3}{4}$$

$$4x = 0.6435\ldots,\ 3.78509\ldots$$

$$0.785\ldots \le x < 1.571\ldots$$

$$x = 0.9463$$

ii. $x = \sin^2 2y \qquad (\sin 2y)^2$

$$2 \times 2\cos 2y \sin 2y$$

$$\frac{dx}{dy} = 4\sin 2y \cos 2y = 2\sin 4y$$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\operatorname{cosec} 4y$$

(6) a. $x^2 + x - 6 \overline{\big)\, x^4 + x^3 - 3x^2 + 7x - 6}$

$$x^2 \qquad x^4 + x^3 - 6x^2$$

$$3x^2 + 7x - 6$$

$$3 \qquad 3x^2 + 3x - 18$$

$$4x + 12$$

$$x^2 + 3 + \frac{4x + 12}{x^2 + x - 6}$$

$$= x^2 + 3 + \frac{4(x+3)}{(x+3)(x-2)} = x^2 + 3 + 4(x-2)^{-1}$$

b. $\frac{dy}{dx} = 2x - 4(x-2)^{-2}$

$$x = 3 \qquad \frac{dy}{dx} = 6 - 4(1)^{-2} = 2$$

$$m = -\frac{1}{2}$$

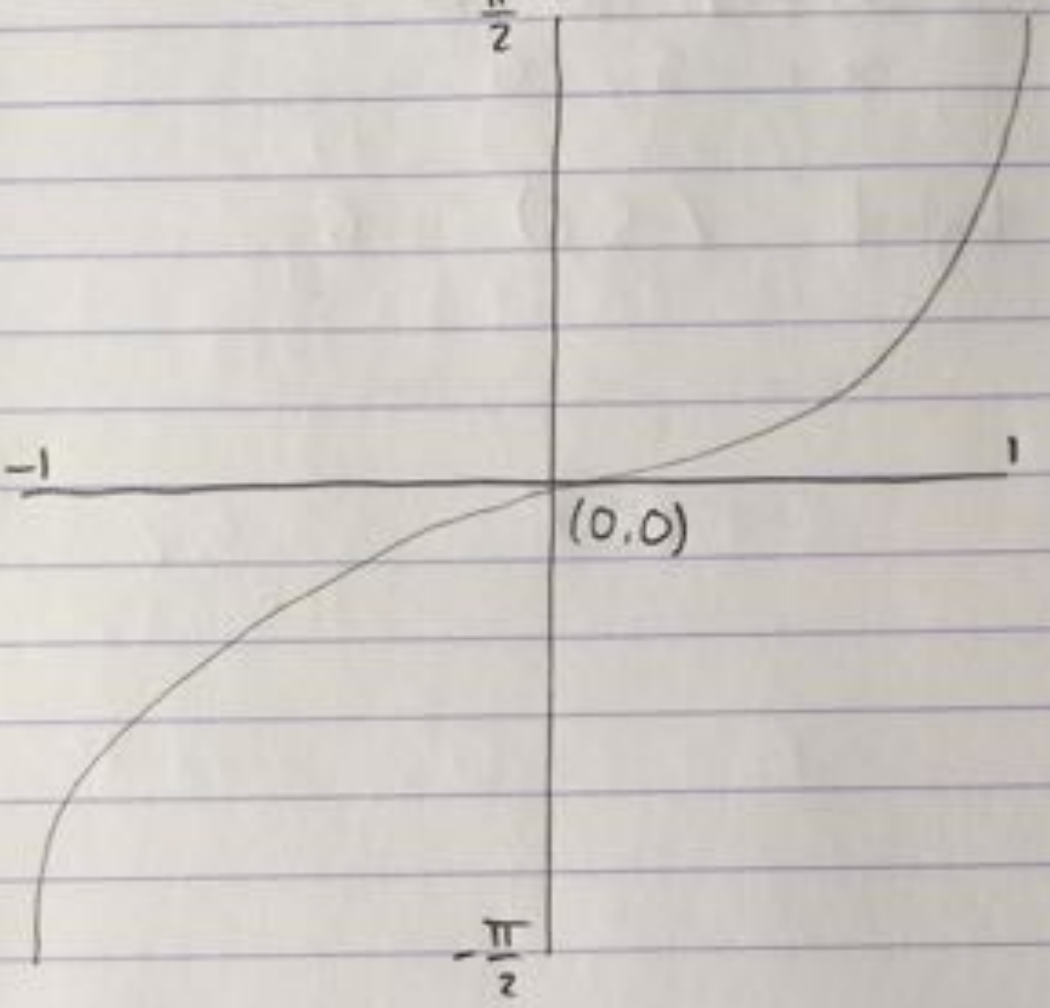
$$y = 9 + 3 + 4(1)^{-1} = 16$$

$$y - 16 = -\frac{1}{2}(x - 3)$$

$$= -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{35}{2}$$

7 a.



b. $3\sin^{-1}(x+1) = -\pi$

$$\sin^{-1}(x+1) = -\frac{\pi}{3}$$

$$x+1 = \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$x = -1-\frac{\sqrt{3}}{2}$$

(8) a. $\dfrac{2\cos 2x}{\sin 2x} + \dfrac{\sin x}{\cos x}$

$$= \frac{2(\cos^2 x - \sin^2 x) + 2\sin^2 x}{2\sin x \cos x} = \frac{2\cos^2 x}{2\sin x \cos x} = \frac{\cos x}{\sin x}$$

$$= \cot x$$

b. $3\cot x = \operatorname{cosec}^2 x - 2$

$$-2 + \operatorname{cosec}^2 x = 1 + \cot^2 x - 2$$

$$\cot^2 x - 3\cot x - 1 = 0$$

$$\cot x = \frac{3 \pm \sqrt{13}}{2}$$

$$\tan x = \frac{2}{3 \pm \sqrt{13}}$$

$$x = -2.848,\ -1.277,\ 0.294,\ 1.865$$

(add or subtract π to initial answers)

9 a. $x = De^{-0.2t}$

$D = 15$, $t = 4$

$x = 15e^{-0.8} = 6.740$ 3d.p

b. $(D = 15, t = 7) + (D = 15, t = 2)$

$= 15e^{-1.4} + 15e^{-0.4}$

$= 13.754$ to 3.d.p.

c. $15e^{-0.2T} + 15e^{-0.2(T+5)} = 7.5$

$15(e^{-0.2T} + e^{-1}(e^{-0.2T})) = 7.5$

$e^{-0.2(T+5)} = e^{-0.2T-1} = e^{-1}(e^{-0.2T})$

$e^{-0.2T}(1 + e^{-1}) = \frac{1}{2}$

$e^{-0.2T} = \frac{1}{2(1+e^{-1})}$

$-0.2T = \ln \frac{1}{2(1+e^{-1})}$

$0.2T = \ln(2 + 2e^{-1})$

$T = 5\ln(2 + 2e^{-1})$