Paper Reference(s)

9801/01

Edexcel

Mathematics

Advanced Extension Award

Thursday 30 June 2016 – Morning

Time: 3 hours

Materials required for examination	**Items included with question papers**
Answer book (AB16)	Nil
Graph paper (ASG2)	
Mathematical Formulae (Pink)	

Candidates may NOT use a calculator in answering this paper.

Instructions to Candidates

In the boxes on the answer book provided, write the name of the examining body (Edexcel), your centre number, candidate number, the paper title (Mathematics), the paper reference (9801), your surname, initials and signature.
Check that you have the correct question paper.
Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{6}$, $3\sqrt{2}$.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 100, of which 7 marks are for style, clarity and presentation.
There are 8 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

1. The function f is given by

$$f(x) = x^2 - 4x + 9 \qquad x \in \mathbb{R},\ x \geqslant 3$$

(a) Find the range of f.

(2)

The function g is given by

$$g(x) = \frac{10}{x+1} \qquad x \in \mathbb{R},\ x \geqslant 4$$

(b) Find an expression for gf(x).

(1)

(c) Find the domain and range of gf.

(4)

(Total 7 marks)

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2. Find the value of

$$\arccos\left(\frac{1}{\sqrt{2}}\right) + \arcsin\left(\frac{1}{3}\right) + 2\arctan\left(\frac{1}{\sqrt{2}}\right)$$

Give your answer as a multiple of π.

(arccos x is an alternative notion for $\cos^{-1} x$ etc.)

(Total 7 marks)

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3. The points A, B, C, D and E are five of the vertices of a rectangular cuboid and AE is a diagonal of the cuboid. With respect to a fixed origin O, the position vectors of A, B, C and D are **a**, **b**, **c** and **d** respectively, where

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix},\ \mathbf{b} = \begin{pmatrix} 0 \\ -3 \\ -8 \end{pmatrix},\ \mathbf{c} = \begin{pmatrix} 4 \\ -1 \\ -10 \end{pmatrix} \text{ and } \mathbf{d} = \begin{pmatrix} -4 \\ 2 \\ -11 \end{pmatrix}$$

(a) Find the position vector of E.

(6)

The volume of a tetrahedron is given by the formula

$$\text{volume} = \frac{1}{3}(\text{area of base}) \times (\text{height})$$

(b) Find the volume of the tetrahedron $ABCD$.

(3)

(Total 9 marks)

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4. (a) Given that $x > 0$, $y > 0$, $x \neq 1$ and $n > 0$, show that

$$\log_x y = \log_{x^n} y^n$$

(2)

(b) Solve the following, leaving your answers in the form 2^p, where p is a rational number.

(i) $\log_2 u + \log_4 u^2 + \log_8 u^3 + \log_{16} u^4 = 5$

(ii) $\log_2 v + \log_4 v + \log_8 v + \log_{16} v = 5$

(iii) $\log_4 w^2 + \dfrac{3\log_8 64}{\log_2 w} = 5$

(9)

(Total 11 marks)

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5. (a) Show that

$$\sum_{r=0}^{n} x^{-r} = \frac{x}{x-1} - \frac{x^{-n}}{x-1} \qquad \text{where } x \neq 0 \text{ and } x \neq 1$$

(2)

(b) Hence find an expression in terms of x and n for $\sum_{r=0}^{n} rx^{-(r+1)}$ for $x \neq 0$ and $x \neq 1$

Simplify your answer.

(4)

(c) Find $\sum_{r=0}^{n} \left(\dfrac{3+5r}{2^r}\right)$

Give your answer in the form $a - \dfrac{b+cn}{2^n}$, where a, b and c are integers.

(7)

(Total 13 marks)

6.

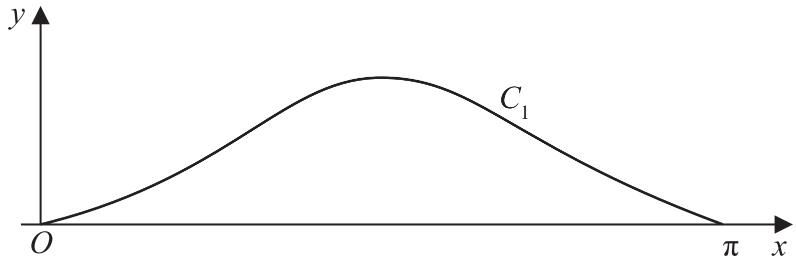

Figure 1

Figure 1 shows a sketch of the curve C_1 with equation

$$y = \cos(\cos x) \sin x \qquad \text{for} \qquad 0 \leqslant x \leqslant \pi$$

(a) Find $\dfrac{dy}{dx}$ **(3)**

(b) Hence verify that the turning point is at $x = \dfrac{\pi}{2}$ and find the y coordinate of this point. **(2)**

(c) Find the area of the region bounded by C_1 and the positive x-axis between $x = 0$ and $x = \pi$ **(4)**

Figure 2 shows a sketch of the curve C_1 and the curve C_2 with equation

$$y = \sin(\cos x) \sin x \qquad \text{for} \qquad 0 \leqslant x \leqslant \pi$$

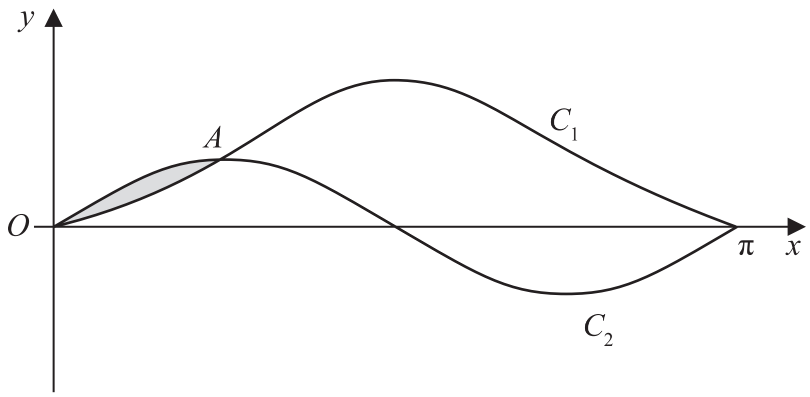

Figure 2

The curves C_1 and C_2 intersect at the origin and the point $A(a, b)$, where $a < \pi$

(d) Find a and b, giving b in a form not involving trigonometric functions. **(5)**

(e) Find the area of the shaded region between C_1 and C_2 **(8)**

(Total 22 marks)

7. (a) Find the set of values of k for which the equation

$$\frac{x^2 + 3x + 8}{x^2 + x - 2} = k$$

has no real roots.

(6)

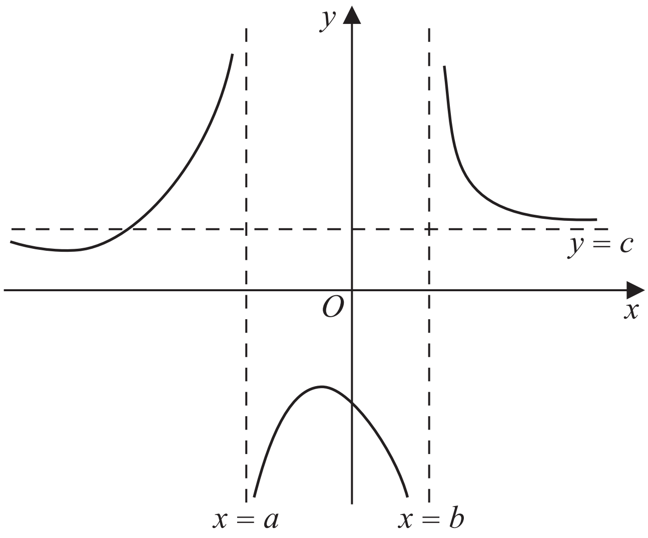

Figure 3

Figure 3 shows a sketch of the curve C_1 with equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = \dfrac{x^2 + 3x + 8}{x^2 + x - 2}$

The curve has asymptotes $x = a$, $x = b$ and $y = c$, where a, b and c are integers.

(b) Find the value of a, the value of b and the value of c.

(4)

(c) Find the coordinates of the points of intersection of C_1 with the line $y = 2$

(3)

(d) Find all the integer pairs (r, s) that satisfy $s = \dfrac{r^2 + 3r + 8}{r^2 + r - 2}$

(4)

The curve C_2 has equation $y = \mathrm{g}(x)$ where $\mathrm{g}(x) = \dfrac{2x^2 - 4x + 6}{x^2 - 3x}$

(e) Show that, for suitable integers m and n, $\mathrm{g}(x)$ can be written in the form $\mathrm{f}(x + m) + n$.

(4)

(f) Sketch C_2 showing any asymptotes and stating their equations.

(3)

(Total 24 marks)

FOR STYLE, CLARITY AND PRESENTATION: 7 MARKS

TOTAL FOR PAPER: 100 MARKS

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