

$$2) (x-a)^2 \mid p(x)$$

$$\Rightarrow p(x) = g(x) (x-a)^2 \quad \text{where } g(x) \text{ is a function in } x$$

$$\Rightarrow p'(x) = g'(x) (x-a)^2 + 2(g(x))(x-a)$$

$$\Rightarrow p'(a) = g'(a) (0)^2 + 2g(a)(0)$$

$$= 0$$

$$\text{Consider } p(x) = h(x) (x-a)^4 \quad \text{where } h(x) \text{ is a function in } x$$

$$\Rightarrow p'(x) = h'(x) (x-a)^4 + 4(x-a)^3 h(x)$$

$$\Rightarrow p''(x) = h''(x) (x-a)^4 + 4h'(x) (x-a)^3 + 4(x-a)^3 h'(x) + 12h(x) (x-a)^2$$

$$= h''(x) (x-a)^4 + 8h'(x) (x-a)^3 + 12h(x) (x-a)^2$$

$$\Rightarrow p'''(x) = h'''(x) (x-a)^4 + 24h''(x) (x-a)^3 + 8h''(x) (x-a)^3 + 4h'''(x) (x-a)^3 + 24h'(x) (x-a)^2 + 12h'(x) (x-a)^2 + 12h''(x) (x-a)^3 + 36h'(x) (x-a)^2 + 24h(x) (x-a)$$

$$\Rightarrow p'''(a) = h'''(a) (0)^4 + 12h''(a) (0)^3 + 36h'(a) (0)^2 + 24h(a) (0)$$

$$= 0$$

$$\therefore (x-a)^4 \mid p(x) \Rightarrow p'''(a) = 0$$

$$\frac{d^3}{dx^3} [x^6 + 4x^5 - 5x^4 - 40x^3 - 40x^2 + 32x + k]$$

$$= 120x^3 + 240x^2 - 120x - 240$$

$$= 120(x^3 + 2x^2 - x - 2)$$

$$= 120(x+2)(x^2-1)$$

$$= 120(x+2)(x-1)(x+1)$$

$$\text{Let } p(x) = x^6 + 4x^5 - 5x^4 - 40x^3 - 40x^2 + 32x + k$$

$$\therefore p'''(x) = 0 \quad \text{if } x = -2, x = 1 \text{ or } x = -1$$

$$\begin{aligned} \text{Consider } p'(x) &= 6x^5 + 20x^4 - 20x^3 - 120x^2 - 80x + 32 \\ &= (x+2)(6x^4 + 8x^3 - 36x^2 - 48x + 16) \\ &= (x+2)f(x) \end{aligned}$$

$$\text{where } f(x) = 6x^4 + 8x^3 - 36x^2 - 48x + 16$$

$$\therefore f(1) = -54$$

$$\& f(-1) = 26$$

$$p'(1) \neq 0 \quad \& \quad p'(-1) \neq 0$$

$$\therefore x = -2 \quad \because p'''(-2) = 0 \quad \& \quad p'(-2) = 0$$

$$\therefore (x+2)^4 \mid p(x)$$

$$\Rightarrow p(-2) = 0$$

$$\begin{aligned} \therefore 64 + 4(-32) - 5(16) - 40(-8) - 40(4) + 32(-2) + k &= 0 \\ -64 + 240 - 224 + k &= 0 \end{aligned}$$

$$\begin{aligned} \therefore k &= 288 - 240 \\ &= 48 \end{aligned}$$