1. (a) $\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$ M1 B1 A1 aef 3

Note

M1: An attempt to factorise the numerator.
B1: Correct factorisation of denominator to give $(x + 5)(x - 3)$.
Can be seen anywhere.

(b) $\ln\left(\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ M1

$\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15} = \mathrm{e}$ dM1

$\dfrac{2x - 1}{x - 3} = \mathrm{e} \Rightarrow \quad 3\mathrm{e} - 1 = x(\mathrm{e} - 2)$ M1

$\Rightarrow \quad x = \dfrac{3\mathrm{e} - 1}{\mathrm{e} - 2}$ A1 aef cso 4

Note

M1: Uses a correct law of logarithms to combine at least two terms.
This usually is achieved by the subtraction law of logarithms to give
$\ln\left(\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$
The product law of logarithms can be used to achieve
$\ln(2x^2 + 9x - 5) = \ln(\mathrm{e}(x^2 + 2x - 15))$.
The product and quotient law could also be used to achieve
$\ln\left(\dfrac{2x^2 + 9x - 5}{e(x^2 + 2x - 15)}\right) = 0$

dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e.
Note that this mark is dependent on the previous method mark being awarded.
M1: Collect x terms together and factorise.
Note that this is not a dependent method mark.

A1: $\dfrac{3e-1}{e-2}$ or $\dfrac{3e^1-1}{e^1-2}$ or $\dfrac{1-3e}{2-e}$. **aef**

Note that the answer needs to be in terms of e. The decimal answer is 9.9610559…

Note that the solution must be correct in order for you to award this final accuracy mark.

[7]

2. $\dfrac{x+1}{3x^2-3} - \dfrac{1}{3x+1}$

$= \dfrac{x+1}{3(x^2-3)} - \dfrac{1}{3x+1}$

	$x^2 - 1 \rightarrow (x + 1)(x - 1)$ or	
	$3x^2 - 3 \rightarrow (x + 1)(3x - 3)$ or	**Award**
$\dfrac{x+1}{3(x+1)(x-1)} - \dfrac{1}{3x+1}$	$3x^2 - 3 \rightarrow (3x + 3)(x - 1)$	**below**
	seen or implied anywhere in candidate's working.	
$= \dfrac{1}{3(x-1)} - \dfrac{1}{3x+1}$		
$= \dfrac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	Attempt to combine.	M1
or $\dfrac{3x+1}{3(x-1)(3x+1)} - \dfrac{3(x-1)}{3(x-1)(3x+1)}$	Correct result.	A1
	Decide to award M1 here!!	**M1**
	Either $\dfrac{4}{3(x-1)(3x+1)}$	
$= \dfrac{4}{3(x-1)(3x+1)}$	or $\dfrac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\dfrac{4}{(3x-3)(3x+1)}$	A1 **aef**
	or $\dfrac{4}{9x^2-6x-3}$	

[4]

3. (a) $f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$

$x \in \mathbb{R}, x \neq -4, x \neq 2.$

$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ An attempt to combine to one fraction M1

Correct result of combining all three fractions A1

$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$

$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$ Simplifies to give the correct numerator. Ignore omission of denominator A1

$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$ An attempt to factorise the numerator. dM1

$= \dfrac{(x-3)}{(x-2)}$ Correct result A1 cso **AG** 5

(b) $g(x) = \dfrac{e^x - 3}{e^x - 2}$ $x \in \mathbb{R}, x \neq \ln 2.$

Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{array} \right\}$

$g'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ Applying $\dfrac{vu' - uv'}{v^2}$ M1

Correct differentiation A1

$= \dfrac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$

$= \dfrac{e^x}{(e^x - 2)^2}$ Correct result A1 **AG**

cso 3

(c) $g'(x) = 1 \Rightarrow \; = \dfrac{e^x}{(e^x - 2)^2} = 1$				
$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1		
$e^x = e^{2x} - 2e^x - 2e^x + 4$				
$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1		
$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1		
$e^x = 4$ or $e^x = 1$				
$x = \ln 4$ or $x = 0$	both $x = 0, \ln 4$	A1	4	
				[12]

4. (a)

$\dfrac{2x+2}{x^2-2x-3} - \dfrac{x+1}{x-3} = \dfrac{2x+2}{(x-3)(x+1)} - \dfrac{x+1}{x-3}$			
$= \dfrac{2x+2-(x+1)(x+1)}{(x-3)(x+1)}$		M1 A1	
$= \dfrac{(x+1)(1-x)}{(x-3)(x+1)}$		M1	
$\dfrac{1-x}{x-3}$	Accept $-\dfrac{x-1}{x-3}, \dfrac{x-1}{3-x}$	A1	4

Alternative

$\dfrac{2x+2}{x^2-2x-3} = \dfrac{2(x+1)}{(x-3)(x+1)} = \dfrac{2}{x-3}$	M1 A1	
$\dfrac{2}{x-3} - \dfrac{x+1}{x-3} = \dfrac{2-(x+1)}{x-3}$	M1	
$\dfrac{1-x}{x-3}$	A1	4

(b)

$\dfrac{d}{dx}\left(\dfrac{1-x}{x-3}\right) = \dfrac{(x-3)(-1)-(1-x)1}{(x-3)^2}$		M1 A1	
$= \dfrac{-x+3-1+x}{(x-3)^2} = \dfrac{2}{(x-3)^2}$ *	cso	A1	3

Alternatives

(1) $\mathrm{f}(x) = \dfrac{1-x}{x-3} = -1 - \dfrac{2}{x-3} = -1 - 2(x-3)^{-1}$

$\mathrm{f}'(x) = (-1)(-2)(x-3)^{-2}$ M1 A1

$= \dfrac{2}{(x-3)^2}$ * cso A1 3

(2) $\mathrm{f}(x) = (1-x)(x-3)^{-1}$

$\mathrm{f}'(x) = (-1)(x-3)^{-1} + (1-x)(-1)(x-3)^{-2}$ M1

$= -\dfrac{1}{x-3} - \dfrac{1-x}{(x-3)^2} = \dfrac{-(x-3)-(1-x)}{(x-3)^2}$ A1

$= \dfrac{2}{(x-3)^2}$ * A1 3

[7]

5.

$$\begin{array}{r|l}
 & 2x^2 \quad -1 \\ \hline
x^2 - 1 & 2x^4 \quad -3x^2 + x + 1 \\
 & 2x^4 \quad -2x^2 \\ \hline
 & \quad -x^2 + x + 1 \\
 & \quad -x^2 \qquad + 1 \\ \hline
 & \qquad x
\end{array}$$

M1

$a = 2$ stated or implied A1

$c = -1$ stated or implied A1

$2x^2 - 1 + \dfrac{x}{x^2 - 1}$

$a = 2$, $b = 0$, $c = -1$, $d = 1$, $e = 0$

$d = 1$ and $b = 0$, $e = 0$ stated or implied A1

[4]

6. (a) $2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage B1

$f(x) = \dfrac{(2x = 3)(2x-1) - (9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors M1, A1ft

(need not be single fraction)

Simplifying numerator to quadratic form M1

Correct **numerator** $= \dfrac{4x^2 + 2x - 12}{[(2x-1)(x+2)]}$ A1

Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e. M1

$= \dfrac{4x-6}{2x-1}$ (*) A1cso 7

Alt. (a) $2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage B1

$f(x) = \dfrac{(2x+3)(2x^2+3x-2) - (9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ M1A1ft

$= \dfrac{4x^3 + 10x^2 - 8x - 24}{(x+2)(2x^2+3x-2)}$

$= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$

$= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$ or $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$ o.e.

Any one linear factor × quadratic factor in **numerator** M1, A1

$= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$ o.e. M1

$= \dfrac{2(2x-3)}{2x-1}$ $\dfrac{4x-6}{2x-1}$ (*) A1

1^{st} M1 in either version is for correct method

1^{st} A1 Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$

or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ (fractions)

2^{nd} M1 in (main a) is for forming 3 term quadratic in **numerator**

3^{rd} M1 is for factorising their quadratic (usual rules); factor of 2 need not be extracted

(*) A1 is given answer so is cso

Alt: (a) 3^{rd} M1 is for factorising resulting quadratic

(b) Complete method for f′(x); e.g. $f'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e. M1A1

$= \dfrac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$ A1 3

Not treating f^{-1} (for f′) as misread

SC: For M allow ± given expression or one error in product rule
Alt: Attempt at $f(x) = 2 - 4(2x-1)^{-1}$ and diff. M1: $k(2x-1)^{-2}$ A1;
A1 as above
Accept $8(4x^2 - 4x + 1)^{-1}$.
Differentiating original function – mark as scheme.

[10]

7. (a) $f(x) = \dfrac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ M1A1, A1

$= \dfrac{x^2+4x+4-3x-6+3}{(x+2)^2} = \dfrac{x^2+x+1}{(x+2)^2}$* cso A1 4

(b) $x^2 + x + 1 = \left(x+\dfrac{1}{2}\right)^2 + \dfrac{3}{4}, > 0$ for all values of x. M1A1, A1 3

Alternative
$\dfrac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\dfrac{1}{2} \Rightarrow x^2+x+1 = \dfrac{3}{4}$ M1A1
A parabola with positive coefficient of x^2 has a minimum
$\Rightarrow x^2 + x + 1 > 0$
Accept equivalent arguments A1 3

(c) $f(x) = \dfrac{\left(x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}}{(x+2)^2}$

Numerator is positive from (b)
$x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive)
Hence $f(x) > 0$ B1 1

[8]

8. (a) f(–2) = 16 + 8 – 8 (= 16) > 0 B1

f(–1) = 1 + 4 – 8 (= –3) < 0 B1

Change of sign (and continuity) $\Rightarrow$ root in interval (–2, –1) B1ft 3

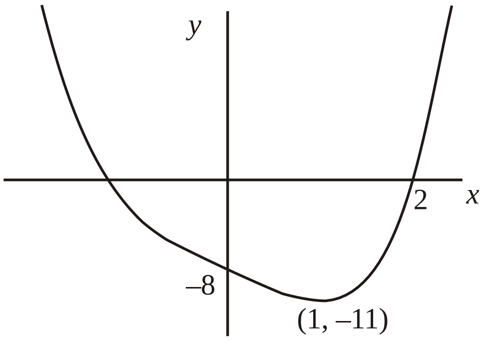
ft their calculation as long as there is a sign change

(b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \Rightarrow x = 1$ M1 A1

Turning point is (1, –11) A1 3

(c) $a = 2$, $b = 4$, $c = 4$ B1 B1 B1 3

(d)

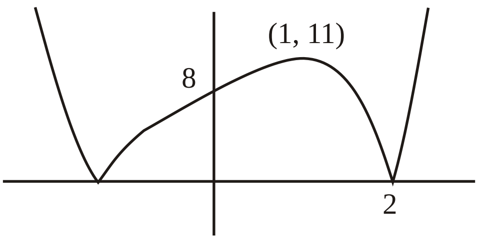


Shape B1

ft their turning point in correct quadrant only B1 ft

2 and –8 B1 3

(e)



Shape B1 1

[13]

9. (a) $\frac{(3x-2)(x-1)}{(x+1)(x-1)}$, $\frac{3x+2}{x+1}$ M1B1, A1 3

M1 attempt to factorise numerator, usual rules

B1 factorising denominator seen anywhere in (a),

A1 given answer

If factorisation of denom. not seen, correct answer implies B1

(b) Expressing over common denominator

$$\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$$ M1

[Or "Otherwise": $\frac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$]

Multiplying out numerator and attempt to factorise M1

[$3x^2 + 2x - 1 \equiv (3x-1)(x+1)$]

$$\frac{3x-1}{x}$$ Al 3

[6

10. $\frac{x(2x+3)}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$ B1, B1

$= \frac{x(x-1)-6}{(x-2)(x+1)}$ M1 A1ft

$= \frac{(x+3)(x-2)}{(x-2)(x+1)}$ M1 A1

$= \frac{x+3}{x+1}$ A1 7

Alternative 1:

$\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$ B1

$= \frac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$ M1 A1ft

$= \frac{(2x^3+5x^2-9x-18)}{(2x+3)(x-2)(x+1)}$ A1

$= \frac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ M1

$= \frac{(x-2)(2x+3)(x+3)}{(2x+3)(x-2)(x+1)}, = \frac{x+3}{x+1}$ A1, A1

Alternative 2:

$$\frac{2x^2+3x}{(2x+3)(x-2)}-\frac{6}{(x^2-x-2)}$$

$$= \frac{x(2x+3x)}{(2x+3)(x-2)}-\frac{6}{x^2-x-2}$$ B1

$$= \frac{x(x^2-x-2)-6(x-2)}{(x-2)(x^2-x-2)}, = \frac{x^3-x^2-2x-6x+12}{(x-2)(x^2-x-2)}$$ M1 A1ft

$$= \frac{x^3-x^2-8x+12}{(x-2)(x^2-x-2)}$$ A1

$$= \frac{(x-2)(x^2+x-6)}{(x-2)(x^2-x-2)}$$ M1

$$= \frac{(x+3)(x-2)}{(x-2)(x+1)}, = \frac{x+3}{x+1}$$ A1, A1

[7]

11. (a) $f(x) = \dfrac{5x+1}{(x+2)(x-1)}-\dfrac{3}{x+2}$ B1

factors of quadratic denominator

$$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$$

common denominator M1

simplifying to linear numerator M1

$$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$$ AG A1cso 4

(b) $y = \dfrac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow$ M1

$xy = 2 + y$ or $x - 1 = \dfrac{2}{y}$ A1

$f^{-1}(x) = \dfrac{2+x}{x}$ or equiv. A1 3

	(c)	$\text{fg}(x) = \dfrac{2}{x^2+4}$ (attempt) $\left[\dfrac{2}{\text{"}g\text{"}-1}\right]$	M1	
		Setting $\dfrac{2}{x^2+4} = \dfrac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	DM1; A1	3
				[10]
12.	(a)	$\dfrac{(x-3)(x+2)}{x(x-3)}; = \dfrac{(x+2)}{x}$ or $1+\dfrac{2}{x}$	B1,B1,B1	3
		B1 numerator, B1 denominator ; *B1 either form of answer*		
	(b)	$\dfrac{(x+2)}{x} = x+1 \Rightarrow x^2 = 2$	M1 A1ft	
		M1 for equating f(x) to x + 1 and forming quadratic. *A1 candidate's correct quadratic*		
		$x = \pm\sqrt{2}$	A1	3
				[6]
13.	(a)	$\dfrac{2x+5}{x+3} - \dfrac{1}{(x+3)(x+2)} = \dfrac{(2x+5)(x+2)-1}{(x+3)(x+2)}$	M1	
		$= \dfrac{2x^2+9x+9}{(x+3)(x+2)}$	A1	
		$= \dfrac{(2x+3)(x+3)}{(x+3)(x+2)}$	M1 A1	
		$= \dfrac{2x+3}{x+2}$	A1	5
	(b)	$2 - \dfrac{1}{x+2} = \dfrac{2(x+2)-1}{x+2} = \dfrac{2x+3}{x+2}$ or the reverse	M1 A1	2
	(c)	T_1: Translation of –2 in x direction	B1	
		T_2: Reflection in the x-axis	B1	
		T_3: Translation of (+)2 in y direction	B1	
		All three fully correct	B1	4
				[11]

One alternative is
T_1: Translation of –2 in x direction
T_2: Rotation of 90° clockwise about O
T_3: Translation of –2 in x direction

14. $\dfrac{(x-3)(x-5)}{(x-3)(x+3)} \times \dfrac{2x(x+3)}{(x-5)^2}$ (3 × factorising) B1 B1 B1

$= \dfrac{2x}{x-5}$ B1

[4]

15. (a) $2 + \dfrac{3}{x+2} \left(= \dfrac{2(x+2)+3}{x+2}\right) = \underline{\underline{\dfrac{2x+7}{x+2}}}$ or $\dfrac{2(x+2)+3}{x+2}$ B1 1

(b) $y = 2 + \dfrac{3}{x+2}$ <u>OR</u> $y = \dfrac{2x+7}{x+2}$

$y - 2 = \dfrac{3}{x+2}$ $y(x+2) = 2x + 7$ M1

$yx - 2x = 7 - 2y$

$x + 2 = \dfrac{3}{y-2}$ $x(y-2) = 7 - 2y$ M1

$x = \dfrac{3}{y-2} - 2$ $x = \dfrac{7-2y}{y-2}$

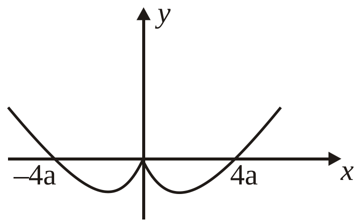
$\therefore\ \underline{\underline{f^{-1}(x) = \dfrac{3}{x-2} - 2}}$ $\underline{\underline{f^{-1}(x) = \dfrac{7-2x}{x-2}}}$ o.e A1 3

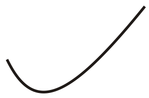
Notes
M1 $y = f(x)$ and <u>1st step</u> towards $x =$.
M1 One step from $x =$.
A1 y or $f^{-1}(x) =$ in terms of x.

(c) Domain of $f^{-1}(x)$ is $x \in \mathbb{R}, x \neq 2$ B1 1
[NB $x \neq +2$]

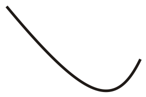
[5]

16. (a)





$x > 0$ B1



$x < 0$ B1 ft

$(4a, 0)$ & $(–4a, 0)$ and shape at $(0, 0)$ B1 3

(b) $f(2a) = (2a)^2 – 4a(2a) = 4a^2 – 8a^2 = \underline{–4a^2}$ B1

$f(–2a)$ [= $f(2a)$ ($\because$ *even function*)] $= \underline{–4a^2}$ B1 ft 2

B1 ft their f(2a)

(c) $a = 3$ and $f(x) = 45 \Rightarrow 45 = x^2 – 12x \quad (x > 0)$ M1

$0 = x^2 – 12x – 45$

$0 = (x – 15)(x + 3)$ M1

$x = 15$ (or $–3$) A1

$\therefore$ Solutions are $\underline{x = \pm 15}$ <u>only</u> A1 4

M1 Attempt 3TQ in x

M1 Attempt to solve

A1 At least x = 15 can ignore x = –3

A1 To get final A1 must make clear <u>only</u> answers are ±15.

[9]

17. (a) $\dfrac{2}{x-3} + \dfrac{13}{(x-3)(x+7)}$ M1

$= \dfrac{2(x+7)+13}{(x-3)(x+7)} = \dfrac{2x+27}{(x-3)(x+7)}$ M1 A1 3

(b) $2x + 27 = x^2 + 4x – 21$ M1

$x^2 + 2x – 48 = (x + 8)(x – 6) = 0$

$x = –8, 6$ M1 A1 3

[6]

18. (a) $\dfrac{x^2+4x+3}{x^2+x} = \dfrac{(x+3)(x+1)}{x(x+1)}$ M1

Attempt to factorise numerator or denominator

$= \dfrac{x+3}{x}$ or $1 + \dfrac{3}{x}$ or $(x+3)x^{-1}$ A1 2

(b) LHS $= \log_2\left(\dfrac{x^2+4x+3}{x^2+x}\right)$ M1 (*)

Use of log a – log b

RHS = 2^4 or 16 B1

$x + 3 = 16x$ M1 (*)

Linear or quadratic equation in x

(*) *dep*

$x = \frac{3}{15}$ or $\frac{1}{5}$ or 0.2 A1 4

[6]

19. $x^2 - 9 = (x-3)(x+3)$ seen B1

Attempt at forming single fraction

$\dfrac{x(x-3)+(x+12)(x+1)}{(x+1)(x+3)(x-3)}$; $= \dfrac{2x^2+10x+12}{(x+1)(x+3)(x-3)}$ M1; A1

Factorising numerator $= \dfrac{2(x+2)(x+3)}{(x+1)(x+3)(x-3)}$

or equivalent $= \dfrac{2(x+2)}{(x+1)(x-3)}$ M1 M1 A1

[6]

20. $2x^2 + 7x + 6 = (x+2)(2x+3)$ M1 A1

$\dfrac{3x^2}{(2+x)(3+2x)} \times \dfrac{7(3+2x)}{3x^5}$

$= \dfrac{7}{(2+x)x^3}$

some correct algebraic cancelling M1 A14

[4]