

11. Taking moments about axis of cylinder we have

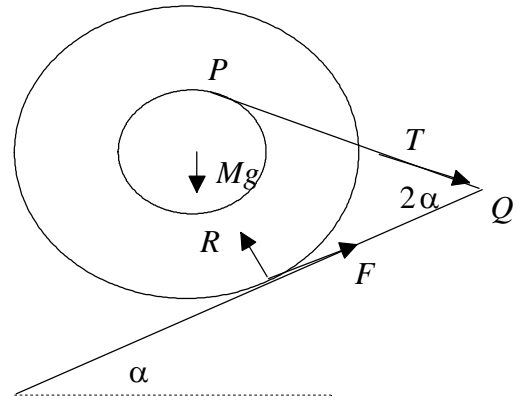
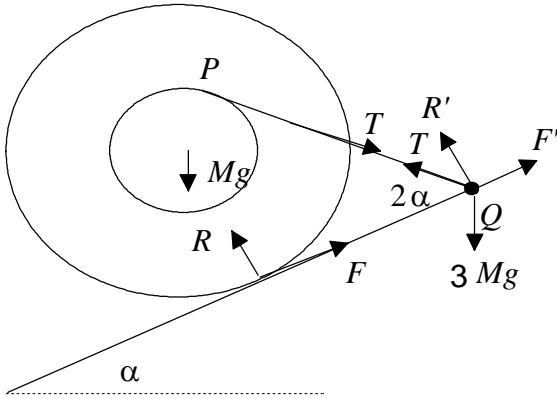
$$aT = 2aF$$

Resolving horizontally $T \cos \alpha + F \cos \alpha = R \sin \alpha$

$$\Rightarrow T + F = R \tan \alpha$$

$$\Rightarrow F = R \tan \alpha - 2F \Rightarrow \frac{F}{R} = \frac{1}{3} \tan \alpha$$

i.e. $\mu \geq \frac{1}{3} \tan \alpha$ as required.



If Q is now fastened to a particle of mass $3M$
 Resolving along and perpendicular to plane at Q
 $F' = 3Mg \sin \alpha + T \cos 2\alpha$ and
 $R' = 3Mg \cos \alpha - T \sin 2\alpha$
 so $\frac{F'}{R'} = \frac{3Mg \sin \alpha + T \cos 2\alpha}{3Mg \cos \alpha - T \sin 2\alpha}$

Resolving along and perpendicular to plane for cylinder, we have

$$F + T \cos 2\alpha = Mg \sin \alpha \text{ and } R = Mg \cos \alpha + T \sin 2\alpha, \text{ also } T = 2F$$

$$\text{so } T(1 + 2 \cos 2\alpha) = 2Mg \sin \alpha \Rightarrow T = \frac{2Mg \sin \alpha}{1 + 2 \cos 2\alpha}$$

$$\alpha = \tan^{-1} \frac{1}{2} \Rightarrow \sin \alpha = \frac{1}{\sqrt{5}}, \cos \alpha = \frac{2}{\sqrt{5}}, \sin 2\alpha = \frac{4}{5} \text{ and } \cos 2\alpha = \frac{3}{5}$$

$$\text{so } T = \frac{2Mg}{\sqrt{5} \left(1 + \frac{6}{5}\right)} = \frac{10Mg}{11\sqrt{5}} \text{ and so } \frac{F'}{R'} = \frac{\frac{3}{\sqrt{5}} + \frac{30}{55\sqrt{5}}}{\frac{6}{\sqrt{5}} - \frac{40}{55\sqrt{5}}} = \frac{165+30}{330-40} = \frac{195}{290} = \frac{49}{58}$$

so minimum coefficient of friction at Q is $\frac{49}{58}$.