Please read in conjunction with the information provided in the introductory section above.

MATHEMATICS TEST SPECIFICATION

1 AIMS

The AEA in mathematics aims to provide a sense of achievement and a stimulating mathematical challenge through:

- encouraging students to use what they have been taught;
- encouraging students to think beyond what they have been taught;
- encouraging students to develop confidence, stamina and fluency in working through unfamiliar and/or unstructured problems which might demand multi-step analysis or the exploration of different possibilities;
- building chains of logical reasoning and using concepts of proof;
- testing critical thinking and the critical evaluation of a mathematical argument;
- rewarding elegance, clarity and insight in the solution of mathematical problems.

2 CONTENT

The AEA in Mathematics will not require knowledge beyond the pure mathematics core, but will extend mathematical thinking beyond the level required for GCE Advanced level Mathematics.

Background Knowledge:

- the arithmetic of integers (including HCFs and LCMs), of fractions, and of real numbers;
- the laws of indices for positive integer exponents;
- solution of problems involving ratio and proportion (including similar triangles, and links between length, area and volume of similar figures);
- elementary algebra (including multiplying out brackets, factorising quadratics with integer coefficients - to include $a^2 - b^2$ - and solution of simultaneous linear equations by eliminating a variable);
- changing the subject of a simple formula or equation;
- the equation $y = mx + c$ for a straight line; gradient and intercept;
- the distance between two points in 2-D with given co-ordinates;
- solution of triangles using trigonometry;
- volume of cone and sphere;
- the following properties of a circle:
    - the angle in a semicircle is a right angle;
    - the perpendicular from the centre to a chord bisects the chord;
    - the perpendicularity of radius and tangent.

Knowledge, Understanding and Skills
Proof

- construction and presentation of rigorous mathematical arguments through appropriate use of precise statements and logical deduction.
- correct understanding and use of mathematical language and grammar in respect of terms such as 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient', and notation such as $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.
- methods of proof, including proof by contradiction and disproof by counter-example.

Algebra and functions

- laws of indices for all rational exponents;
- use and manipulation of surds;
- quadratic functions and their graphs. The discriminant of a quadratic function. Completing the square. Solution of quadratic equations;
- simultaneous equations: one linear and one quadratic, analytical solution by substitution;
- solution of linear and quadratic inequalities;
- algebraic manipulation of polynomials, including expanding and collecting like terms, and factorisation; use of the Factor Theorem; simplification of rational expressions including factorising and cancelling, and algebraic division;
- definition of a function. Domain and range of functions. Composition of functions. Inverse functions;
- graphs of functions and their inverses; curves defined by simple equations. Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs of functions to solve equations;
- The modulus function;
- knowledge of the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\,\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$ and combinations of these transformations;
- rational functions. Partial fractions (denominators not more complicated than repeated linear terms);
- the Remainder Theorem.

Co-ordinate geometry in the (*x*,*y*) plane

- equation of a straight line in the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$, conditions for two straight lines to be parallel or perpendicular to each other;
- co-ordinate geometry of the circle. Equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$;
- cartesian and parametric equations of curves and conversion between the two forms.

Sequences and series

- sequences, including those given by a formula for the *n*th term, and those generated by a simple recurrence relation of the form $x_{n+1} = \mathrm{f}(x_n)$;
- arithmetic series, including the formula for the sum of the first *n* natural numbers;
- the sum of a finite geometric series; the sum to infinity of a convergent geometric series;
- binomial expansion of $(1 + x)^n$ for positive integer n. The notations n! and $\binom{\boldsymbol{n}}{\boldsymbol{r}}$;
- binomial series for any rational n.

Trigonometry

- radian measure, arc length, area of sector;
- sine, cosine and tangent functions and their graphs, symmetries and periodicity;
- knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains;
- knowledge and use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, and $\sin^2\theta + \cos^2\theta = 1$ and its equivalents;
- knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$;
- solution of simple trigonometric equations in a given interval.

Exponentials and logarithms

- the function e^x and its graph;
- exponential growth and decay;
- The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x; laws of logarithms:

  $\log_a x + \log_a y \equiv \log_a (xy)$; $\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$; $k\log_a x \equiv \log_a (x^k)$;
- the solution of equations of the form $a^x = b$.

Differentiation

- the derivative of $f(x)$ as the gradient of the tangent to $y = f(x)$ at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.
- differentiation of x^n, e^x, $\ln x$ and their sums and differences; differentiation of $\sin x$, $\cos x$, $\tan x$ and their sums and differences;
- applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions;
- differentiation using the product rule, the quotient rule, the chain rule and by the use of

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

- differentiation of simple functions defined implicitly or parametrically;
- formation of simple differential equations.

Integration

- indefinite integration as the reverse of differentiation;
- integration of x^n, e^x, $1/x$; integration of $\sin x$, $\cos x$;
- evaluation of definite integrals and interpretation of the definite integral as the area under a curve;
- evaluation of volume of revolution;
- simple cases of integration by substitution and integration by parts - these methods as the reverse processes of the chain and product rules respectively;
- simple cases of integration using partial fractions;
- analytical solution of simple first order differential equations with separable variables.

Numerical methods

- location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous;
- approximate solution of equations using simple iterative methods;
- numerical integration of functions (eg using trapezium rule).

Vectors

- vectors in two and three dimensions;
- magnitude of a vector;
- algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations;
- position vectors, the distance between two points, vector equations of lines;
- the scalar product, its use for calculating the angle between two lines.

3 ASSESSMENT OBJECTIVE

The AEA in mathematics will assess candidates' abilities to solve a range of unfamiliar problems that can be generated from the content specified above. Candidates will be assessed on their ability to apply and communicate effectively their understanding of mathematics (including for example problem-solving and the use of deductive logic), using the skills of critical analysis (including critical comment on an incorrect argument), evaluation and synthesis.

Marks will also be awarded for style and clarity of mathematical presentation.

4 SCHEME OF ASSESSMENT

Supervision and duration of assessment
The AEA in Mathematics will be externally assessed. There is a single common, timed written examination of 3 hours.

The question paper in mathematics will consist of about seven questions. Questions may be multi-step with confidence building parts or unstructured. Some may be of an unusual nature that might include topics from GCSE and logic-based items. Questions maybe open-ended.

There are no optional questions and full-marks can only be achieved by outstanding answers to all the questions.

Seven percent of the marks on the mathematics paper as a whole will be assigned for style and clarity of mathematical presentation. The examiners will seek to reward elegance of solution, insight in reaching a solution, rigour in developing a mathematical argument and excellent use of notation. These marks will be awarded across the paper and not attached in advance to individual questions. However, examiners may reward all the marks to an exceptionally brilliant solution to any particular question. Each candidate will be rewarded with 0-7% of the total mark for the style and clarity of his or her presentation.

Use of resources
The use of scientific or graphic calculators will not be allowed nor will computer algebra systems.

Candidates will be required to remember the same formulae as for GCE Advanced level Mathematics. These are set out in Appendix 1. The candidates will also be expected to be familiar with the Mathematical Notation agreed for GCE Advanced level Mathematics as set out in Appendix 2.

5 PERFORMANCE LEVEL DESCRIPTORS

Distinction
Candidates:

- demonstrate understanding and command of most of the topics tested.

They will usually:

- handle complex mathematical expressions accurately;
- exhibit insight and clarity of thought;
- adopt effective and imaginative mathematical strategies to produce logically coherent and elegant solutions to problems;
- set out formal proofs, generalise and pick out special cases;
- detect and correct faulty logic;
- cope with unfamiliar situations and unstructured questions.

Merit
Candidates:

- demonstrate understanding and command of many of the topics tested.

They will often:

- handle complex mathematical expressions accurately;
- exhibit insight and clarity of thought;
- adopt effective and imaginative mathematical strategies to produce logically coherent and elegant solutions to problems;
- set out formal proofs, generalise and pick out special cases;
- detect and correct faulty logic;
- cope with unfamiliar situations and unstructured questions.

APPENDIX 1 - FORMULAE FOR AS AND A LEVEL MATHEMATICS SYLLABUSES

This appendix lists formulae which candidates are expected to remember and which may not be included in formulae booklets.

Quadratic equations

$a x^2 + b x + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of logarithms

$\log_a x + \log_a y \equiv \log_a (x y)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

Arithmetic series

$u_n = a + (n - 1) d$

$S_n = ½ n (a + l) = ½ n [2a + (n - 1) d]$

Geometric series

$u_n = a r^{n-1}$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$

$S_\infty = \dfrac{a}{1 - r}$ for $|r| < 1$

Trigonometry

In the triangle ABC

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

$area = ½ a b \sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\text{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$

Differentiation

Function	Derivative
x^n	$n x^{n-1}$
$\sin kx$	$k \cos kx$
$\cos kx$	$- k \sin kx$
$\tan kx$	$k \sec^2 kx$
e^{kx}	$k e^{kx}$
$\ln x$	$\dfrac{1}{x}$

$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)\,g(x) - f(x)\,g'(x)}{(g(x))^2}$
$f(g(x))$	$f'(g(x))\,g'(x)$

Integration

Function	Integral
x^n	$\dfrac{1}{n+1} x^{n+1} + c \qquad n \neq -1$
$\cos kx$	$\dfrac{1}{k} \sin kx + c$
$\sin kx$	$-\dfrac{1}{k} \cos kx + c$
$\sec^2 kx$	$\dfrac{1}{k} \tan kx + c$
e^{kx}	$\dfrac{1}{k} e^{kx} + c$
$\dfrac{1}{x}$	$\ln \lvert x \rvert + c, \qquad x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))\,g'(x)$	$f(g(x)) + c$

Area

area under a curve $= \int_a^b y \, dx \qquad (y \geq 0)$

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

APPENDIX 2 - MATHEMATICAL NOTATION

1. Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x : \ldots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
$\varnothing$	the empty set
ε	the universal set
A'	the complement of the set A
N	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbf{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbf{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbf{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n - 1\}$
$\mathbf{Q}$	the set of rational numbers, $\left\{ \frac{p}{q} : p \in \mathbf{Z}, q \in \mathbf{Z}^+ \right\}$
$\mathbf{Q}^+$	the set of positive rational numbers, $\{x \in \mathbf{Q} : x > 0\}$
$\mathbf{Q}^+_0$	the set of positive rational numbers and zero, $\{x \in \mathbf{Q} : x \geq 0\}$
$\mathbf{R}$	the set of real numbers
$\mathbf{R}^+$	the set of positive real numbers, $\{x \in \mathbf{R} : x > 0\}$
$\mathbf{R}^+_0$	the set of positive real numbers and zero, $\{x \in \mathbf{R} : x \geq 0\}$
C	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the Cartesian product of sets A and B i.e. $A \times B = \{(a, b): a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathbf{R} : a \leq x \leq b\}$
$[a, b)$, $[a, b[$	the interval $\{x \in \mathbf{R} : a \leq x < b\}$
$(a, b]$, $]a, b]$	the interval $\{x \in \mathbf{R} : a < x \leq b\}$
(a, b), $]a, b[$	the open interval $\{x \in \mathbf{R} : a < x < b\}$
$y\,R\,x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2. Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq$, $\ngtr$	is less than or equal to, is not greater than
$>$	is greater than
$\geq$, $\nless$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3. Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$\sum_{i=1}^{n} a$	$a_1 + a_2 + \ldots + a_n$
$\prod_{i=1}^{n} a$	$a_1 \times a_2 \times \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\begin{pmatrix} n \\ r \end{pmatrix}$	the binomial coefficient for $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbf{Z}^+$
	$\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbf{Q}$

4. Functions

f(x)	the value of the function f at x
$f : A \to B$	f is a function under which each element of set A has an image in set B
$f : x \to y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
g o f, gf	the composite function of f and g which is defined by (g o f) (x) or g f (x) = g (f (x))
$\lim_{x \to a} f(x)$	the limit of f(x) as x tends to a
Δx, δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
$f'(x)$, $f''(x)$, … , $f^{(n)}(x)$	the first, second, ... , nth derivatives of f(x) with respect to x
$\int y\,dx$	the indefinite integral of y with respect to x
$\int_a^b y\,dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\dot{x}$, $\ddot{x}$, …	the first, second, ... derivatives of x with respect to t

5. Exponential and Logarithmic Functions

e	base of natural logarithms
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x, $\log_e x$	natural logarithm of x
lg x, $\log_{10} x$	logarithm of x to base 10

6. Circular Functions and their inverses

sin, cos, tan cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$ or arcsin, arccos, arctan, arcosec, arcsec, arccot	the inverse circular functions

9. Vectors

a	the vector **a**
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment *AB*
â	a unit vector in the direction of **a**
i, j, k	unit vectors in the directions of the Cartesian co-ordinate axes
$\lvert \mathbf{a} \rvert$, a	the magnitude of **a**
$\left\lvert \overrightarrow{AB} \right\rvert$, AB	the magnitude of $\overrightarrow{AB}$
a . b	the scalar product of **a** and **b**