**1.** (a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

**(3)**

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find $x$ in terms of e.

**(4)**
**(Total 7 marks)**

**2.** Express

$$\frac{x+1}{3x^2-3}-\frac{1}{3x+1}$$

as a single fraction in its simplest form.

**(Total 4 marks)**

**3.** The function f is defined by

$$\mathrm{f}(x)=1-\frac{2}{(x+4)}+\frac{x-8}{(x-2)(x+4)}, \ x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $\mathrm{f}(x)=\dfrac{x-3}{x-2}$

**(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \qquad x \in \mathbb{R}, x \neq \text{1n } 2$$

(b) Differentiate g($x$) to show that $g'(x) = \frac{e^x}{(e^x - 2)^2}$,

**(3)**

(c) Find the exact values of x for which g′($x$) = 1

**(4)**
**(Total 12 marks)**

**4.**

$$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}$$

(a) Express f ($x$) as a single fraction in its simplest form.

**(4)**

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

**(3)**
**(Total 7 marks)**

**5.** Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants $a$, $b$, $c$, $d$ and $e$.

**(Total 4 marks)**

**6.**

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \qquad x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{4x-6}{2x-1}$.

**(7)**

(b) Hence, or otherwise, find $f'(x)$ in its simplest form.

**(3)**
**(Total 10 marks)**

**7.**

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \; x \neq -2.$$

(a) Show that $f(x) = \frac{x^2+x+1}{(x+2)^2}, x \neq -2$.

**(4)**

(b) Show that $x^2 + x + 1 > 0$ for all values of $x$.

**(3)**

(c) Show that $f(x) > 0$ for all values of $x$, $x \neq -2$.

**(1)**
**(Total 8 marks)**

**8.**

$$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval [–2, –1].

**(3)**

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

**(3)**

(c) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, $a$, $b$ and $c$.

**(3)**

(d) Sketch the graph of $y = f(x)$.

**(3)**

(e) Hence sketch the graph of $y = |f(x)|$.

**(1)**
**(Total 13 marks)**

**9.** (a) Simplify $\dfrac{3x^2 - x - 2}{x^2 - 1}$

**(3)**

(b) Hence, or otherwise, express $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

**(3)**
**(Total 6 marks)**

**10.** Express

$$\frac{2x^2 + 3x}{(2x+3)(x-2)} - \frac{6}{x^2 - x - 2}$$

as a single fraction in its simplest form.

**(Total 7 marks)**

**11.** The function f is defined by

$$\text{f}: x \to \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $\text{f}(x) = \dfrac{2}{x-1}$, $x > 1$.

**(4)**

(b) Find $\text{f}^{-1}(x)$.

**(3)**

The function g is defined by

$$\text{g}: x \to x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $\text{fg}(x) = \frac{1}{4}$.

**(3)**
**(Total 10 marks)**

**12.** $$\text{f}(x) = \frac{x^2-x-6}{x^2-3x}, \quad x \neq 0, \quad x \neq 3.$$

(a) Express $\text{f}(x)$ in its simplest form.

**(3)**

(b) Hence, or otherwise, find the exact solutions of $\text{f}(x) = x + 1$.

**(3)**
**(Total 6 marks)**

**13.**

$$f(x) = \frac{2x+5}{x+3} - \frac{1}{(x+3)(x+2)}, \quad x > -2.$$

(a) Express f($x$) as a single fraction in its simplest form.

**(5)**

(b) Hence show that $f(x) = 2 - \dfrac{1}{x+2}, \quad x > -2.$

**(2)**

The curve $y = \dfrac{1}{x}$, $x > 0$, is mapped onto the curve $y = f(x)$, using three successive transformations $T_1$, $T_2$ and $T_3$, where $T_1$ and $T_3$ are translations.

(c) Describe fully $T_1$, $T_2$ and $T_3$.

**(4)**
**(Total 11 marks)**

**14.** Express as a single fraction in its simplest form

$$\frac{x^2 - 8x + 15}{x^2 - 9} \times \frac{2x^2 + 6x}{(x-5)^2}.$$

**(Total 4 marks)**

**15.** The function f is given by

$$f : x \mapsto 2 + \frac{3}{x+2}, \quad x \in \mathbb{R}, \quad x \neq -2.$$

(a) Express $2 + \dfrac{3}{x+2}$ as a single fraction.

**(1)**

(b) Find an expression for $f^{-1}(x)$.

**(3)**

(c) Write down the domain of $f^{-1}$.

**(1)**
**(Total 5 marks)**

**16.** The function f is even and has domain $\mathbb{R}$. For $x \geq 0$, $f(x) = x^2 - 4ax$, where $a$ is a positive constant.

(a) In the space below, sketch the curve with equation $y = f(x)$, showing the coordinates of all the points at which the curve meets the axes.

**(3)**

(b) Find, in terms of $a$, the value of $f(2a)$ and the value of $f(-2a)$.

**(2)**

Given that $a = 3$,

(c) use algebra to find the values of $x$ for which $f(x) = 45$.

**(4)**

**(Total 9 marks)**

**17.** (a) Express as a fraction in its simplest form

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}.$$

**(3)**

(b) Hence solve

$$\frac{2}{x-3}+\frac{13}{x^2+4x-21}=1.$$

**(3)**
**(Total 6 marks)**

**18.** (a) Simplify $\frac{x^2+4x+3}{x^2+x}$.

**(2)**

(b) Find the value of $x$ for which $\log_2(x^2+4x+3)-\log_2(x^2+x)=4$.

**(4)**
**(Total 6 marks)**

**19.** Express $\frac{x}{(x+1)(x+3)}+\frac{x+12}{x^2-9}$ as a single fraction in its simplest form.

**(Total 6 marks)**

**20.** Express

$$\frac{3x^2}{(2x^2+7x+6)}\times\frac{7(3+2x)}{3x^5}$$

as a single fraction in its simplest form.

**(Total 4 marks)**