1.	(a)	Either $y = 2$ or $(0, 2)$	B1	1
	(b)	When $x = 2$, $y = (8 - 10 + 2)e^{-2} = 0e^{-2} = 0$	B1	
		$(2x^2 - 5x + 2) = 0 \Rightarrow (x-2)(2x-1) = 0$	M1	
		Either $x = 2$ (for possibly B1 above) or $\underline{x = \frac{1}{2}}$.	A1	3

Note

If the candidate believes that $e^{-x} = 0$ solves to $x = 0$ or gives an extra solution of $x = 0$, then withhold the final accuracy mark.

	(c)	$\frac{dy}{dx} = (4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	M1 A1 A1	3

Note

M1: (their u')e^{-x} + $(2x^2 - 5x + 2)$(their v')
A1: Any one term correct.
A1: Both terms correct.

	(d)	$(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	M1	
		$2x^2 - 9x + 7 = 0 \Rightarrow (2x - 7)(x - 1) = 0$	M1	
		$x = \frac{7}{2}, 1$	A1	
		When $x = \frac{7}{2}$, $y = 9e^{-\frac{7}{2}}$, when $x = 1$, $y = -e^{-1}$	dd M1 A1	5

Note

1st M1: For setting their $\frac{dy}{dx}$ found in part (c) equal to 0.
2nd M1: Factorise or eliminate out e^{-x} correctly and an attempt to factorise a 3-term quadratic or apply the formula to candidate's $ax^2 + bx + c$.
See rules for solving a three term quadratic equation on page 1 of this Appendix.
3rd ddM1: An attempt to use at least one x-coordinate on $y = (2x^2 - 5x + 2)e^{-x}$.
Note that this method mark is dependent on the award of the two previous method marks in this part.
Some candidates write down corresponding y-coordinates without any working. It may be necessary on some occasions to use your calculator to check that at least one of the two y-coordinates found is correct to awrt 2 sf.

Final A1: Both$\{x = 1\}$, $y = -e^{-1}$ and $\{x = \frac{7}{2}\}$, $y = 9e^{-\frac{7}{2}}$. **cao**
Note that both exact values of y are required.

[12]

2. (a) $\dfrac{(x+5)(2x-1)}{(x+5)(x-3)} = \dfrac{(2x-1)}{(x-3)}$ M1 B1 A1 aef 3

Note

M1: An attempt to factorise the numerator.
B1: Correct factorisation of denominator to give $(x + 5)(x - 3)$.
Can be seen anywhere.

(b) $\ln\left(\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ M1

$\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15} = \mathrm{e}$ dM1

$\dfrac{2x - 1}{x - 3} = \mathrm{e} \Rightarrow \quad 3\mathrm{e} - 1 = x(\mathrm{e} - 2)$ M1

$\Rightarrow \; x = \dfrac{3\mathrm{e} - 1}{\mathrm{e} - 2}$ A1 aef cso 4

Note

M1: Uses a correct law of logarithms to combine at least two terms.
This usually is achieved by the subtraction law of logarithms to give

$\ln\left(\dfrac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$

The product law of logarithms can be used to achieve
$\ln(2x^2 + 9x - 5) = \ln(\mathrm{e}(x^2 + 2x - 15))$.
The product and quotient law could also be used to achieve

$\text{In}\left(\dfrac{2x^2+9x-5}{e\left(x^2+2x-15\right)}\right)=0$

dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e.
Note that this mark is dependent on the previous method mark being awarded.
M1: Collect x terms together and factorise.
Note that this is not a dependent method mark.

A1: $\dfrac{3e-1}{e-2}$ *or* $\dfrac{3e^1-1}{e^1-2}$ *or* $\dfrac{1-3e}{2-e}$. **aef**

Note that the answer needs to be in terms of e. The decimal answer is 9.9610559…

Note that the solution must be correct in order for you to award this final accuracy mark.

[7]

3.	(i)	(a)	$\ln(3x-7)=5$			
			$e^{\ln(3x-7)}=e^5$	Takes e of both sides of the equation. This can be implied by $3x-7=e^5$.	M1	
				Then rearranges to make x the subject.	dM1	
			$3x-7=e^5 \Rightarrow$			
			$x=\dfrac{e^5+7}{3}\{=51.804...\}$	***Exact answer*** of $\dfrac{e^5+7}{3}$.	A1	3
		(b)	$3^x e^{7x+2}=15$			
			$\ln(3^x e^{7x+2})=\ln 15$	Takes ln (or logs) of both sides of the equation.	M1	
			$\ln 3^x+\ln e^{7x+2}=\ln 15$	Applies the addition law of logarithms.	M1	
			$x\ln 3+7x+2=\ln 15$	$x\ln 3+7x+2=\ln 15$	A1 oe	
			$x(\ln 3+7)=-2+\ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1	
			$x=\dfrac{-2+\ln 15}{7+\ln 3}\{=0.0874...\}$	***Exact answer*** of $\dfrac{-2+\ln 15}{7+\ln 3}$	A1 oe	5

(ii) (a) $\mathrm{f}(x) = \mathrm{e}^{2x} + 3, x \in \mathbb{R}$			
$y = \mathrm{e}^{2x} + 3 \Rightarrow y - 3 = \mathrm{e}^{2x}$	Attempt to make x (or swapped y) the subject	M1	
$\Rightarrow \ln(y-3) = 2x$			
$\Rightarrow \frac{1}{2}\ln y - 3 = x$	Makes e^{2x} the subject and takes ln of both sides	M1	
Hence $\mathrm{f}^{-1}(x) = \underline{\frac{1}{2}\ln(x-3)}$	$\frac{1}{2}\ln(x-3)$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{\mathrm{f}^{-1}(x) = \frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**	
$\mathrm{f}^{-1}(x)$: Domain: $\underline{x > 3}$ or $\underline{(3, \infty)}$	Either $\underline{x > 3}$ or $\underline{(3, \infty)}$ or <u>Domain > 3.</u>	B1	4
(b) $\mathrm{g}(x) = \ln(x-1), x \in \mathbb{R}, x > 1$			
$\mathrm{fg}(x) = \mathrm{e}^{2\ln(x-1)} + 3$ $\{= (x-1)^2 + 3\}$	An attempt to put function g into function f.	M1	
	$\mathrm{e}^{2\ln(x-1)} + 3$ or $(x-1)^2 + 3$ or $x^2 - 2x + 4$.	A1 isw	
$\mathrm{fg}(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$	Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or <u>Range>3</u> or <u>fg(x)>3</u>.	B1	3
			[15]

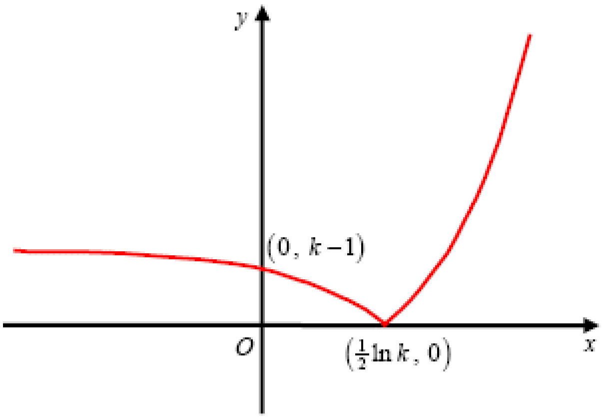
4. (a) $P = 80\mathrm{e}^{\frac{t}{5}}$

$t = 0 \Rightarrow P = 80\,\mathrm{e}^{\frac{o}{5}} = 80(1) = \underline{80}$	$\underline{80}$	B1	1

(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$	Substitutes P = 1000 and rearranges equation to make $e^{\frac{t}{5}}$ the subject.	M1	
	$\therefore t = 5\ln\left(\frac{1000}{80}\right)$			
	$t = 12.6286...$	awrt 12.6 or 13 years Note $t = 12$ or t = awrt 12.6 $\Rightarrow$ $t = 12$ will score A0	A1	2
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{t}{5}}$ and $k \neq 80$.	M1	
		$16\ e^{\frac{1}{5}t}$	A1	2
(d)	$50 = 16e^{\frac{t}{5}}$			
	$\therefore t = 5\ln\left(\frac{50}{16}\right)$ { 5.69717...}	Using 50 = $\frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$	M1	
	$P = 80\ e^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80e^{\frac{1}{5}(5.69717...)}$	Substitutes their value of t back into the equation for P.	dM1	
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250	A1	3

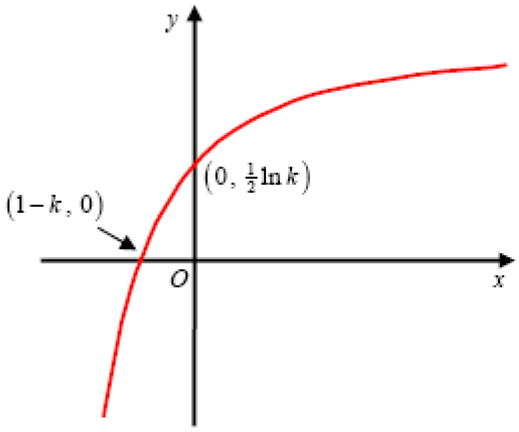
[8]

5. (a)



Curve retains shape when $x > \frac{1}{2}\ln k$	B1	
Curve reflects through the *x*-axis when $x > \frac{1}{2}\ln k$	B1	
$(0, k - 1)$ and $(\frac{1}{2}\ln k, 0)$ marked in the correct positions.	B1	3

(b)



Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)	B1	
$(1 - k, 0)$ and $(0, \frac{1}{2}\ln k)$	B1	2

(c) Range of f: <u>f(*x*) > –*k*</u> or <u>*y* > –*k*</u> or <u>(–*k*, ∞)</u>	Either <u>f(*x*) > –*k*</u> or <u>*y*> –*k*</u> or <u>(– *k*, ∞)</u>or <u>f > – *k*</u> or <u>Range > – *k*</u> .	B1	1

	(d)	$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y+k) = 2x$ $\Rightarrow \frac{1}{2}\ln(y+k) = x$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides	M1 M1	
		Hence $\underline{f^{-1}(x) = \frac{1}{2}\ln(x+k)}$	$\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$	<u>A1</u> cao	3
	(e)	$f^{-1}(x)$: Domain: <u>$x > -k$</u> or <u>$(-k, \infty)$</u>	Either <u>$x > -k$</u> or <u>$(-k, \infty)$</u> or Domain $> -k$ or x "ft one sided inequality" their part (c) RANGE answer	B1ft	1
					[10]
6.	(a)	$f(x) = 1 - \dfrac{2}{(x+4)} + \dfrac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R}, x \neq -4, x \neq 2.$			
		$f(x) = \dfrac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$	An attempt to combine to one fraction Correct result of combining all three fractions	M1 A1	
		$= \dfrac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$			
		$= \dfrac{x^2 + x - 12}{[(x+4)(x-2)]}$	Simplifies to give the correct numerator. Ignore omission of denominator	A1	
		$= \dfrac{(x+4)(x-3)}{[(x+4)(x-2)]}$	An attempt to factorise the numerator.	dM1	
		$= \dfrac{(x-3)}{(x-2)}$	Correct result	A1 cso **AG**	5

(b)	$g(x) = \dfrac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq \ln 2.$			
	Apply quotient rule: $\left\{\begin{matrix} u = e^x - 3 & v = e^x - 2 \\ \dfrac{du}{dx} = e^x & \dfrac{dv}{dx} = e^x \end{matrix}\right\}$			
	$g'(x) = \dfrac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$	M1	
		Correct differentiation	A1	
	$= \dfrac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$			
	$= \dfrac{e^x}{(e^x - 2)^2}$	Correct result	A1 **AG**	
			cso	3
(c)	$g'(x) = 1 \Rightarrow = \dfrac{e^x}{(e^x - 2)^2} = 1$			
	$e^x = (e^x - 2)^2$	Puts their differentiated numerator equal to their denominator.	M1	
	$e^x = e^{2x} - 2e^x - 2e^x + 4$			
	$\underline{e^{2x} - 5e^x + 4} = 0$	$\underline{e^{2x} - 5e^x + 4}$	A1	
	$(e^x - 4)(e^x - 1) = 0$	Attempt to factorise or solve quadratic in e^x	M1	
	$e^x = 4$ or $e^x = 1$			
	$x = \ln 4$ or $x = 0$	both $x = 0, \ln 4$	A1	4

[12]

7. (a)	$g(x) \geq 1$		B1	1
(b)	$fg(x) = f(e^{x^2}) = 3e^{x^2} + \ln e^{x^2}$		M1	
	$= x^2 + 3e^{x^2}$ *		A1	2
	$(fg : x \longmapsto x^2 + 3e^{x^2})$			
(c)	$fg(x) \geq 3$		B1	1

(d)		$\frac{d}{dx}(x^2 + 3e^{x^2}) = 2x + 6xe^{x^2}$	M1 A1	
		$2x + 6xe^{x^2} = x^2 e^{x^2} + 2x$		
		$e^{x^2}(6x - x^2) = 0$	M1	
	$e^{x^2} \neq 0,$	$6x - x^2 = 0$	A1	
		$x = 0, 6$	A1 A1	6

[10]

8.	(a)	$f'(x) = 3e^x + 3xe^x$	M1 A1	
		$3e^x + 3xe^x = 3ex(1+x) = 0$		
		$x = -1$	M1A1	
		$f(-1) = -3e^{-1} - 1$	B1	5
	(b)	$x_1 = 0.2596$	B1	
		$x_2 = 0.2571$	B1	
		$x_3 = 0.2578$	B1	3

(c) Choosing (0.257 55, 0.257 65) or an appropriate tighter interval.

$f(0.257\,55) = -0.000\,379\ ...$

$f(0.257\,65) = 0.000109\ \ldots$ A1

Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ $*$ cso A1 3

($\Rightarrow$ $x = 0.2576$, is correct to 4 decimal places)

Note:

$x = 0.257\,627\,65\ \ldots$ is accurate

[11]

9.	(a)	$e^{2x+1} = 2$		
		$2x + 1 = \ln 2$	M1	
		$x = \frac{1}{2}(\ln 2 - 1)$	A1	2

(b)	$\frac{dy}{dx} = 8e^{2x+1}$	B1		
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1		
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1		
	$y = 16x + 16 - 8\ln 2$	A1	4	
				[6]

10.	(a)	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$	M1		
		$x = 2$ (only this answer)	A1cso	2	
		Answer $x = 2$ with no working or no incorrect working seen: M1A1			
		Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0			
		$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1			
	(b)	$(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form)	M1		
		$(e^x - 3)(e^x - 1) = 0$			
		$e^x = 3$ or $e^x = 1$ Solving quadratic	M1dep		
		$x = \ln 3$, $x = 0$ (or $\ln 1$)	M1A1	4	
		1st M1 for attempting to multiply through by e^x : Allow y, X, even x, for e^x			
		2nd M1 is for solving quadratic as far as getting two values for e^x or y or X etc			
		3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)			
		A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)			
					[6]

11.	(a)	$D = 10$, $t = 5$, $x = 10e^{-\frac{1}{8}\times 5}$		M1	
		$= 5.353$	awrt	A1	2
	(b)	$D = 10 + 10e^{-\frac{5}{8}}$, $t = 1$, $x = 15.3526... \times e^{-\frac{1}{8}}$		M1	
		$x = 13.549$ (*)		A1cso	2

Alt. (b) $x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $x = 13.549$ (*) A1cso

(Main scheme M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or {10 + their(a)} $e^{-\frac{1}{8}}$

N.B. The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(c) $15.3526... e^{-\frac{1}{8}T} = 3$ M1

$e^{-\frac{1}{8}T} = \dfrac{3}{15.3526...} = 0.1954...$

$-\dfrac{1}{8}T = \ln 0.1954...$ M1

$T = 13.06...$ or 13.1 or 13 A1 3

1st M is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{T}{8}} = 3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}} = k$ $(k > 0)$ to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation
(two equal to 3 s.f.)
A1 as scheme

[7]

12. (a) 425 °C B1 1

(b) $300 = 400e^{-0.05\,t} + 25 \Rightarrow 400e^{-0.05\,t} = 275$ M1

sub. T = 300 and attempt to rearrange to $e^{-0.05t} = a$*, where* $a \in Q$

$e^{-0.05t} = \dfrac{275}{400}$ A1

M1 correct application of logs M1

$t = 7.49$ A1 4

(c) $\dfrac{dT}{dt} = -20\ e^{-0.05t}$ (M1 for $ke^{-0.05t}$) M1 A1

At $t = 50$, rate of decrease = (±) 1.64 °C / min A1 3

	(d)	$T > 25$, (since $e^{-0.05t} \to 0$ as $t \to \infty$)	B1	1	
					[9]
13.		$\frac{dy}{dx} = \frac{1}{x}$	M1 A1		
		At $x = 3$, gradient of normal = $\frac{-1}{\frac{1}{3}} = -3$	M1		
		$y - \ln 1 = -3(x - 3)$	M1		
		$y = -3x + 9$	A1	5	
					[5]
14.	(a)	Setting $p - 300$ at $t = 0 \Rightarrow 300 = \frac{2800a}{1+a}$	M1		
		$(300 = 2500a)$; $a = 0.12$ (c.s.o.) (*)	dM1 A1	3	
	(b)	$1850 = \frac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$; $e^{0.2t} = 16.2\ldots$	M1A1		
		Correctly taking logs to $0.2\,t = \ln k$	M1		
		$t = 14$ (13.9..)	A1	4	
	(c)	Correct derivation:	B1	1	
		(Showing division of num. and den. by $e^{0.2t}$; using a)			
	(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1		
		$p \to \frac{336}{0.12} = 2800$	A1	2	
					[10]
15.	(a)	$\log 3^x = \log 5$	M1		
		taking logs			
		$x = \frac{\log 5}{\log 3}$ or $x \log 3 = \log 5$	A1		
		$= 1.46$ cao	A1	3	

	(b)	$2 = \log_2 \dfrac{2x+1}{x}$	M1	
		$\dfrac{2x+1}{x} = 4$ or equivalent;	B1	
		4		
		$2x + 1 = 4x$	M1	
		multiplying by x to get a linear equation		
		$x = \dfrac{1}{2}$	A1	4
	(c)	$\sec x = 1/\cos x$	B1	
		$\sin x = \cos x \Rightarrow \tan x = 1$ $x = 45$	M1, A1	3
		use of tan x		

[10]

16.	(a)	$I = 3x + 2e^x$	B1	
		Using limits correctly to give 1 + 2e. (c.a.o.)	M1 A1	3
		must subst 0 and 1 and subtract		
	(b)	$A = (0, 5)$;	B1	
		y = 5		
		$\dfrac{dy}{dx} = 2e^x$	B1	
		Equation of tangent: $y = 2x + 5$; $c = -2.5$	M1; A1	4
		attempting to find eq. of tangent and subst in y = 0, must be linear equation		
	(c)	$y = \dfrac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \Rightarrow 4y - 2 = 5x - xy$	M1; A1	
		putting y = and att. to rearrange to find x .		
		$g^{-1}(x) = \dfrac{4x-2}{5-x}$ or equivalent	A1	3
		must be in terms of x		
	(d)	gf(0) = g(5); =3	M1; A1	2
		att to put 0 into f and then their answer into g		

[12]

17.	(a)	(i)	$\underline{x = a^y}$	B1	1
		(ii)	In both sides of (i) i.e $\ln x = \ln a^y$ or (y =) $\log_a x = \dfrac{\ln x}{\ln a}$		
			$= \underline{y \ln a}$ * $\Rightarrow y \ln a = \ln x$	B1_{cso}	1

B1 $x = e^{y\ln a}$ is BO

B1 Must see ln a^y or use of change of base formula.

(b) $y = \dfrac{1}{\ln a} \bullet \ln x$, $\Rightarrow \dfrac{dy}{dx}, = \underline{\dfrac{1}{\ln a} \times \dfrac{1}{x}}$ * M1, A1_{cso} 2

ALT. $\left[\text{or } \dfrac{1}{x} = \dfrac{dy}{dx} \cdot \ln a \quad , \Rightarrow \underline{\dfrac{dy}{dx} = \dfrac{1}{x \ln a}} \; * \right]$

M1, $A1_{cso}$ M1 needs some correct attempt at differentiating.

(c) $\log_{10} 10 = 1 \Rightarrow$ A is (10, $\underline{1}$) $y_A = 1$ B1

from(b) $\underline{m = \dfrac{1}{10 \ln a} \text{ or } \dfrac{1}{10 \ln 10}}$ or 0.043 (or better) B1

equ of target $y - 1 = m(x - 10)$

i.e $\underline{y - 1 = \dfrac{1}{10 \ln 10}(x - 10) \text{ or } y = \dfrac{1}{10 \ln 10} x + 1 - \dfrac{1}{\ln 10}}$ (o.e) A1 4

B1 Allow either

M1 ft their y_A and m

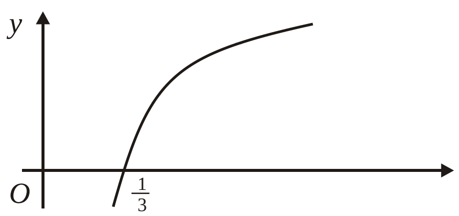
(d) $y = 0$ *in* (c) $\Rightarrow 0 = \dfrac{x}{10 \ln 10} + 1 - \dfrac{1}{\ln 10} \Rightarrow x, = 10 \ln 10 \left(\dfrac{1}{\ln 10} - 1 \right)$ M1

$\underline{x = 10 - 10 \ln 10}$ or $\underline{10(1 - \ln 10)}$ or $\underline{10 \ln 10 (\dfrac{1}{\ln 10} - 1)}$ A1 2

M1 Attempt to solve correct equation. Allow if a not = 10.

[10]

18. (a)



Shape B1

$p = \frac{1}{3}$ or $\{\frac{1}{3}, 0\}$ seen B1 2

	(b)	Gradient of tangent at $Q = \frac{1}{q}$	B1	
		Gradient of normal = $-q$	M1	
		Attempt at equation of OQ [$y = -qx$] and substituting $x = q$, $y = \ln 3q$		
		or attempt at equation of tangent [$y - 3 \ln q = -q(x - q)$] with $x = 0$, $y = 0$		
		or equating gradient of normal to $(\ln 3q)/q$	M1	
		$q^2 + \ln 3q = 0$ (*)	A1	4
	(c)	$\ln 3x = -x^2 \Rightarrow 3x = e^{-x^2}$; $\Rightarrow x = \frac{1}{3}e^{-x^2}$	M1; A1	2
	(d)	$x_1 = 0.298280$; $x_2 = 0.304957$, $x_3 = 0.303731$, $x_4 = 0.303958$	M1; A1	
		Root = 0.304 (3 decimal places)	A1	3

[11]

19.	$\frac{dy}{dx} = 6x^{\frac{1}{2}} - \frac{1}{x}$	M1 A1	
	At $x = 1$, $\frac{dy}{dx} = 5$; $y = 4 - \ln 5$	A1; B1	
	Tangent is $y - 4 + \ln 5 = 5(x - 1)$	M1	
	At $y = 0$, $x = \frac{1 + \ln 5}{5} = \frac{\ln e + \ln 5}{5} = \frac{1}{5}\ln 5e$	M1 A1	

[7]

20.	(a)	A is (2, 0); B is (0, $e^{-2} - 1$)	B1; B1	2
	(b)	$y = e^{x-2} - 1$		
		Change over x and y, $x = e^{y-2} - 1$	M1	
		$y - 2 = \ln(x + 1)$	M1	
		$y = 2 + \ln(x + 1)$	A1	
		$f^{-1} : x / 2 + \ln(x + 1)$, $x > -1$	A1 A1	5

(c) $f(x) - x = 0$ is equivalent to $e^{x-2} - 1 - x = 0$

Let $g(x) = e^{x-2} - 1 - x$

$g(3) = -1.28\ldots$

$g(4) = \quad 2.38\ldots$

Sign change $\Rightarrow$ root α — M1 A1 — 2

(d) $x_{n+1} = 2 + \ln(x_n + 1)$, $x_1 = 3.5$ — M1

$x_2 = 3.5040774$ — A1

$x_3 = 3.5049831$ — A1

$x_4 = 3.5051841$

$x_5 = 3.5052288$

Needs convincing argument on 3 d.p. accuracy

Take 3.5053 and next iteration is reducing 3.50525… — M1

Answer: 3.505 (3 d.p.) — A1 — 5

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21. (i) $e^{2x+3} = 6$

$2x + 3 = \ln 6$ — M1

$x = \frac{1}{2}(\ln 6 - 3)$ — M1 A1 — 3

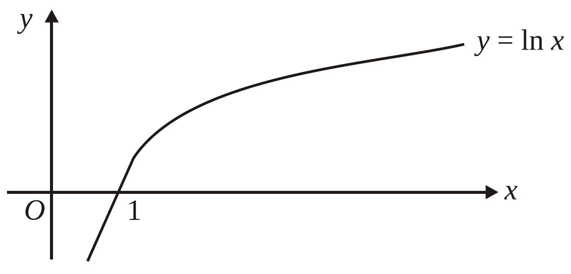
(ii) $\ln(3x + 2) = 4$

$3x + 2 = e^4$ — M1

$x = \frac{1}{3}(e^4 - 2)$ — M1 A1 — 3

[6]

22. (a)



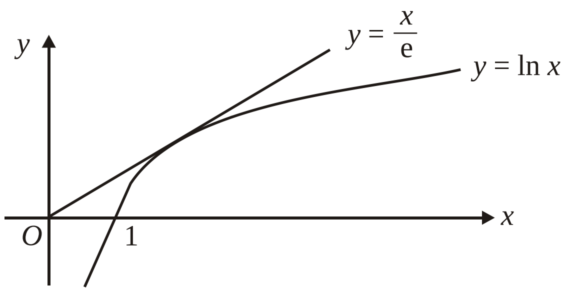
B1 shape

B1 x-intercept labelled — 2

(b) $\frac{dy}{dx} = \frac{1}{x}$ so tangent line to (e, 1) is $y = \frac{1}{e}x + C$ — M1

the line passes through (e, 1) so $1 = e\frac{1}{e} + C$ and $C = 0$ — M1

The line passes through the origin. — A1 — 3



(c) All lines $y = mx$ passing through the origin and having a gradient > 0 lie above the x-axis.

Those having a gradient < $\frac{1}{e}$ will lie below the line. — B1

$y = \frac{x}{e}$ so it cuts $y = \ln x$ between $x = 1$ and $x = e$. — B1 — 2

(d) $x_0 = 1.86$

$x_1 = e^{\frac{x_n}{3}} = 1.859$ — M1

$x_2 = 1.858$ — A1

$x_3 = 1.858$

$x_4 = 1.858$

$x_5 = 1.857$ — A1 — 3

(e) When $x = 1.8575$, $\ln x - \frac{1}{3}x = 0.000\,064\,8\ldots > 0$ — M1

When $x = 1.8565$, $\ln x = -0.000\,140\ldots < 0$ — A1

Change of sign implies there is a root between. — A1 — 3

[13]