1. (a) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

(2)

(b) Hence find, for -180° ≤ θ < 180°, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1$$

Give your answers to 1 decimal place.

(3)
(Total 5 marks)

2. (a) Express 2 sin θ – 1.5 cos θ in the form R sin (θ – α), where R > 0 and 0 < α < $\frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(b) (i) Find the maximum value of 2 sin θ – 1.5 cos θ.

(ii) Find the value of θ, for 0 ≤ $\theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \le t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)
(Total 15 marks)

3. (a) Express $5 \cos x - 3 \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(b) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$,giving your answers to 2 decimal places.

(5)
(Total 9 marks)

4. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(Total 7 marks)

5. (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$.

(2)

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2$$

(6)
(Total 8 marks)

6. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1\text{: } y = 3\sin 2x$$

$$C_2\text{: } y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)
(Total 12 marks)

7. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)
(Total 6 marks)

8. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

(Total 13 marks)

9. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \leq t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

(Total 12 marks)

10. $$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

(Total 12 marks)

11. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leq \theta < 180°$, the equation

$$2\cot^2\theta - 9\text{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

(6)

(Total 8 marks)

12. (a) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

(Total 11 marks)

13. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

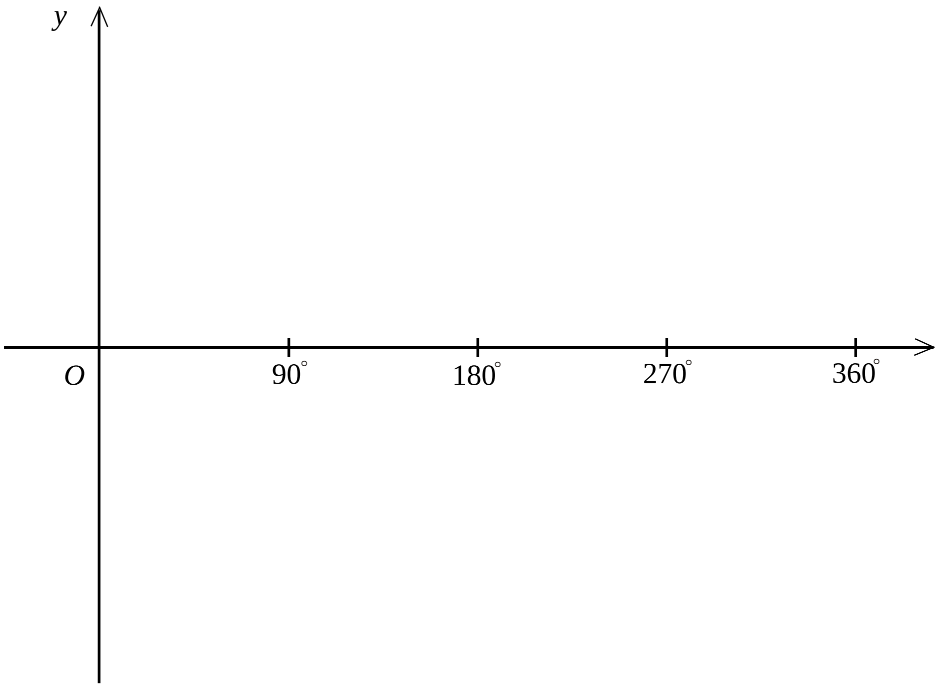
(Total 11 marks)

14. (a) Prove that

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}2\theta, \qquad \theta\neq 90n°.$$

(4)

(b) On the axes below, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.



(2)

(c) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=3,$$

giving your answers to 1 decimal place.

(6)
(Total 12 marks)

15. (a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

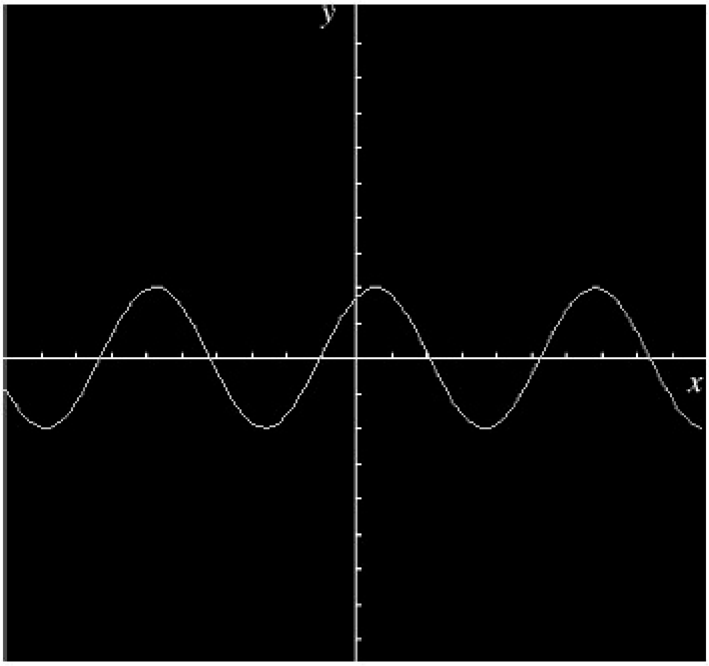
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(b) Given that $\sin\theta = \dfrac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)
(Total 7 marks)

16.



The diagram above shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3}\cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \leq x < 2\pi$, for which $y = 1$.

(4)
(Total 8 marks)

17. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x,\ -1 \le x \le 1, \text{ and } 0 \le y \le \pi$$

(a) express $\arcsin x$ in terms of y.

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)
(Total 6 marks)

18. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)
(Total 10 marks)

19. (a) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\dfrac{dy}{dx} = \sin 2x$

(4)
(Total 12 marks)

20. (a) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi, \ n \in \mathbb{Z}$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)
(Total 12 marks)

21. (a) Differentiate with respect to x

(i) x^2e^{3x+2},

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)
(Total 13 marks)

22. $f(x) = 12\cos x - 4\sin x$.

Given that $f(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leq x \leq 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)
(Total 12 marks)

23. (a) Given that $2\sin(\theta + 30)° = \cos(\theta + 60)°$, find the exact value of $\tan\theta°$.

(5)

(b) (i) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(ii) Hence solve, for $0 \le x < 2\pi$,

$$\cos 2x = \sin x,$$

giving your answers in terms of π.

(5)

(iii) Show that $\sin 2y \tan y + \cos 2y \equiv 1$, for $0 \le y < \frac{1}{2}\pi$.

(3)
(Total 15 marks)

24. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(b) Solve, for $0 \le \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)
(Total 8 marks)

25. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(b) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

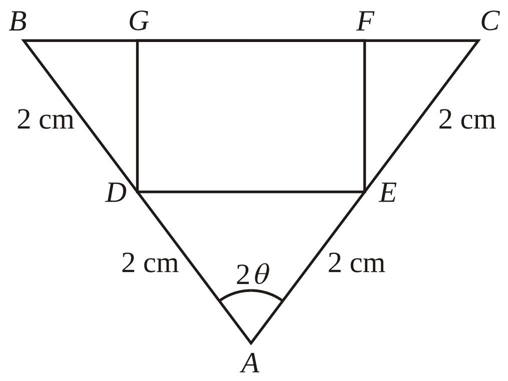
(d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)
(Total 15 marks)

26.



This diagram shows an isosceles triangle ABC with $AB = AC = 4$ cm and $\angle BAC = 2\theta$.

The mid-points of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is P cm.

(a) Show that $DE = 4 \sin \theta$.

(2)

(b) Show that $P = 8 \sin\theta + 4 \cos\theta$.

(2)

(c) Express P in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(4)

Given that $P = 8.5$,

(d) find, to 3 significant figures, the possible values of θ.

(5)
(Total 13 marks)

27. (i) Given that $\sin x = \frac{3}{5}$, use an appropriate double angle formula to find the exact value of $\sec 2x$.

(4)

(ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \qquad (x \neq \frac{n\pi}{2}, n \in \mathbb{Z}).$$

(4)
(Total 8 marks)

28. (i) (a) Express $(12 \cos \theta - 5 \sin \theta)$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 4,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(3)

(ii) Solve

$$8 \cot \theta - 3 \tan \theta = 2,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(5)
(Total 12 marks)

, $n \in \mathbb{Z}$**29.** (a) Prove that

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} \equiv \cos 2\theta.$$

(4)

(b) Hence, or otherwise, prove

$$\tan^2 \frac{\pi}{8} = 3 - 2\sqrt{2}.$$

(5)
(Total 9 marks)

30. (i) Given that $\cos(x + 30)° = 3 \cos(x - 30)°$, prove that $\tan x° = -\frac{\sqrt{3}}{2}$.

(5)

(ii) (a) Prove that $\frac{1-\cos 2\theta}{\sin 2\theta} \equiv \tan\theta$.

(3)

(b) Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(1)

(c) Using the result in part (a), or otherwise, find the other two solutions, $0 < \theta < 360°$, of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(4)
(Total 13 marks)

31. (a) Express $\sin x + \sqrt{3} \cos x$ in the form $R \sin (x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Show that the equation $\sec x + \sqrt{3} \operatorname{cosec} x = 4$ can be written in the form

$$\sin x + \sqrt{3} \cos x = 2 \sin 2x.$$

(3)

(c) Deduce from parts (a) and (b) that $\sec x + \sqrt{3} \operatorname{cosec} x = 4$ can be written in the form

$$\sin 2x - \sin (x + 60°) = 0.$$

(1)

(d) Hence, using the identity $\sin X - \sin Y = 2 \cos \dfrac{X+Y}{2} \sin \dfrac{X-Y}{2}$, or otherwise, find the values of x in the interval $0 \leq x \leq 180°$, for which $\sec x + \sqrt{3} \operatorname{cosec} x = 4$.

(5)
(Total 13 marks)

32. On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \leq x \leq 1,$

(b) $y = \sec x, \quad -\dfrac{\pi}{3} \leq x \leq \dfrac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \dfrac{\pi}{3}$ and $x = -\dfrac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

33. (a) Prove that for all values of x,

$$\sin x + \sin (60° - x) \equiv \sin (60° + x).$$

(4)

(b) Given that $\sin 84° - \sin 36° = \sin \alpha°$, deduce the exact value of the acute angle α.

(2)

(c) Solve the equation

$$4 \sin 2x + \sin (60° - 2x) = \sin (60° + 2x) - 1$$

for values of x in the interval $0 \leq x < 360°$, giving your answers to one decimal place.

(5)

(Total 11 marks)

34. Find, giving your answers to two decimal places, the values of w, x, y and z for which

(a) $e^{-w} = 4$,

(2)

(b) $\arctan x = 1$,

(2)

(c) $\ln (y + 1) - \ln y = 0.85$

(4)

(d) $\cos z + \sin z = \frac{1}{3}, -\pi < z < \pi$.

(5)

(Total 13 marks)

35. (a) Using the formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B,$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

(i) $\sin (A + B) - \sin (A - B) = 2 \cos A \sin B$,

(2)

(ii) $\cos (A - B) - \cos (A + B) = 2 \sin A \sin B$.

(2)

(b) Use the above results to show that

$$\frac{\sin(A+B)-\sin(A-B)}{\cos(A-B)-\cos(A+B)} = \cot A.$$

(3)

Using the result of part (b) and the exact values of sin 60° and cos 60°,

(c) find an exact value for cot 75° in its simplest form.

(4)
(Total 11 marks)

36. In a particular circuit the current, I amperes, is given by

$$I = 4 \sin \theta - 3 \cos \theta, \qquad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R \sin (\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha < 360°$,

(a) find the value of R, and the value of α to 1 decimal place.

(4)

(b) Hence solve the equation $4 \sin \theta - 3 \cos \theta = 3$ to find the values of θ between 0 and 360°.

(5)

(c) Write down the greatest value for I.

(1)

(d) Find the value of θ between 0 and 360° at which the greatest value of I occurs.

(2)
(Total 12 marks)