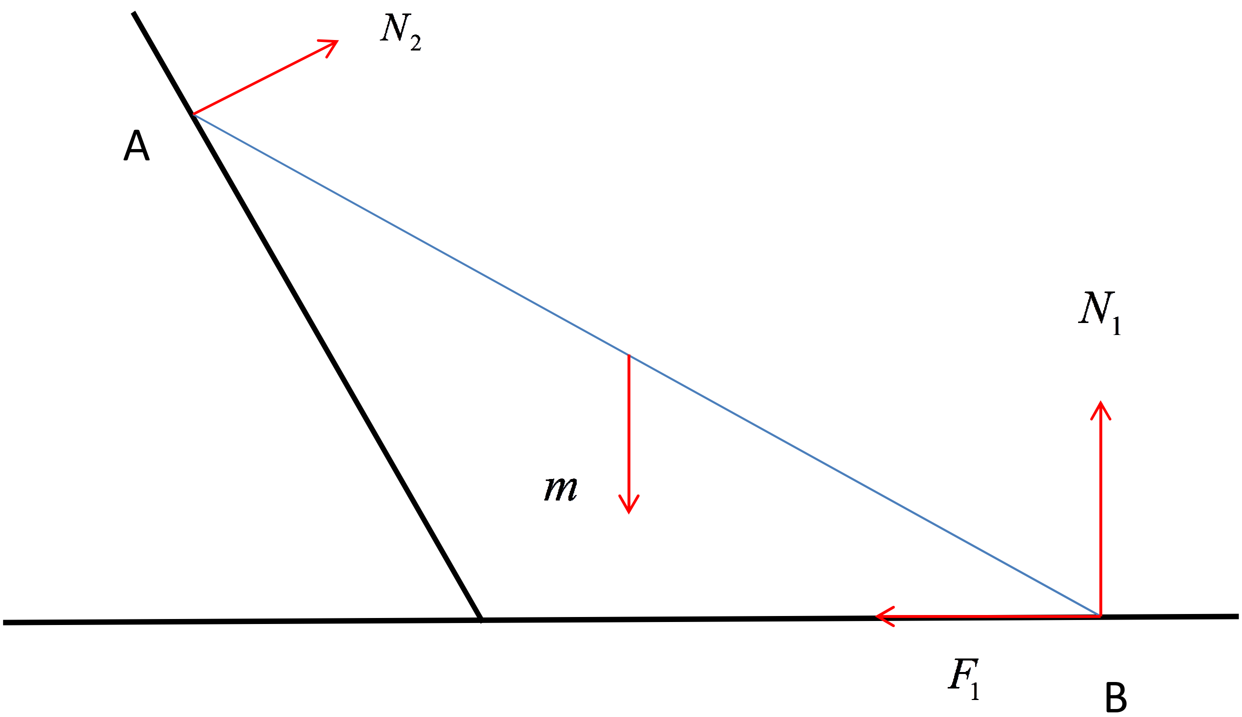
Question 11



Part (i)

Suppose the labourer of weight m is in the middle of the uniform ladder and the ladder is about to slip.

Taking moments about point A we have :-

$$2m\times\frac{l}{2}\cos\frac{\pi}{6}+F_1\times l\sin\frac{\pi}{6}=N_1\times l\cos\frac{\pi}{6}\Rightarrow m+F_1\tan\frac{\pi}{6}=N_1 \qquad (1.1)$$

Resolving perpendicular to N_2 we have :-

$$N_1\cos\frac{\pi}{6}+F_1\cos\frac{\pi}{3}=2m\cos\frac{\pi}{6}\Rightarrow N_1+F_1\tan\frac{\pi}{6}=2m \qquad (1.2)$$

Hence substituting for m from 1.2 into 1.1 we have :-

$$\frac{1}{2}\left(N_1+F_1\tan\frac{\pi}{6}\right)+F_1\tan\frac{\pi}{6}=N_1\Rightarrow\frac{3}{2}F_1\tan\frac{\pi}{6}=\frac{1}{2}N_1\Rightarrow\frac{F_1}{N_1}=\frac{1}{\sqrt{3}}$$

Part (ii)

Let the labourer be a distance of x from the point B and in that position the ladder is again about to slip.

Equation 1.2 is unaltered by the actual position of the labourer.

However we must alter equation 1.1 as follows :-

$$m\times\frac{l}{2}\cos\frac{\pi}{6}+m\times(l-x)\cos\frac{\pi}{6}+F_1\times l\sin\frac{\pi}{6}=N_1\times l\cos\frac{\pi}{6}$$

$$\Rightarrow\frac{ml}{2}+m(l-x)+F_1l\tan\frac{\pi}{6}=N_1l$$

$$\Rightarrow m\left(\frac{3l}{2}-x\right)+F_1l\tan\frac{\pi}{6}=N_1l$$

Using the substitution $m=\frac{1}{2}\left(N_1+F_1\tan\frac{\pi}{6}\right)$ from 1.2 we have :-

$$\frac{1}{2}\left(N_1+F_1\tan\frac{\pi}{6}\right)\left(\frac{3l}{2}-x\right)+F_1l\tan\frac{\pi}{6}=N_1l$$

$$\Rightarrow\frac{1}{2}F_1\tan\frac{\pi}{6}\left(\frac{3l}{2}-x\right)+F_1l\tan\frac{\pi}{6}=N_1l-\frac{1}{2}N_1\left(\frac{3l}{2}-x\right)$$

$$\Rightarrow F_1\tan\frac{\pi}{6}\left(\frac{7l}{4}-\frac{x}{2}\right)=N_1\left(\frac{l}{4}+\frac{x}{2}\right)$$

$$\Rightarrow\frac{F}{N_1}=\sqrt{3}\left(\frac{l+2x}{7l-2x}\right)$$

As x increases from 0 to l we clearly have an increasing function (the top line gets bigger and the bottom line smaller).

Therefore the maximum value for the coefficient of friction so that the labourer can reach the top of the ladder occurs when $x=l\Rightarrow\mu=\frac{3\sqrt{3}}{5}$
