

1.

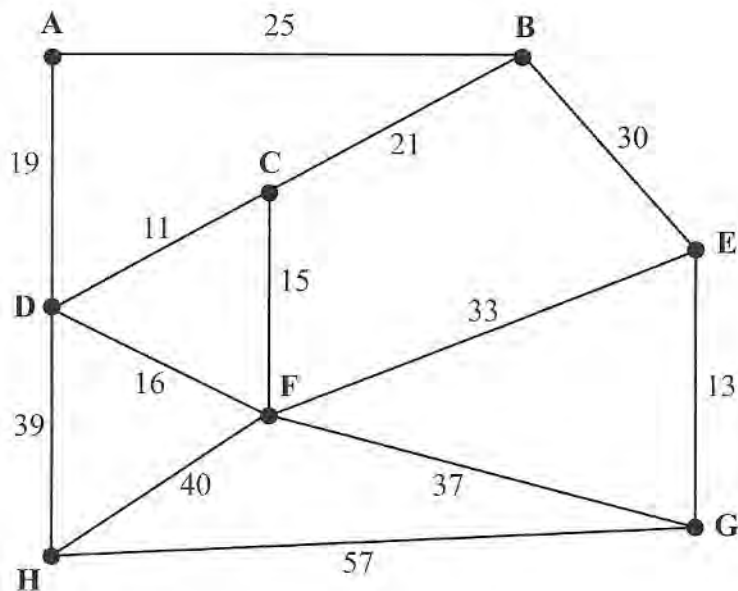


Figure 1

Figure 1 represents the distances, in km, between eight vertices, A, B, C, D, E, F, G and H in a network.

- Use **Kruskal's** algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree. (3)
- Starting at A, use **Prim's** algorithm to find the minimum spanning tree. You must clearly state the order in which you selected the arcs of your tree. (3)
- Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book. (1)
- State the weight of the tree. (1)

(Total 8 marks)

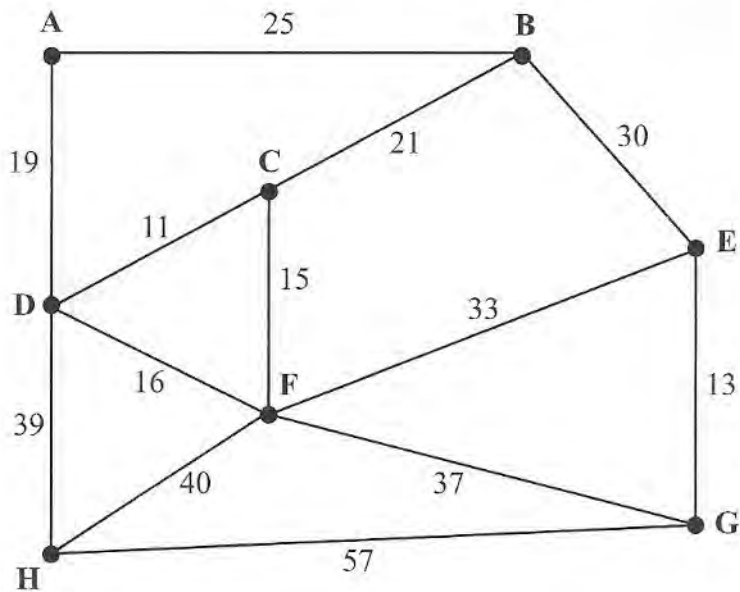
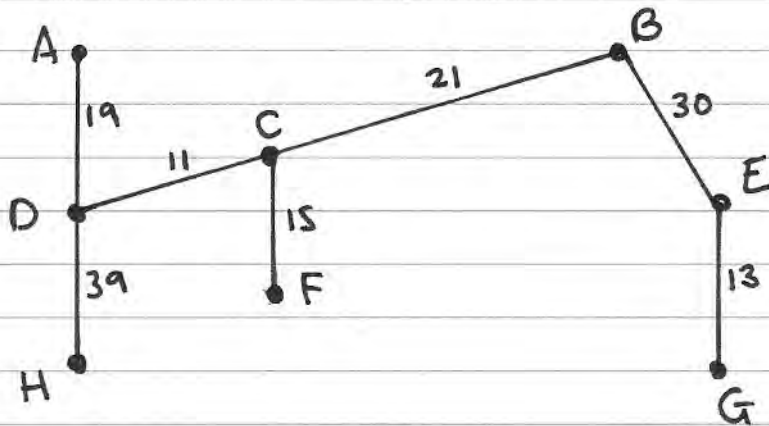


Figure 1

a) Kruskal's

CD, EG, CF, DF ~~rej~~, AD, CB, AB ~~rej~~, BE, EF ~~rej~~,
FG ~~rej~~, DH

b) Prim's : AD, DC, CF, CB, BE, EG, DH



Weight
= 148

2.

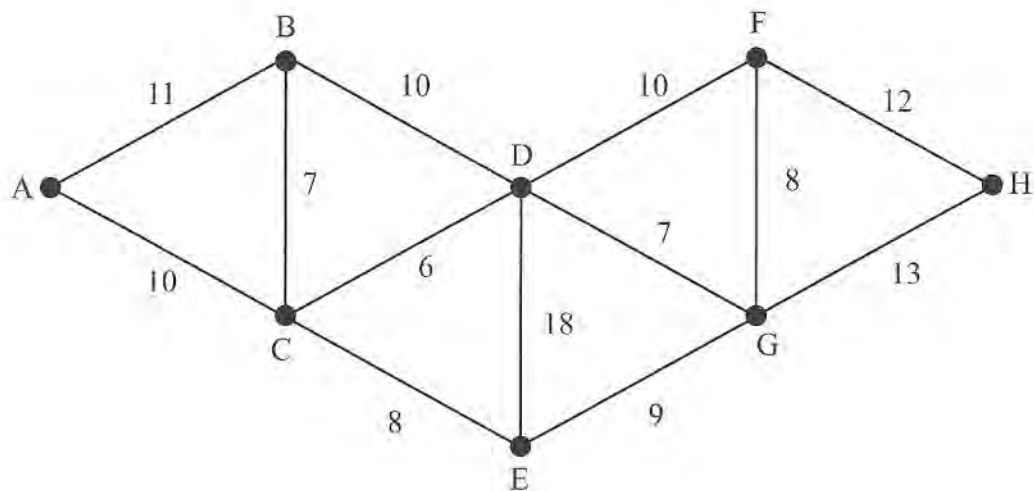


Figure 2

[The weight of the network is 129 miles]

Figure 2 models a network of canals. The number on each arc gives the length, in miles, of that canal.

Brett needs to travel along each canal to check that it is in good repair. He wishes to minimise the length of his route.

- (a) Use the route inspection algorithm to find the length of his route. State the arcs that should be repeated. You should make your method and working clear.

(6)

A canal between B and F, of length 12 miles, is to be opened and needs to be included in Brett's inspection route.

- (b) Determine if the addition of this canal will increase or decrease the length of Brett's minimum route. You must make your reasoning clear.

(2)

(Total 8 marks)

2.

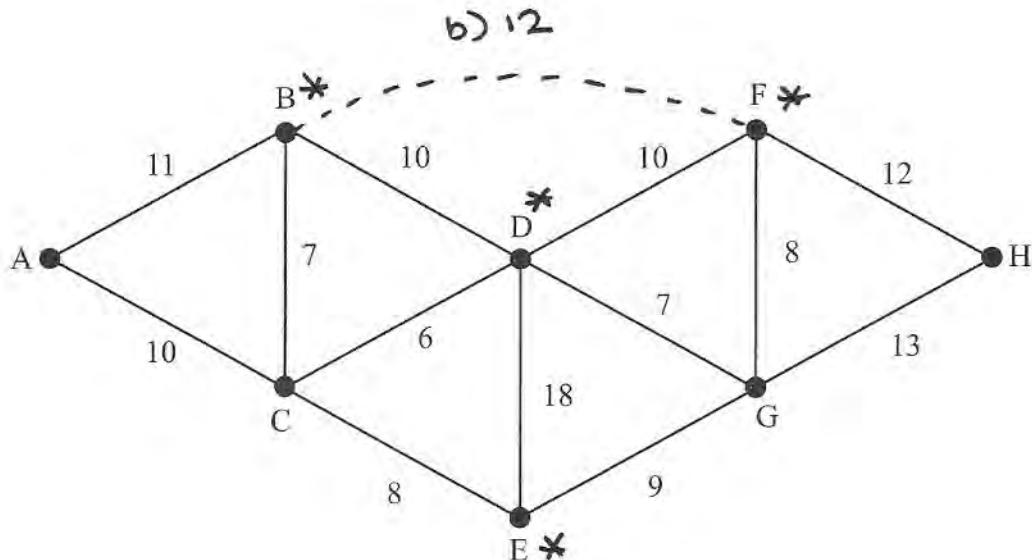


Figure 2

a)

$$BD \underset{G}{10} \quad EF \underset{G}{17} \Rightarrow 27$$

traverse

$$B \underset{C}{E} \underset{C}{15} \quad DF \underset{D}{10} \Rightarrow 25 *$$

BC, CE, DF
twice

$$B \underset{D}{F} \underset{D}{20} \quad DE \underset{C}{14} \Rightarrow 34$$

$$\text{length} = 129 + 25 \\ = \underline{154 \text{ miles}}$$

b) B, f now even, so only $D \rightarrow E$ needs to be repeated $\therefore$

roads DC, CE will be traversed twice
so his new route will be of length
 $129 + 14 + 12 = 155 \text{ miles}$

$\therefore$ So his new route will increase by 1 mile

3.

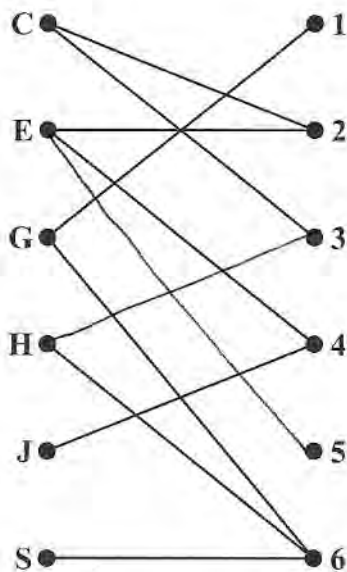


Figure 3

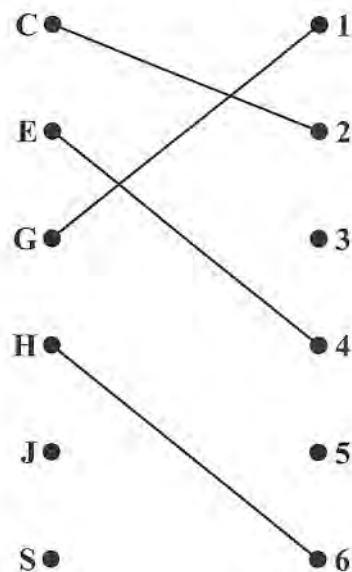


Figure 4

Define the terms

(a) bipartite graph,

(2)

(b) matching.

(2)

Figure 3 shows the possible allocations of six people, Charles (C), Emily (E), George (G), Harriet (H), Jack (J) and Shen (S), to six tasks, 1, 2, 3, 4, 5 and 6.

Figure 4 shows an initial matching.

(c) Starting from this initial matching, use the maximum matching algorithm to find an improved matching. You should list the alternating path you use and state your improved matching.

(3)

Emily has task 5 added to her possible allocations and Harriet has task 3 added to her possible allocations.

(d) Starting from the improved matching found in part (c), use the maximum matching algorithm to find a complete matching. You should list the alternating path you use and state your improved matching.

(3)

(Total 10 marks)

3.

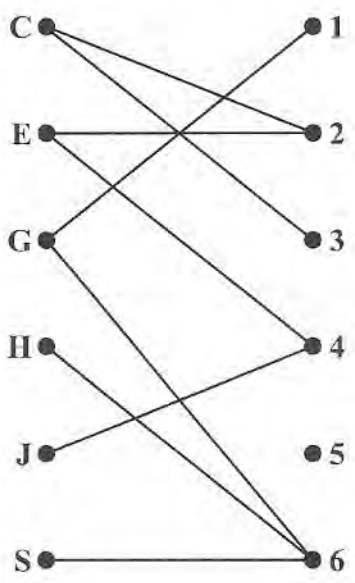


Figure 3

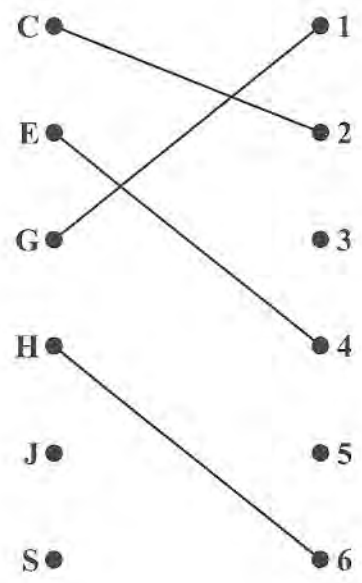


Figure 4

a) a bipartite graphs consists of two distinct sets of nodes; nodes in set X are connected to nodes in set Y by arcs. However, nodes in set X can not be connected to other nodes in set X, similarly for those in set Y.

b) a matching is when some nodes in set X are connected to nodes in set Y in a 1-1 manner

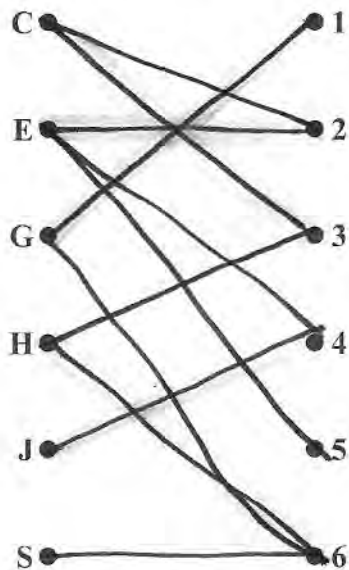
c) $J=4 = E=2 = C=3$
 change status
 $J=4 - E=2 - C=3$

Improved matching
 $J=4 \ E=2 \ C=3 \ G=1 \ H=6$

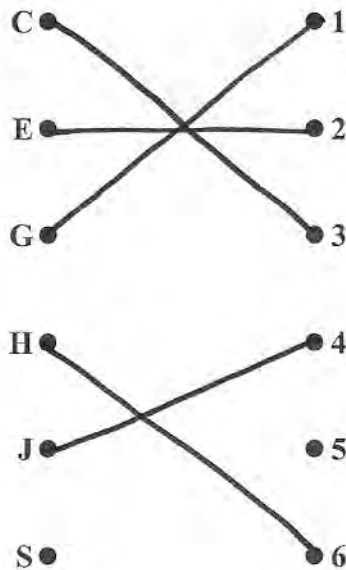
Question 3 continued

d)

new bipartite



Improved matching
(e)



$$S-6=H-3=C-2=E-5$$

change status

$$S=6-H=3-C=2-E=5$$

maximum matching

$$C=2 \ E=5 \ G=1 \ H=3 \ J=4 \ S=6$$

4.

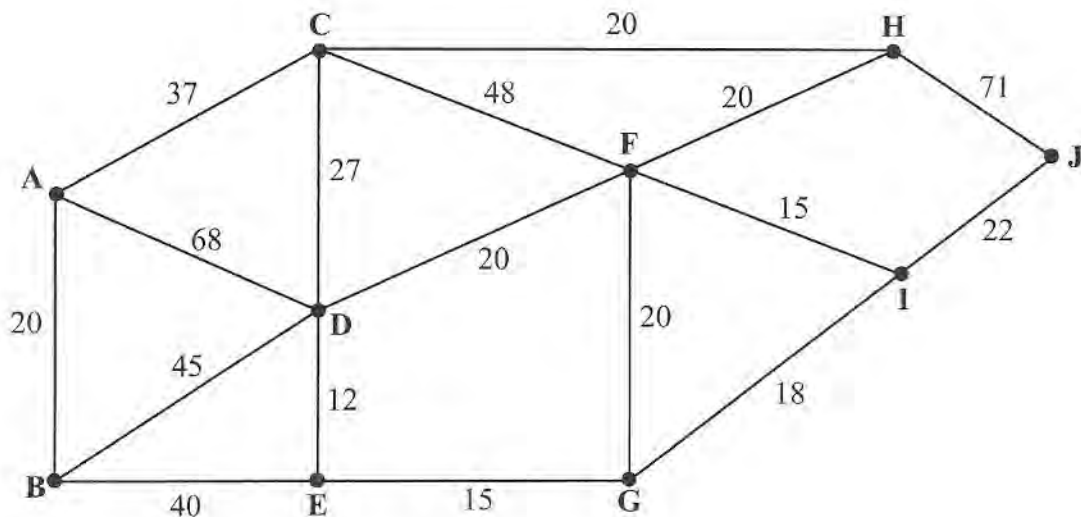


Figure 5

Figure 5 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Olivia wishes to travel from A to J as quickly as possible.

- (a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J. State the shortest route.

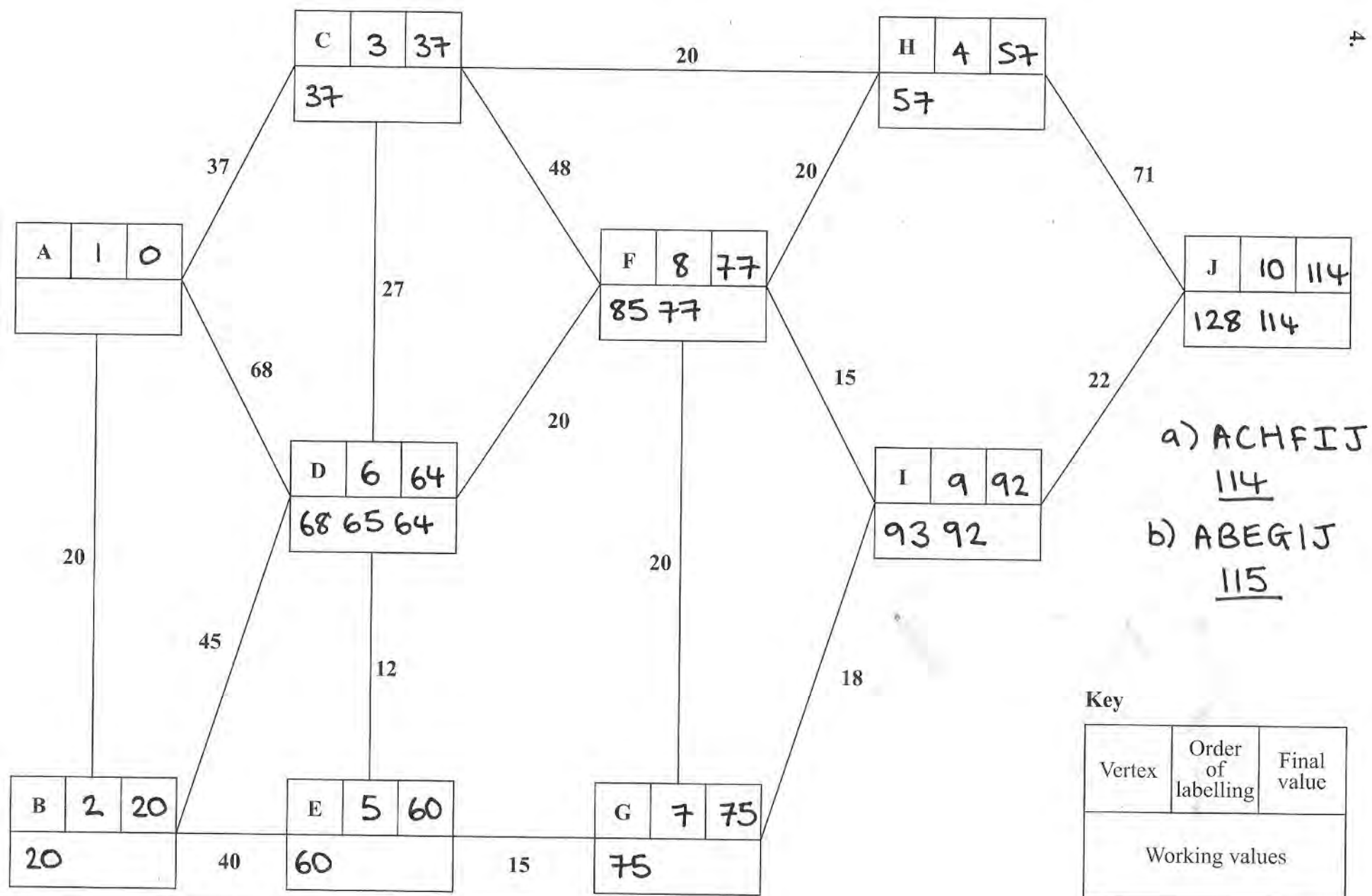
(7)

On a particular day Olivia must include G in her route.

- (b) Find a route of minimal time from A to J that includes G, and state its length

(2)

(Total 9 marks)



5.

5 1 8 13 16 5 8 2 15 12 10

- (a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 20. (3)
- (b) The list of numbers is to be sorted into **descending** order. Use a bubble sort to obtain the sorted list, giving the state of the list after each **complete** pass. (5)
- (c) Apply the first-fit decreasing bin packing algorithm to your ordered list to pack the numbers into bins of size 20. (3)
- (d) Determine whether your answer to (c) uses the minimum number of bins. You must justify your answer. (2)

(Total 13 marks)

Bin 1: 5, 1, 8, 5

Bin 2: 13, 2

Bin 3: 16

Bin 4: 8, 12

Bin 5: 15

Bin 6: 10

6 bins, 25 units of waste

b)

	5	1	8	13	16	5	8	2	15	12	10
①	5	8	13	16	5	8	2	15	12	10	1
②	8	13	16	5	8	5	15	12	10	2	1
③	13	16	8	8	5	15	12	10	5	2	1
④	16	13	8	8	15	12	10	5	5	2	1
⑤	16	13	8	15	12	10	8	5	5	2	1
⑥	16	13	15	12	10	8	8	5	5	2	1
⑦	16	15	13	12	10	8	8	5	5	2	1
⑧	16	15	13	12	10	8	8	5	5	2	1

8 passes

c) Bin 1: 16, 2, 1

Bin 2: 15, 5

Bin 3: 13, 5

Bin 4: 12, 8

Bin 5: 10, 8

5 bins, 5 units of waste

$$d) \text{ lower bound} = \frac{95}{20} = 4.75$$

$\Rightarrow$ lower bound = 5 bins, so the answer to (c) does use the minimum number of bins

Edgar has recently bought a field in which he intends to plant apple trees and plum trees.

He can use linear programming to determine the number of each type of tree he should plant.

Let x be the number of apple trees he plants and y be the number of plum trees he plants.

Two of the constraints are

$$x \geq 40$$

$$y \leq 50$$

These are shown on the graph in Figure 6, where the rejected region is shaded out.

- (a) Use these two constraints to write down two statements that describe the number of apple trees and plum trees Edgar can plant.

(1)

Two further constraints are

$$3x + 4y \leq 360$$

$$x \leq 2y$$

- (b) Add two lines and shading to Diagram 1 in your answer book to represent these inequalities. Hence determine the feasible region and label it R.

(4)

Edgar will make a profit of £60 from each apple tree and £20 from each plum tree. He wishes to maximise his profit, P.

- (c) Write down the objective function.

(1)

- (d) Use an objective line to determine the optimal point of the feasible region, R. You must make your method clear.

(4)

- (e) Find Edgar's maximum profit.

(1)

(Total 11 marks)

6.

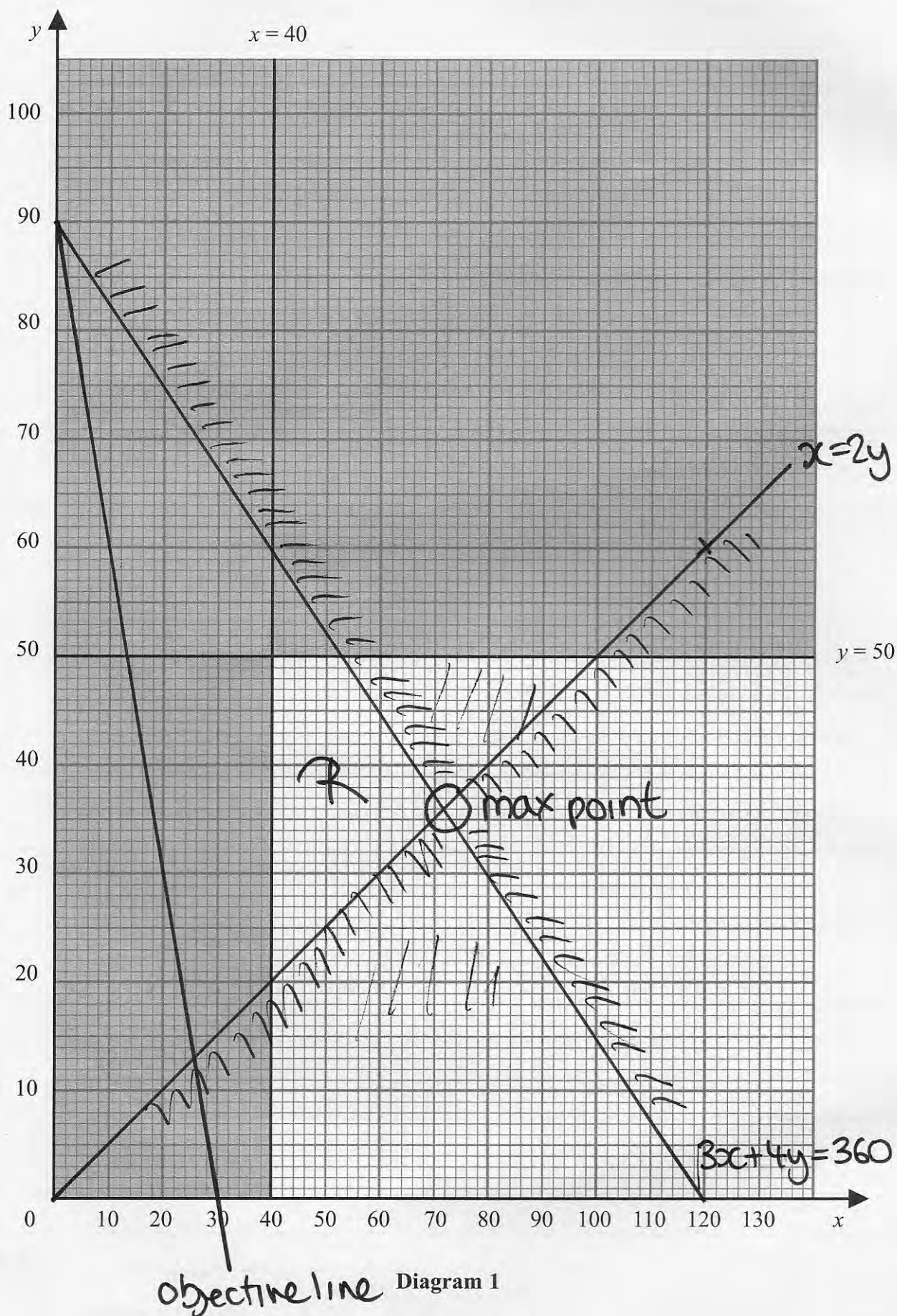


Diagram 1

a) $x \geq 40 \Rightarrow$ at least 40 apple trees must be planted

$y \leq 50 \Rightarrow$ at most 50 plum trees should be planted

c) max Profit $P(\pounds) = 60x + 20y$

d) objective line $(20,0) \rightarrow (0,60)$
 $\times 3 \quad (60,0) \rightarrow (0,180) \quad \times$
 $\div 2 \quad (30,0) \rightarrow (0,90)$

optimal point $(72,36)$

check $x=2y$ meets $3x+4y=360$

$\Rightarrow 6y+4y=360 \Rightarrow 10y=360 \Rightarrow y=36 \Rightarrow x=72 \checkmark$

e) max profit $= 60 \times 72 + 20 \times 36 = \underline{\pounds 5040}$

7.

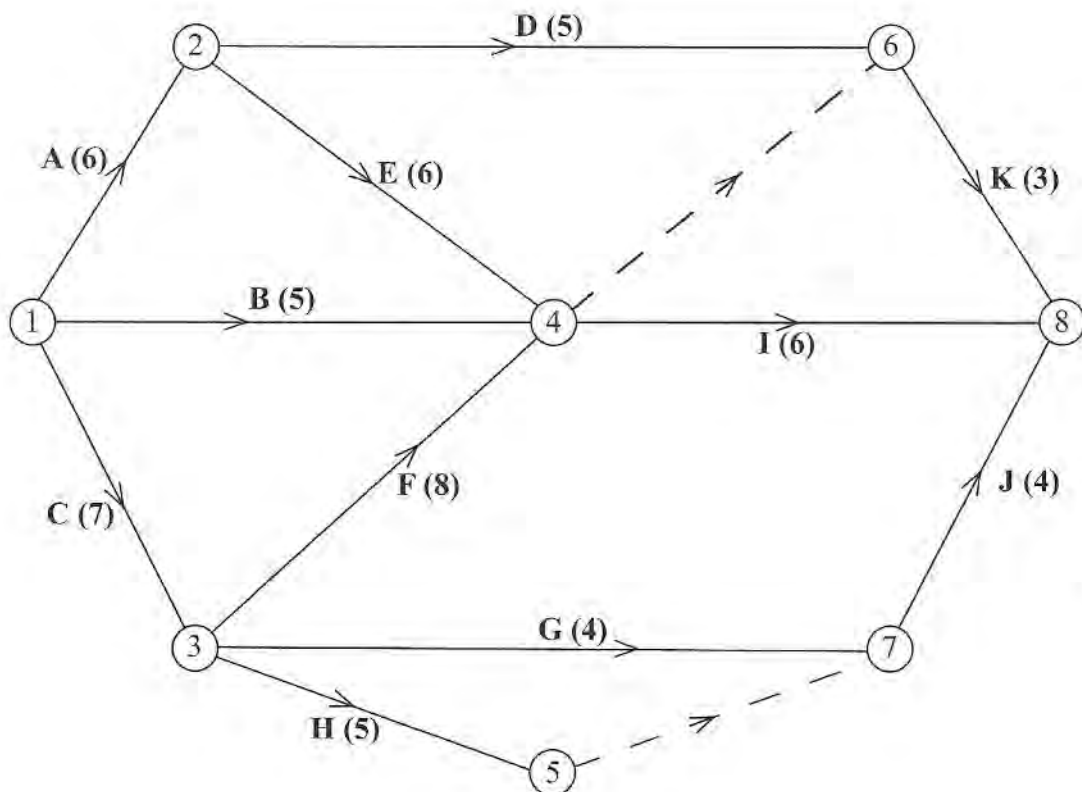


Figure 7

A project is modelled by the activity network shown in Figure 7. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

- (a) Explain the significance of the dummy activity
 - (i) from event 4 to event 6,
 - (ii) from event 5 to event 7

(3)
- (b) Calculate the early time and the late time for each event. Write these in the boxes in the answer book.

(4)
- (c) Calculate the total float on each of activities D and G. You must make the numbers you use in your calculations clear.

(3)
- (d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time.

(2)
- (e) On the grid in your answer book, draw a cascade (Gantt) chart for this project.

(4)

(Total 16 marks)

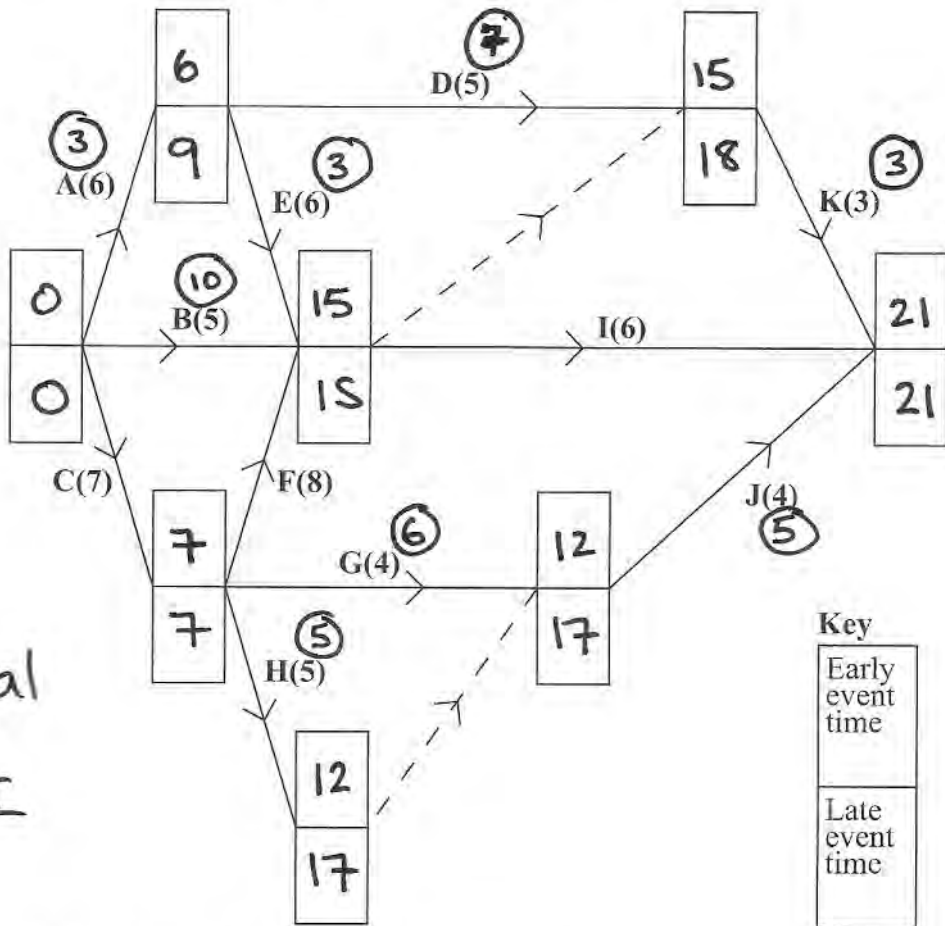
TOTAL FOR PAPER: 75 MARKS

END

7. (a) (i) Dummy required from ④ → ⑥ since activity I depends on activities B, E, F but activity K depends on activities B, E, F and D

(ii) Dummy required from ⑤ → ⑦ since no two activities may share the same start event and end event

(b)



critical path
CFI

(c) Float D $\Rightarrow 18 - 5 - 6 = 7$
Float G $\Rightarrow 17 - 4 - 7 = 6$

d) lower bound $= \frac{59}{21} = 2.81 \rightarrow \underline{3 \text{ workers}}$

0 2 4 6 8 10 12 14 16 18 20 22

