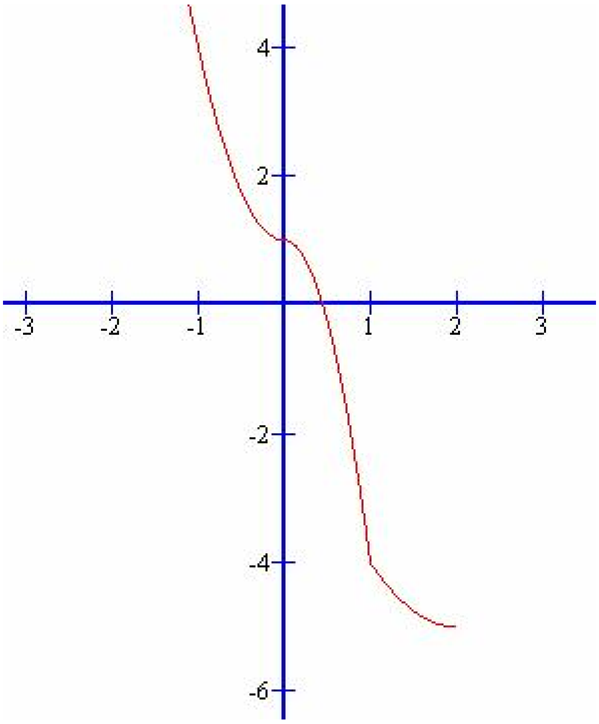

3. $f(x) = 2|x^2 - x| + |x^2 - 1| - 2|x^2 + x|$

critical values of x are $x = 0$ and $x = \pm 1$

$x < -1 \Rightarrow f(x) = 2(x^2 - x) + x^2 - 1 - 2(x^2 + x) = x^2 - 4x - 1$

$f(-1) = 4$

$-1 < x < 0 \Rightarrow f(x) = 3x^2 + 1$

$f(0) = 1$

$0 < x < 1 \Rightarrow f(x) = -5x^2 + 1$

$f(1) = -4$

$x > 1 \Rightarrow f(x) = x^2 - 4x - 1$

so graph is a sequence of portions of the above quadratic curves as shown

$\frac{d}{dx}\left(x^2 - 4x - 1\right) = 2x - 4 = 0$ when $x = 2$

$\frac{d}{dx}\left(3x^2 + 1\right) = 6x = 0$ when $x = 0$

$\frac{d}{dx}\left(-5x^2 + 1\right) = -10x = 0$ when $x = 0$

curve has a point of inflexiuon at $x = 0,\ y = 1$

and a minimum at $x = 2,\ y = -5$

f is continuous everywhere as is obvious from the graph.

Not differentiable at $x = 1, y = -4$, gradients either side are -10 on the left but -2 on the right.

$y = f(x) = x^2 - 4x - 1 \Rightarrow x = \frac{1}{2}\left(4 \pm \sqrt{16 + 4(y+1)}\right) = 2 \pm \sqrt{5 + y}$

$y = 3x^2 + 1 \Rightarrow x = \pm\sqrt{\frac{y-1}{3}},\quad y = 1 - 5x^2 \Rightarrow x = \pm\sqrt{\frac{1-y}{5}}$

so inverse functions are: in $x < -1,\ f^{-1}(x) = 2 - \sqrt{5 + x}$; in $-1 < x < 0;\ f^{-1}(x) = -\sqrt{\frac{x-1}{3}}$

in $0 < x < 1;\ f^{-1}(x) = \sqrt{\frac{1-x}{5}}$ and in $x > 1;\ f^{-1}(x) = 2 - \sqrt{5 + x}$
