Types of numbers

Square roots and cube roots

$\sqrt{\ }$ is the square root sign. Taking the square root is the opposite of squaring.

For example, $\sqrt{25} = \pm 5$ since $5^2 = 25$, or $(-5)^2 = 25$.

$\sqrt[3]{\ }$ is the cube root sign. Taking the cube root is the opposite of cubing.

For example, $\sqrt[3]{8} = 2$ since $2^3 = 8$.

Squares and cubes

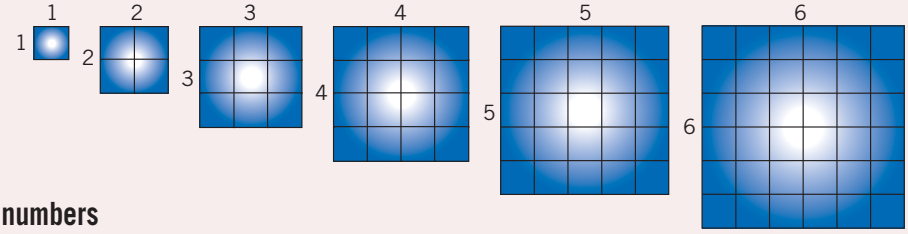
Square numbers

Any number raised to the **power 2** gives a **square number**. For example, $6^2 = 6 \times 6 = 36$ (six squared).

Square numbers include:

1	4	9	16	25	36	49	64	81	100	...
(1×1)	(2×2)	(3×3)	(4×4)	(5×5)	(6×6)	(7×7)	(8×8)	(9×9)	(10×10)	

Square numbers can be illustrated by drawing squares:



You need to know up to 15^2.

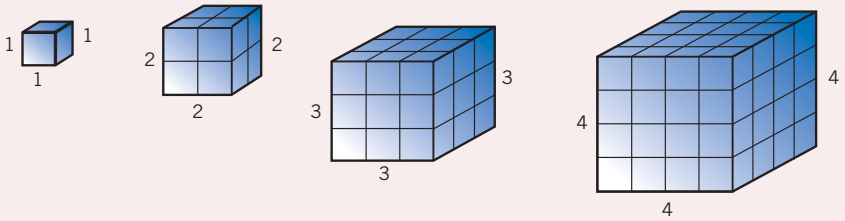
Cube numbers

Any number raised to the **power 3** gives a **cube number**. For example, $5^3 = 5 \times 5 \times 5 = 125$ (five cubed).

Cube numbers include:

1	8	27	64	125	216 ...	1000 ...
$(1 \times 1 \times 1)$	$(2 \times 2 \times 2)$	$(3 \times 3 \times 3)$	$(4 \times 4 \times 4)$	$(5 \times 5 \times 5)$	$(6 \times 6 \times 6)$	$(10 \times 10 \times 10)$

Cube numbers can be illustrated by drawing cubes:



It is important that you recognise square and cube numbers because they often appear in number sequences.

Multiples

Multiples are the numbers that appear in multiplication tables.

For example, multiples of 8 are 8, 16, 24, 32, 40, ...

Reciprocals

The **reciprocal** of a number $\frac{a}{x}$ is $\frac{x}{a}$ $(= x \div a)$.

Multiplying a number by its reciprocal always gives 1.

Zero has no reciprocal.

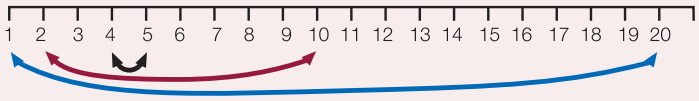
Examples

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.
- The reciprocal of 4 is $\frac{1}{4}$. (4 is the same as $\frac{4}{1}$.)
- To find the reciprocal of $1\frac{2}{3}$, first put it in the form $\frac{a}{x}$ ($1\frac{2}{3} = \frac{5}{3}$), then invert it ($\frac{3}{5}$).

Factors and primes

Factors

Factors are whole numbers which divide exactly into another number. For example, the factors of 20 are 1, 2, 4, 5, 10, 20. To find all the factors of a number, start at 1 and divide by each whole number in turn. Factors can be split up into factor pairs. For example, for the factors of 20:



So $1 \times 20 = 20$ $2 \times 10 = 20$ $4 \times 5 = 20$

Prime numbers

A **prime number** is a number which has only two factors, 1 and itself. Note that 1 is not a prime number. The prime numbers up to 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

Prime factors

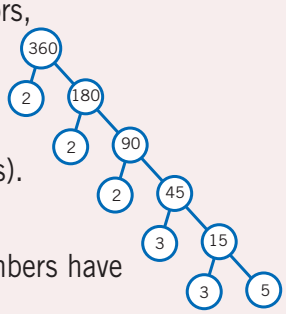
These are factors that are prime.
All whole numbers can be written as products of their prime factors.

Example

The diagram below shows the prime factors of 360.

- Divide 360 by its first prime factor, 2.
- Divide 180 by its first prime factor, 2.
- Keep on going until the final number is prime.

As a product of its prime factors, 360 can be written as:
$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$
or $2^3 \times 3^2 \times 5 = 360$
in **index** notation (using powers).



Highest common factor (HCF)

The **largest factor** that two numbers have in common is called the **HCF**.

Example

Find the HCF of 84 and 360.

- Write the numbers as products of their prime factors.
  $84 = 2 \times 2 \times 3 \times 7$
  $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
- Ring the common factors
- These give the HCF $= 2 \times 2 \times 3 = 12$

Lowest common multiple (LCM)

The **LCM** is the lowest number that is a multiple of two or more numbers.

Example

Find the LCM of 6 and 8.

- Write the numbers as products of their prime factors.
  $8 = 2 \times 2 \times 2$
  $6 = 2 \times 3$
- 8 and 6 have a common prime factor of 2. It is only counted once.
- The LCM of 6 and 8 is $2 \times 2 \times 2 \times 3 = 24$

KEY TERMS

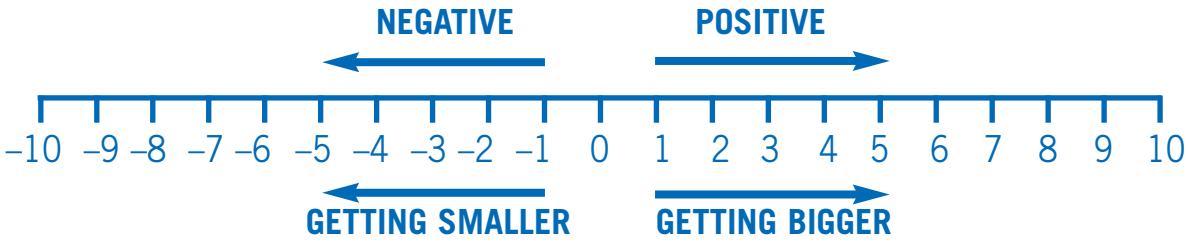
Make sure you understand these terms before moving on!

- square number
- cube number
- multiple
- reciprocal
- factor
- prime number
- HCF
- LCM

QUICK TEST

1. List the prime numbers up to 20.
2. Find the HCF and LCM of 24 and 60.
3. Find a) $\sqrt{64}$ b) $\sqrt[3]{216}$
4. Write down the reciprocals of
   a) $\frac{9}{12}$ b) $\frac{x}{p}$

Positive and negative numbers



Directed numbers

Directed numbers are numbers that are either **positive** or **negative**.
Positive numbers are above zero, negative are below zero.

Examples

–10 is smaller than –8. $-10 < -8$
–4 is bigger than –8. $-4 > -8$
2 is bigger than –6. $2 > -6$

Directed numbers are often seen on the weather forecast in winter. Quite often the **temperature** is below 0°C.

Example

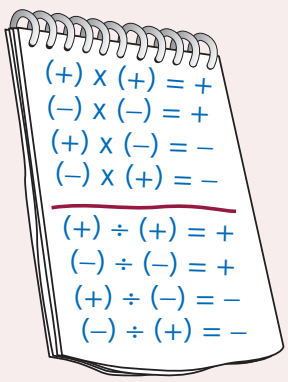
On this weather map, Aberdeen is the coldest at –8°C.
London is 6 degrees warmer than Manchester.

Integers

The **integers** are the set of numbers {. . ., –3, –2, –1, 0, 1, 2, 3, ...}.
When referring to integers, the term **integral value** is used.
A number that is **non–integral** is not an integer.

Multiplying and dividing directed numbers

Multiply and divide the numbers as normal.
Then find the sign for the answer using these rules:



- Two **like** signs (both + or both –) give a positive answer.
- Two **unlike** signs (one + and the other –) give a negative answer.

Examples

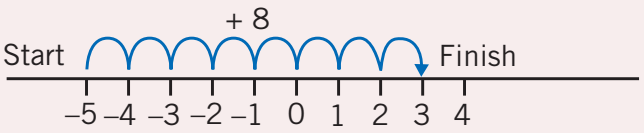
$-6 \times (+4) = -24$ $-12 \div (-3) = 4$
$-6 \times (-3) = 18$ $20 \div (-4) = -5$

These rules of multiplication/division need to be remembered.
You can also easily use these laws when multiplying out brackets in algebra.

Adding and subtracting directed numbers

Example

The temperature at 6 a.m. was –5°C. By 10 a.m. it had risen 8 degrees.
So the new temperature was 3°C.

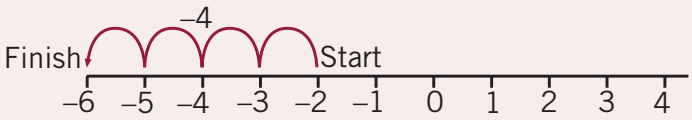


Example

Find the value of –2 – 4.

This represents the sign of the number. Start at –2.

This represents the operation of subtraction. Move 4 places to the left.



So –2 – 4 = –6.

- When the number to be added (or subtracted) is **negative**, the normal direction of movement is **reversed**.

Example

–4 – (–3) is the same as –4 + 3 = –1

The negative changes the **direction**. Move 3 places to the **right**.

- When two (+) or two (–) signs are together, these rules are used:

+(+) → +
–(–) → +
} **like** signs give a **positive**

+(–) → –
–(+) → –
} **unlike** signs give a **negative**.

Examples

–6 + (–2) = –6 – 2 = –8 –2 – (+6) = –2 – 6 = –8
4 – (–3) = 4 + 3 = 7 9 + (–3) = 9 – 3 = 6

If you find working with directed numbers difficult, sketch a quick number line to help you.

Negative numbers on the calculator

The [+/–] or [(–)] key on the calculator gives a **negative** number.

For example, to get –6, press [6] [+/–] or [(–)] [6].

Example

–4 – (–2) = –2
is keyed in the calculator like this:

[4] [+/–] [–] [2] [+/–] [=]

sign operation sign

Make sure you know how to enter negative numbers in your calculator.

KEY TERMS

Make sure you understand these terms before moving on!

- directed numbers
- integers

QUICK TEST

1. If the temperature was –12°C at 2 a.m., and it rose by 15 degrees by 11 a.m., what was the temperature at 11 a.m.?

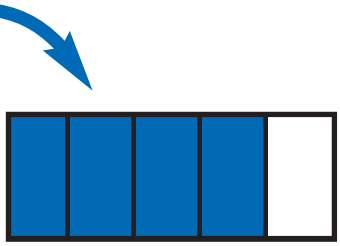
2. Work these out, without a calculator.

a) –2 – (–6) b) –9 + (–7) c) –2 × 6
d) –9 + (–3) e) –20 ÷ (–4) f) –18 ÷ (–3)
g) 4 – (–3) h) –7 + (–3) i) –9 × –4

Fractions

A fraction is a part of a whole.
The top number is the *numerator*.
The bottom one is the *denominator*.
A fraction such as $\frac{4}{5}$ is called a *proper fraction*.
A fraction such as $\frac{24}{17}$ is called an *improper fraction*.
$2\frac{1}{2}$ is called a mixed number.

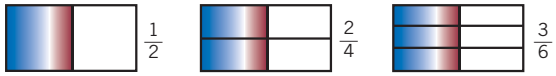
$\frac{4}{5}$ means 4 parts out of 5.



Equivalent fractions

Equivalent fractions have the same value.

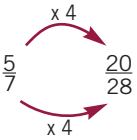
Example



From the diagrams it can be seen that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

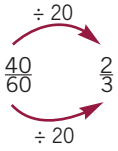
They are equivalent fractions. Fractions can be changed to their equivalents by **multiplying** or **dividing** both the numerator and denominator by the same amount.

Examples



Change $\frac{5}{7}$ to its equivalent fraction with a denominator of 28.

- Multiply top and bottom by 4.
- So $\frac{5}{7}$ is equivalent to $\frac{20}{28}$.



Change $\frac{40}{60}$ to its equivalent fraction with a denominator of 3.

Divide top and bottom by 20.
So $\frac{40}{60}$ is equivalent to $\frac{2}{3}$.

Using the fraction key on the calculator

a b/c is the fraction key on the calculator.

Example

$\frac{12}{18}$ is keyed in as 1 2 a b/c 1 8. This is displayed as 12_18 or 12r18.
The calculator will automatically cancel down fractions when the = key is pressed.
For example, $\frac{12}{18}$ becomes 2_3 or 2r3. This means two-thirds.

A display of 1_4_9 means $1\frac{4}{9}$. If you now press shift a b/c, it converts back to an improper fraction, 13_9 ($\frac{13}{9}$).

Multiplication and division of fractions

When multiplying and dividing fractions, write out whole or mixed numbers as improper fractions before starting.

Example

$1\frac{2}{9} \times \frac{4}{7} = \frac{11}{9} \times \frac{4}{7} = \frac{44}{63}$

← Multiply numerators together.
← Multiply denominators together.

Change a division into a multiplication by turning the second fraction upside down and multiplying both fractions together; to divide by a fraction, **multiply by the reciprocal**.

Example

$\frac{7}{9} \div \frac{12}{18} = \frac{7}{9} \times \frac{18}{12}$

$= \frac{126}{108}$

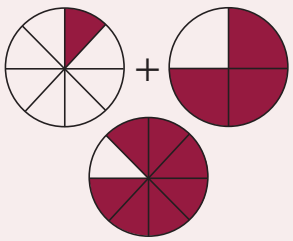
$= 1\frac{1}{6}$

Take the **reciprocal** of the **second fraction**.

Rewrite the answer as a mixed number.

Addition and subtraction of fractions

These examples show the basic principles of adding and subtracting fractions.



Example

$\frac{1}{8} + \frac{3}{4}$

- First make the denominators the same:

$\frac{3}{4} = \frac{6}{8}$ (x2 above, x2 below)

$\frac{3}{4}$ is **equivalent** to $\frac{6}{8}$.

$= \frac{1}{8} + \frac{6}{8}$

- Replace $\frac{3}{4}$ with $\frac{6}{8}$.

$= \frac{7}{8}$

- Add the numerators 1 + 6 = 7.
  Do not add the denominators; the denominator stays the same.

Example

$\frac{9}{12} - \frac{1}{3}$

- First make the denominators the same:

$\frac{1}{3} = \frac{4}{12}$ (x4 above, x4 below)

$\frac{1}{3}$ is equivalent to $\frac{4}{12}$.

$= \frac{9}{12} - \frac{4}{12}$

- Replace the $\frac{1}{3}$ with $\frac{4}{12}$.

$= \frac{5}{12}$

- Subtract the numerators but not the denominators; the denominator stays the same .

On a calculator, you would type in: 9 a⅞ 12 – 1 a⅞ 3 =

Proportional changes with fractions and percentages

Increase and decrease

There are two methods. Use the one that is familiar to you. Remember that 'of' means multiply.

Example

Last year a gym had 290 members.
This year there are $\frac{3}{5}$ more.
How many members are there now?

Method 1

$\frac{3}{5} \times 290 = 174$ — Work out $\frac{3}{5}$ **of** 290.

290 + 174 = 464 people — Add this to the original number.

Method 2

Increasing by $\frac{3}{5}$ is the same as multiplying by $1\frac{3}{5}$ $(1 + \frac{3}{5})$:

$1\frac{3}{5} \times 290 = 464$

On the calculator, key in 1 a⅞ 3 a⅞ 5 x 290 =

Questions involving fractions are quite common on the non-calculator paper. Learn the quick way of finding a fraction of a quantity.

KEY TERMS

Make sure you understand these terms before moving on!

- numerator
- denominator
- proper fraction
- improper fraction

QUICK TEST

1. Without using a calculator, work out the following:

   a) $\frac{2}{9} + \frac{3}{27}$ b) $\frac{3}{5} - \frac{1}{4}$ c) $\frac{6}{9} \times \frac{72}{104}$ d) $\frac{8}{9} \div \frac{2}{3}$

   e) $\frac{4}{7} - \frac{1}{3}$ f) $\frac{2}{7} \div 1\frac{1}{2}$ g) $\frac{7}{11} \div \frac{22}{14}$ h) $\frac{2}{9} + \frac{4}{7}$

2. Calculate $\frac{2}{9}$ of £180.

3. $\frac{7}{12}$ more rain fell this year than last.
   If 156 mm fell last year, how much fell this year?

Decimals

A *decimal point* is used to separate whole number columns from fractional columns.

Example

Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths
5	9	2	4	.	1	6	3

The 1 means $\frac{1}{10}$, the 6 means $\frac{6}{100}$, the 3 means $\frac{3}{1000}$

Recurring decimals

A decimal that recurs is shown by placing a dot over the first and last digits in a group of numbers that repeat.

Examples

$0.333\ldots = 0.\dot{3}$ $0.17777\ldots = 0.1\dot{7}$ $0.232323\ldots = 0.\dot{2}\dot{3}$

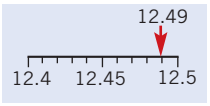
Decimal places (d.p.)

When rounding numbers to a specified number of decimal places:

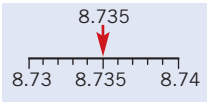
- look at the last number that is wanted (e.g. if rounding 12.367 to 2 d.p., look at the 6 which is in the second d.p.);
- look at the number to the right of it (the number which is not needed – i.e. the 7);
- if it is **5 or more**, then **round up the last digit** (7 is greater than 5, so round up the 6 to a 7);
- if it is **less than 5**, then the digit remains the **same**.

Examples

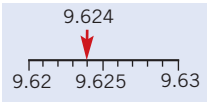
Round 12.49 to 1 d.p.
12.4**9** rounds up to 12.5.



Round 8.735 to 2 d.p.
8.73**5** rounds up to 8.74.



Round 9.624 to 2 d.p.
9.62**4** rounds down to 9.62



Ordering decimals

When ordering decimals:

- First write them all with the same number of digits after the decimal point.
- Then compare whole numbers, digits in the tenths place, digits in the hundredths place, and so on.

Examples

Arrange these numbers in order of size, smallest first:
6.21, 6.023, 6.4, 6.04, 2.71, 9.4
First rewrite them:
6.210, 6.023, 6.400, 6.040, 2.710, 9.400
Then re-order them:
2.710, 6.023, 6.040, 6.210, 6.400, 9.400

Remember, hundredths are smaller than tenths: $\frac{10}{100} = \frac{1}{10}$ ***so*** $\frac{6}{100} < \frac{1}{10}$

Multiplying and dividing by numbers between 0 and 1

When **multiplying** by numbers between 0 and 1, the result is **smaller** than the starting value.
When **dividing** by numbers between 0 and 1, the result is **bigger** than the starting value.

Examples

$6 \times 0.1 = 0.6$
$6 \times 0.01 = 0.06$
$6 \times 0.001 = 0.006$
The results are all smaller than the starting values.

$6 \div 0.1 = 60$
$6 \div 0.01 = 600$
$6 \div 0.001 = 6000$
The results are all bigger than the starting values.

Calculations with decimals

When **adding** and **subtracting** decimals, align the decimal points in a column.

Examples

$$\begin{array}{r} 27.46 \\ 7.291 + \\ \hline 34.751 \\ \hline 1\ \ 1 \end{array}$$

Line up the digits carefully.

Put the decimal points under each other.

$$\begin{array}{r} {\scriptstyle 6\,9\,1} \\ 17.\not{7}\not{\emptyset}0 \\ 12.84 - \\ \hline 4.16 \end{array}$$

The decimal point in the answer will be in line.

When **dividing** decimals, divide as normal, placing the decimal points in line.

Example

$$3\overline{)14.4}\quad 4.8$$

Put the decimal points in line.

When **multiplying** decimals, the answer must have the same number of decimal places as the total number of decimal places in the numbers which are being multiplied.

Examples

Work out 24.6×7.

$$\begin{array}{r} 246 \\ 7\times \\ \hline 1722 \\ \hline 3\,4 \end{array}$$

Multiply 246 by 7 = 1722, ignoring the decimal point. 24.6 has 1 digit after the decimal point. The answer must have 1 decimal place (1 d.p.).

So $24.6 \times 7 = 172.2$

Work out 4.52×0.2

$$\begin{array}{r} 452 \\ 2\times \\ \hline 904 \\ \hline 1 \end{array}$$

Work out 452 × 2, ignoring the decimal points. 4.52 has 2 d.p.; 0.2 has 1 d.p. So the answer must have 3 d.p.

$904 \rightarrow 0.904$ Move the digits 3 places to the right.

So $4.52 \times 0.2 = 0.904$

Remember to check your answer with a calculator.

Multiplying and dividing by numbers between 0 and 1 usually occur on the non-calculator paper – it is wise to practise these by writing out several calculations and then checking the answer with a calculator.

KEY TERMS

Make sure you understand these terms before moving on!

- decimal point
- decimal place

QUICK TEST

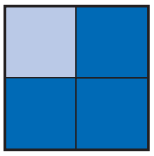
1. Without using a calculator, work out the following.

a) 27.16 + 9.32	b) 29.04 – 11.361	c) 12.8 × 2.1
d) 49.2 ÷ 4	e) 600 × 0.01	f) 520 × 0.1
g) 20 × 0.02	h) 37 × 0.0001	i) 400 ÷ 0.1
j) 450 ÷ 0.01	k) 470 ÷ 0.001	l) 650 ÷ 0.02

2. Round the following numbers to 2 decimal places.
   a) 7.469
   b) 12.0372
   c) 9.365
   d) 10.042
   e) 8.1794

Percentages 1

Percentages **are fractions with a denominator of 100.**
% is the percentage sign.
75% means $\frac{75}{100}$ (this is also equal to $\frac{3}{4}$).


75%

Percentage of a quantity

Again, the word '**of**' means **multiply**.

Example
40% of £600 becomes

$\frac{40}{100} \times 600 = £240$

On the calculator, key in
40 ÷ 100 x 600 =

If this is on the non-calculator paper:
- Work out 10% first by dividing by 10
  600 ÷10 = £60
- Multiply by 4 to get 40%
  4 × 60 = £240

Percentage questions appear frequently at GCSE. If there is a percentage question on the non-calculator paper, first work out what 10% is equal to, as shown in the examples above.

Example
A meal for four costs £92.20.
VAT (value added tax) is charged at 17.5%.
a) How much VAT is there to pay on the meal?
b) What is the final price of the meal?

a) 17.5% of £92.20 = $\frac{17.5}{100} \times 92.20$
= £16.14 to the nearest penny
VAT = £16.14

b) Price of meal = £92.20 + £16.14 = £108.34

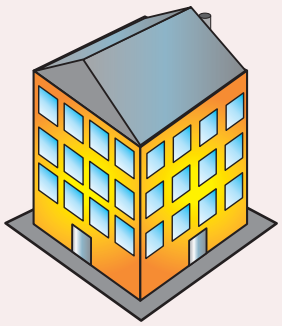
An alternative is to use a **scale factor method**:
An increase of 17.5%, is the same as multiplying by $1.175 = 1 + \frac{17.5}{100}$.

£92.20 × 1.175
= £108.34 (to the nearest penny)

Percentage increase and decrease

The answers to these questions will be a percentage so multiply the change by 100%.

$$\text{\% change} = \frac{\text{change}}{\text{original}} \times 100\%$$



Example
A coat costs £125.
In a sale it is reduced to £85.
What is the percentage reduction?

Reduction = £125 – £85 = £40

% reduction = $\frac{40}{125} \times 100\%$
= 32%

Example
Matthew bought a flat for £45000.
Three years later, he sold it for £62000.
What was his percentage profit?

Profit = £62000 – £45000
= £17000

% Profit = $\frac{17000}{45000} \times 100\%$
= 37.78%

One quantity as a percentage of another

Again, to make the answer a percentage, multiply by 100%.

fraction —**x 100%**→ **percentage**

Example

In a carton of milk, 6.2g of the contents are fat.
If 2.5g of the fat is saturated, what percentage of the total fat content is this?

$\frac{2.5}{6.2}$ x 100% = 40.3% (to 1 d.p.)

On the calculator, key in

2.5 ÷ 6.2 x 100 =

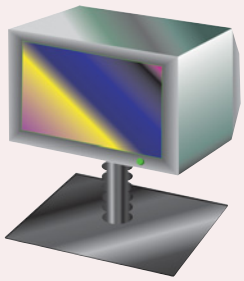
Reverse percentage problems

In reverse percentage problems the **original quantity** is calculated.

Example

The price of a television is reduced by 20% in the sales.
It now costs £250. What was the original price?

- The sale price is 100% – 20% = 80% of the pre-sale price.
  $\frac{80}{100}$ = 0.8
  0.8 × (original price) = £250
  So, original price = $\frac{250}{0.8}$ = £312.50



original price —**x 0.8**→ **sale price**
sale price —**÷ 0.8**→ **original price**

- Is the original price more than the sale price?
- Always check the answer is sensible.

It is 80% of the original price which is being found, not 80% of the new one.

KEY TERMS

Make sure you understand this term before moving on!

- percentage

QUICK TEST

1. Work out 30% of £700.
2. Sarah got 94 out of 126 in a maths test. What was her result as a percentage? (C)
3. Reece weighed 6 lb when he was born. If his weight has increased by 65%, how much does he weigh now? (C)
4. The price of a hi-fi is reduced by 15% in the sales. It now costs £350. What was the original price? (C)
5. Super's football boots $\frac{1}{3}$ off — Joe's football boots 28% off. If a pair of football boots costs £49.99, which shop is selling them cheaper in the sale and what is their price? (C)

(C) *Indicates that a calculator may be used.*

Percentages 2

Repeated percentage change

Example
A car was bought for £8000 in 2004. Each year it depreciated in value by 20%. What was the car worth 3 years later?

Beware! Do not do 3 × 20 = 60% reduction over 3 years!

Method 1

- First find 80% of the original value of the car.

Year 1 $\frac{80}{100} \times £8000 = £6400$

- Then work out the value year by year.

Year 2 $\frac{80}{100} \times £6400 = £5120$ (£6400 depreciated in value by 20%)

Year 3 $\frac{80}{100} \times £5120 = £4096$ after 3 years (£5120 depreciated by 20%)

Method 2

A quick way to work this out uses the **scale factor method**.
Finding 80% of the value of the car is the same as multiplying by 0.8. The scale factor is 0.8.

Year 1 $0.8 \times £8000 = £6400$
Year 2 $0.8 \times £6400 = £5120$
Year 3 $0.8 \times £5120 = £4096$

This is the same as working out $(0.8)^3 \times 8000 = £4096$
A much quicker way, if you understand it!

Simple interest

Simple interest is the interest that is sometimes paid on money in banks and building societies. The interest is paid each year (**per annum** or **p.a.**) and is the same amount each year.

Example
Jonathan has £2500 in his savings account.
Simple interest is paid at 4.4% p.a.
How much does he have in his account at the end of the year?

Increasing by 4.4% is the same as multiplying by 100 + 4.4 = 104.4%.

Total savings = £2500 × $\frac{104.4}{100}$ = £2610

Interest paid = £2610 – £2500 = £110

Note: If the money was in the account for 4 years, simple interest at the end of the 4 years would be 4 × £110 = £440.

Compound interest

Compound interest is the type of interest where the bank pays interest on the interest earned as well as on the original money.

Example
Jonathan has £2500 in his savings account.
Compound interest is paid at 4.4% p.a.
How much will he have in his account after 4 years?

Method 1

Year 1: $\frac{104.4}{100} \times £2500 = £2610$
Year 2: $1.044 \times £2610 = £2724.84$
Year 3: $1.044 \times £2724.84 = £2844.73$
Year 4: $1.044 \times £2844.73 = £2969.90$
Total = £2969.90 (to the nearest penny)

Method 2

Using the scale factor method:

$\frac{104.4}{100}$ = 1.044 is the scale factor
$= 2500 \times (1.044)^4$
Total = £2969.90 (to the nearest penny)

Tax and National Insurance

National Insurance

National Insurance (NI) is usually deducted from a wage as a percentage.

Example

Sue earns £1402.65 a month. National Insurance is deducted at 9%. How much NI must she pay?

9% of £1402.65 = 0.09 × £1402.65
= £126.24

Income Tax

A percentage of a wage or salary is deducted as **income tax**.
Personal allowances must first be deducted, in order to obtain the **taxable income**.

Example

Harold earns £190 per week. The first £62 is not taxable; the remainder is taxed at 24%.
How much income tax does he pay each week?

Taxable income = £190 – £62 = £128
24% tax = 0.24 × £128 = £30.72
Tax per week = £30.72

Being able to answer questions like the examples in this section is important – not only because they often appear on the GCSE examination paper but because you will come across them in everyday life.

There are really only two types of percentage questions:

1. 'Percentage of' – Here you are given the percentage, so you divide by 100.
2. You are asked to find a percentage. Here you need to multiply by 100.

KEY TERMS

Make sure you understand these terms before moving on!

- simple interest
- compound interest
- National Insurance
- income tax

QUICK TEST

1. Charlotte has £4250 in the bank. If the interest rate is 6.8% p.a., how much interest on the savings will she get at the end of the year? (C)
2. A car costs £6000 cash, or can be bought by hire purchase with a 30% deposit followed by 12 monthly instalments of £365. Find: (C)
   a) the deposit
   b) the total amount paid for the car on hire purchase.
3. A flat was bought in 1998 for £62000. The price increased by 20% in 1999 and then by a further 35% in 2000. How much was the flat worth at the end of 2000? (C)
4. Fiona has £3200 in savings. If compound interest is paid at 3% p.a., how much will she have in her account after 3 years? (C)

(C) ***Indicates that a calculator may be used.***

Equivalents

Fractions to decimals to percentages

Equivalent fractions, decimals and percentages are all different ways of expressing the same number.

Fraction		Decimal		Percentage
$\frac{1}{2}$		0.5		50%
$\frac{1}{3}$		$0.\dot{3}$		$33.\dot{3}\%$
$\frac{2}{3}$		$0.\dot{6}$		$66.\dot{6}\%$
$\frac{1}{4}$		0.25		25%
$\frac{3}{4}$	3 ÷ 4 →	0.75	× 100% →	75%
$\frac{1}{5}$		0.2		20%
$\frac{1}{8}$		0.125		12.5%
$\frac{3}{8}$		0.375		37.5%
$\frac{1}{10}$		0.1		10%
$\frac{1}{100}$		0.01		1%

The above table shows:

- Some common fractions and their **equivalents** which you need to learn
- How to convert fractions → decimals → percentages

Get a friend to test you on the equivalences between fractions, decimals and percentages. You need to learn all the ones in this table.

Ordering different numbers

When putting a mixture of fractions, decimals and percentages in order of size, it is best to change them all to decimals first.

Example

$\frac{3}{5}$, 0.65, 0.273, 27%, 62%, $\frac{4}{9}$

0.6, 0.65, 0.273, 0.27, 0.62, $0.\dot{4}4$ — Put into decimals first.

0.27, 0.273, $0.\dot{4}4$, 0.6, 0.62, 0.65 — Place in order of size, smallest first.

27%, 0.273, $\frac{4}{9}$, $\frac{3}{5}$, 62%, 0.65

QUICK TEST

1. Change the following fractions into: (C)
   a) decimals b) percentages
   i) $\frac{2}{7}$ ii) $\frac{3}{5}$ iii) $\frac{8}{9}$
2. Place these numbers in order of size, smallest first.
   $\frac{2}{5}$, 0.42, 0.041, $\frac{1}{3}$, 5%, 26%

(C) ***Indicates that a calculator may be used.***

KEY TERMS

Make sure you understand this term before moving on!

- equivalents

Using a calculator

Order of operations

BIDMAS is a made-up word which helps you to remember the order in which calculations take place.

B I D M A S

Brackets Indices Division Multiplication Addition Subtraction

This just means anything in brackets is done first, then the other operations are done in this order.

Examples

$(2 + 4) \times 3 = 18$ but $2 + 4 \times 3 = 14$, *not* 18, because without the brackets the multiplication is done first.

Calculating powers

y^x or x^y is used for calculating powers such as 2^7.

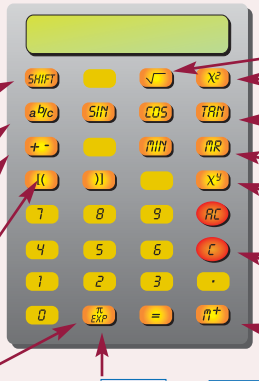
- Use the power key on your calculator to work out 2^7.
- Write down calculator keys used. Check that you obtain the answer 128.

Now try writing down the keys that would be needed for these calculations.
Check that you get the right answers.

a) $\frac{2.9 \times 3.6}{(4.2 + 3.7)} = 1.322$ b) $9^{\frac{1}{3}} \times 4^5 = 2130$ c) $\frac{3 \times (5.2)^2}{9.6 \times (12.4)^3} = 4.432 \times 10^{-3}$

Important calculator keys

This calculator is an imaginary one to show you some of the most important keys. Make sure you are familiar with your own calculator.



Shift or 2nd or Inv allow 2nd functions to be carried out

allows a fraction to be put in the calculator

– or +/– changes positive numbers to negative ones

bracket keys

often puts the ×10 part in when working in standard form

pressing shift or EXP often gives π

square root

square

trigonometric buttons

memory keys

works out powers

cancels only the last key you have pressed

memory key

QUICK TEST

1 Work out these on your calculator:

a) $\frac{27.1 \times 6.4}{9.3 + 2.7}$ b) $\frac{(9.3)^4}{2.7 \times 3.6}$

c) $\sqrt{\frac{25^2}{4\pi}}$ d) $\frac{5}{9}(25 - 10)$

Approximating and checking calculations

Significant figures (s.f. or sig. fig.)

The 1st **significant figure** is the first digit which is not zero. The 2nd, 3rd, 4th, ... significant figures follow on after the 1st digit. They may or may not be zeros.

Examples

6 . 4 0 2 7 has 5 s.f. (1st 2nd 3rd 4th 5th)

0 . 0 0 0 4 7 0 1 has 4 s.f. (1st 2nd 3rd 4th)

To round a number to a given number of significant places, apply the same rule as with decimal places: if the next digit is 5 or more, round up.

Examples

Number	to 3 s.f.	to 2 s.f.	to 1 s.f.
4.207	4.21	4.2	4
4379	4380	4400	4000
0.006209	0.00621	0.0062	0.006

After rounding the last digit, you must fill in the end zeros. For example, 4380 = 4400 to 2 s.f. (not 44).

Take care when rounding that you do not change the place values.

Estimates and approximations

Estimating is a good way of checking answers.

- Round the numbers to 'easy' numbers, usually ones with 1 or 2 significant figures.
- Work out the estimate using these easy numbers.
- Use the symbol ≈, which means '**approximately equal to**'.
- For multiplying or dividing, never approximate a number to zero. Use 0.1, 0.01, 0.001, etc.

Examples

a) $8.93 \times 25.09 \approx 10 \times 25 = 250$

b) $(6.29)^2 \approx 6^2 = 36$

c) $\frac{296 \times 52.1}{9.72 \times 1.14} \approx \frac{300 \times 50}{10 \times 1} = \frac{15000}{10} = 1500$

d) $0.096 \times 79.2 \approx 0.1 \times 80 = 8$

Example

Jack does the calculation $\frac{9.6 \times 103}{(2.9)^2}$

a) Estimate the answer to this calculation, without using a calculator.

b) Jack's answer is 1175.7. Is this the **right order of magnitude** (about the right size)?

a) Estimate: $\frac{9.6 \times 103}{(2.9)^2} \approx \frac{10 \times 100}{3^2} = \frac{1000}{9} \approx \frac{1000}{10} = 100$

b) Jack's answer is not the right order of magnitude. It is 10 times too big.

- When adding and subtracting, very small numbers may be approximated to zero.

Examples

$109.6 + 0.0002 \approx 110 + 0 = 110$

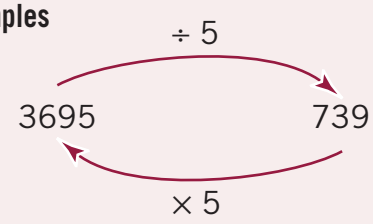
$63.87 - 0.01 \approx 64 - 0 = 64$

Questions which involve approximating are common on the non-calculator paper. For most of these questions, you are expected to round to 1 significant figure. Even if you find the calculation difficult, show your approximations to pick up method marks.

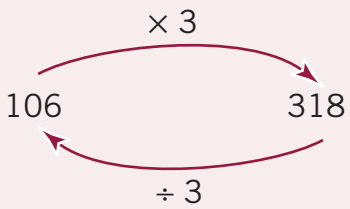
Checking calculations

When checking calculations, the process can be reversed.

Examples



3695 ÷ 5 = 739
Check: 739 × 5 = 3695



106 × 3 = 318
Check: 318 ÷ 3 = 106

Calculations

When solving problems the answers should be rounded sensibly.

Example
95.26 × 6.39 = 608.7114 = 608.71 (2 d.p.)

Round to 2 d.p. because the values in the question are to 2 d.p.

When rounding remainders, consider the context of the question.

Example
Jackie has £9.37. She divides it equally between 5 people. How much does each person receive?

£9.37 ÷ 5 = £1.874
= £1.87
(Round to 2 d.p. as it is money.)

Example
Paint is sold in 8 litre tins. Sandra needs 27 litres of paint. How many tins must she buy?

27 ÷ 8 = 3 remainder 3

Sandra needs 4 tins of paint.
Sandra would not have enough paint with 3 tins – she would be 3 litres short. Hence the number of tins of paint must be rounded up.

You will lose marks if you do not write money to 2 d.p. If the answer to a money calculation is £9.7, always write it to 2 d.p. i.e. £9.70.

Make sure you understand this term before moving on!
■ significant figure

KEY TERMS

QUICK TEST

1. Round the following numbers to 3 significant figures (3 s.f.)
   a) 0.003786 b) 27 490 c) 307 250
2. Estimate the answer to $\frac{(29.4)^2 + 106}{2.2 \times 5.1}$
3. Sukvinder decided to decorate her living room. The total area of the walls was 48 m^2. If one roll of wallpaper covers 5 m^2 of wall, how many rolls of wallpaper did Sukvinder need? (C)
4. Thomas earned £109.25 for working a 23-hour week. How much was he paid per hour? Check your calculation by estimating.

(C) ***Indicates that a calculator may be used.***

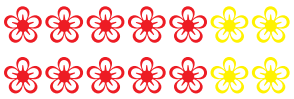
Ratio

Ratios

- A **ratio** is used to compare two or more related quantities.
- 'Compared to' is replaced with two dots **:**
  For example, '16 boys compared to 20 girls' can be written as 16 : 20.
- To simplify ratios, divide both parts of the ratio by their highest common factor.
  For example, 16 : 20 = 4 : 5
  (Divide both sides by 4.)

Examples

- Simplify the ratio 21 : 28.
  21 : 28 = 3 : 4 (Divide both sides by 7)
- The ratio of red flowers to yellow flowers can be written:
  10 : 4
  = 5 : 2



In other words, for every 5 red flowers there are 2 yellow flowers.
To express the ratio 5 : 2 as the ratio n : 1, divide both sides by 2:

$5 : 2 = \frac{5}{2} : \frac{2}{2}$

$= 2.5 : 1$

Sharing a quantity in a given ratio

- Add up the total parts.
- Work out what one part is worth.
- Work out what the other parts are worth.

Example

£20000 is shared between Ewan and Leroy in the ratio 1 : 4.
How much does each receive?
1 + 4 = 5 parts
5 parts = £20000 so, 1 part = $\frac{£20000}{5}$ = £4000
So Ewan gets 1 × £4000 = £4000 and Leroy gets 4 × £4000 = £16000.

Best buys

Use unit amounts to decide which option is the better value for money.

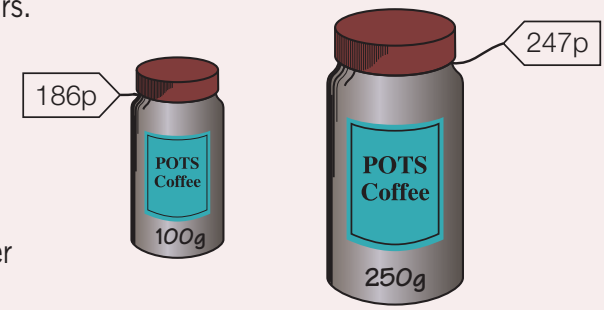
Example

The same brand of coffee is sold in two different-sized jars.
Which jar represents the better value for money?

- Find the cost per gram for both jars.

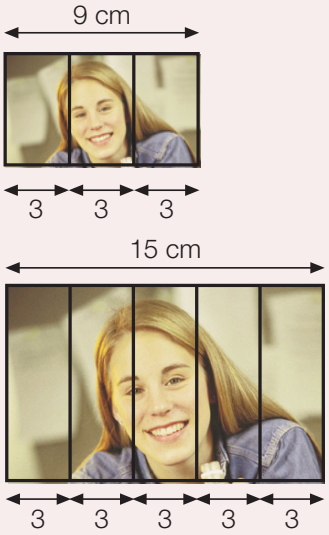
  100g costs 186p so 186 ÷ 100 = 1.86p per gram.
  250g costs 247p so 247 ÷ 250 = 0.988p per gram.

Since the larger jar costs less per gram it offers the better value for money.



Increasing and decreasing in a given ratio

- Divide to get one part.
- Multiply for each new part.



Example

A photograph of length 9 cm is to be enlarged in the ratio 5 : 3.
What is the length of the enlarged photograph?

- Divide 9 cm by 3 to get 1 part.
  $9 \div 3 = 3$
- Multiply this by 5.
  $5 \times 3 = 15$ cm

So the length 9 cm is 15 cm on the enlarged photograph.

Example

A house took 8 people 6 days to build.
At the same rate, how long would it take 3 people?
Time for 8 people = 6 days
Time for 1 person = $8 \times 6 = 48$ days
It takes 1 person longer to build the house.
So time for 3 people = $\frac{48}{3} = 16$ days
3 people will take $\frac{1}{3}$ of the time taken by 1 person.

Example

A recipe for 4 people needs 1600 g of flour.
How much is needed to make the recipe for 6 people?

- Divide 1600 g by 4: 400 g for 1 person.
- Multiply by 6, so 6×400 g = 2400 g for 6 people.

When answering problems of the type shown here, always try and work out what a unit (or one) is worth. You should then be able to work out what any other value is worth.

KEY TERMS

Make sure you understand this term before moving on!

- ratio

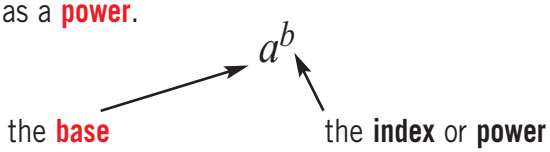
QUICK TEST

1. Write the following ratios in their simplest form:
   a) 12 : 15
   b) 6 : 12
   c) 25 : 10
2. Three sisters share 60 sweets between them in the ratio 2 : 3 : 7. How many sweets does each sister receive?
3. If 15 oranges cost £1.80, how much will 23 identical oranges cost?
4. A map is being enlarged in the ratio 12 : 7. If a road length was 21 cm on the original map, what is the length of the road on the enlarged map?

Indices

Indices

An **index** is sometimes known as a **power**.



Examples

6^4 is read as '6 to the power of 4'. It means $6 \times 6 \times 6 \times 6$.

2^7 is read as '2 to the power 7'. It means $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

- The base has to be the same when the rules of indices are applied.

Rules of indices

You need to learn these rules:

- When multiplying, add the powers.
  $4^7 \times 4^3 = 4^{7+3} = 4^{10}$
- When dividing, subtract the powers.
  $6^9 \div 6^4 = 6^{9-4} = 6^5$
- When raising one power to another, multiply the powers.
  $(7^2)^4 = 7^{2x4} = 7^8$
- Anything raised to the **power zero** is **1,** provided the number is not zero.
  $5^0 = 1$ $6^0 = 1$
  $2.7189^0 = 1$ 0^0 = undefined
- Anything to the **power 1** is **itself.**
  $15^1 = 15$ $1923^1 = 1923$
- All these rules also apply when the powers are negative.

Examples

$6^{-2} \times 6^{12} = 6^{-2+12} = 6^{10}$

$8^{-4} \times 8^{-3} = 8^{-4+-3} = 8^{-7}$

$(6^4)^{-2} = 6^{4 \times -2} = 6^{-8}$

$5^0 = 1$

Negative and fractional powers

At GCSE Higher level there are a few more properties of indices that need to be learnt:

Negative powers

Turn a negative power **upside down** (take the reciprocal) to make the **power positive.**

In general, $a^{-n} = \frac{1}{a^n}$.

Examples

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$ $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ $2x^{-3} = \frac{2}{x^3}$

Fractional powers

These mean **roots**!

In general, $a^{\frac{1}{m}} = \sqrt[m]{a}$.

- The power $\frac{1}{2}$ means square root $\sqrt{\ }$.
- The power $\frac{1}{3}$ means cube root $\sqrt[3]{\ }$.
- The power $\frac{1}{4}$ means fourth root $\sqrt[4]{\ }$, etc.

Examples

$25^{\frac{1}{2}} = \sqrt{25} = 5$ $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

Sometimes powers and fractions are put together.

In general, $a^{\frac{n}{m}} = (\sqrt[m]{a})^n$.

Examples

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

$64^{\frac{5}{6}} = (\sqrt[6]{64})^5 = 2^5 = 32$

If there is a negative fraction, remember to turn it upside down as well (take the reciprocal).

$125^{-\frac{2}{3}} = \frac{1}{125^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{125})^2} = \frac{1}{5^2} = \frac{1}{25}$

$(2x)^{-3} = \frac{1}{(2x)^3} = \frac{1}{8x^3}$

Indices and algebra

The rules that apply with numbers also apply with letters.

Laws of indices

$a^n \times a^m = a^{n+m}$

$a^n \div a^m = a^{n-m}$

$(a^n)^m = a^{n \times m}$

$a^0 = 1$

$a^1 = a$

Indices are a very common topic on the non-calculator paper – learn the rules and you should be OK!

Examples

Note that the numbers are multiplied ...

$4x^2 \times 3x^5 = 12x^7$

... but the powers of the same letter are added

$12x^4 \div 3x^7 = 4x^{-3}$

$(7x^2)^2 = 49x^4$

$x^0 = 1$

$(2x^4)^3 = 8x^{12}$

Simplify the following expressions:

$$\frac{3x^7 \times 4x^9}{6x^4} = \frac{12x^{16}}{6x^4} = 2x^{12}$$

Work this out in 2 stages.

$x^6 \times 4x^3 = 4x^9$

$$\frac{12a^2b^3}{6a^3b^2} = \frac{2b}{a}$$

$$\frac{4a^4b^3}{2ab} = 2a^3b^2$$

KEY TERMS

Make sure you understand these terms before moving on!

- index
- power
- base

QUICK TEST

1. Simplify the following:
   a) $12^4 \times 12^8$ b) $9^{-2} \times 9^{-4}$ c) 4^0
   d) $18^6 \div 18^{-2}$ e) $(4^2)^5$ f) 1^{20}

2. Simplify the following:
   a) $x^4 \times x^9$ b) $2x^6 \times 3x^7$ c) $12x^4 \div 3x^2$
   d) $25x^9 \div 5x^{-2}$ e) $\dfrac{5x^6 \times 4x^9}{10x^3}$

3. Evaluate:
   a) $(64)^{\frac{2}{3}}$ b) 5^{-3} c) $144^{\frac{1}{2}}$ d) $36^{-\frac{1}{2}}$

4. Simplify:
   a) $(4x)^{-2}$ b) $(6x^2y^4)^{-2}$

Standard index form (Standard form)

Standard index form

Standard index form is used to write very large numbers or very small numbers in a simpler way.
When written in **standard form**, a number will be written as:

$$a \times 10^n$$

a must be at least 1 but not greater than 10, i.e. $1 \leqslant a < 10$.
n is the power of 10 by which you multiply (if n is positive),
or divide (if n is negative).

Learn these rules:

- The front number (a) must always be at least 1 but less than 10.
- The power of 10, n:
  If the number is big, n is positive.
  If the number is small, n is negative.

Big numbers

Examples

- Write 6 230 000 in standard form.

  Place the decimal point between the 6 and 2 to give 6.230 000 ($1 \leqslant 6.23 < 10$).

  Work out how many times you multiply by 10 to restore the number.

  6 2 3 0 0 0 0 ($n = 6$)

  In standard form, $6\,230\,000 = 6.23 \times 10^6$

- $4371 = 4.371 \times 10^3$ in standard form.

Small numbers

Examples

- Write 0.003 71 in standard form.

  Place the decimal point between the 3 and 7 to give 3.71 ($1 \leqslant 3.71 < 10$).

  Work out how many times you divide the number by 10.

  0 . 0 0 3 7 1 ($n = 3$)

  In standard form, $0.00371 = 3.71 \times 10^{-3}$

- $0.000\,047\,9 = 4.79 \times 10^{-5}$ in standard form.

Standard form and the calculator

To key a number in standard form into the calculator, use the EXP key.
(Some calculators use EE . Make sure that you check your calculator, as calculators vary greatly.)

Examples

6.23×10^6 can be keyed in as: 6 . 2 3 EXP 6

4.93×10^{-5} can be keyed in as: 4 . 9 3 EXP 5 +/–

Most calculators do not show standard form correctly on the display.

7.632 09 means 7.632×10^9. 4.62 -07 means 4.62×10^{-7}.

Remember to put in the times 10 part if the sign has been left out.

Calculations with standard form

You can use the calculator to do complex calculations in standard form.

Examples

$(2.6 \times 10^3) \times (8.9 \times 10^{12}) = 2.314 \times 10^{16}$

This would be keyed in as:

2	.	6	EXP	3	X	8	.	9	EXP	1	2	=

Check that for $(1.8 \times 10^6) \div (2.7 \times 10^{-3})$ the answer is 6.7×10^8.

Just key in as normal:

2	.	7	EXP	3	+/–

If a calculation with standard form is on the non-calculator paper, the laws of indices can be used when multiplying and dividing numbers written in standard form.

Examples

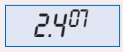
$(2.4 \times 10^{-4}) \times (3 \times 10^7)$
$= (2.4 \times 3) \times (10^{-4} \times 10^7)$
$= 7.2 \times 10^3$

$(12.4 \times 10^{-4}) \div (4 \times 10^7)$
$= (12.4 \div 4) \times (10^{-4} \div 10^7)$
$= 3.1 \times 10^{-11}$

Standard form questions are very common on both the calculator and non-calculator paper.

Watch Out!

Several common mistakes when answering standard form questions are:

- ***Reading a calculator display***  ***incorrectly and writing down*** 2.4^7 ***instead of*** 2.4×10^7.
- ***Forgetting to write the answer in standard form, particularly on the non-calculator paper.***

e.g. $(2 \times 10^6) \times (6 \times 10^3) = (2 \times 6) \times (10^6 \times 10^3)$
$= 12 \times 10^9$
$= 1.2 \times 10^{10}$

QUICK TEST

1. Write the following numbers in standard form:
   a) 630 000 b) 2730 c) 0.000 042 9 d) 0.000 000 63

2. Without a calculator work out the following, leaving your answers in standard form.
   a) $(2 \times 10^5) \times (3 \times 10^7)$ b) $(6.1 \times 10^{12}) \times (2 \times 10^{-4})$
   c) $(8 \times 10^9) \div (2 \times 10^6)$ d) $(6 \times 10^8) \div (2 \times 10^{-10})$

3. Work these out on a calculator. Give your answers to 3 s.f. (C)
   a) $\dfrac{1.279 \times 10^9}{2.94 \times 10^{-2}}$ b) $(1.693 \times 10^4) \times (2.71 \times 10^{12})$

4. Calculate, giving your answer in standard form correct to 3 s.f. (C)
   $\dfrac{(3.72 \times 10^8) - (1.6 \times 10^4)}{3.81 \times 10^{-3}}$

(C) ***Indicates that a calculator may be used.***

Direct and inverse proportion

The notation $\propto$ means 'is directly proportional to'. This is often abbreviated to 'is proportional to'. For example, $y \propto x^3$ is read as 'y is proportional to x cubed'.

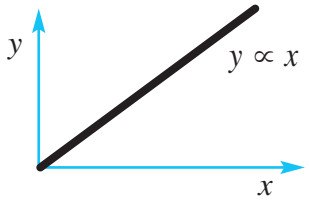
Direct proportion

$y \propto x$ means that when x is multiplied by a number, then so is the corresponding value of y.

Example

		x2		x3		x4	
x	2		4		12		48
y	3		6		18		72
		x2		x3		x4	

For **direct proportion**, the graph of y against x goes through the origin. For $y \propto x$, the graph is a straight line through the origin.



Inverse proportion

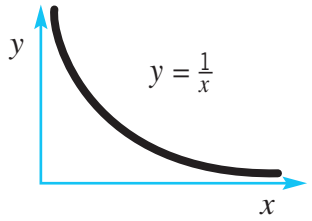
$y \propto \frac{1}{x}$ means that y is **inversely proportional** to x: when x is multiplied by a number, then y is divided by that number, and vice versa.

Example

		x4		x3		÷2	
x	4		16		48		24
y	12		3		1		2
		÷4		÷3		x2	

For **inverse proportion**, the graphs of y against x go to infinity when x or $y = 0$.

The graph of $y = \frac{1}{x}$ looks like this:



Variation

These are common GCSE questions that usually involve statements such as:

'a is proportional to the square of b'
'c is proportional to the square root of d'
'p is inversely proportional to a^2'
'd varies as the square of x'

Example

The value (V) of a diamond varies directly with the square of its weight (w).
A diamond weighing 5.0g is worth £2500.
How heavy is a diamond that is worth £6000?

Step 1 Change the sentence into a proportionality expression using the symbol $\propto$: $V \propto w^2$

Step 2 Replace $\propto$ with '$= k$' to make an equation: $V = kw^2$

Step 3 Substitute the values given in the question in order to find k: $2500 = k \times 5^2$

Step 4 Rearrange the equation to find the value of k.

$\frac{2500}{5^2} = k \quad \therefore k = 100$

Step 5 Put the value of k back into the equation: $V = 100w^2$

Step 6 Now answer the question using the equation you have found.

$6000 = 100 \times w^2$
$\frac{6000}{100} = w^2$
$w = \sqrt{60}$
$w = 7.75$g (2 d.p.)

You use exactly the same steps if y is inversely proportional to x, except the equation in Step 2 would be $y = \frac{k}{x}$.

QUICK TEST

1. y is inversely proportional to the square of x. If $x = 5$ when $y = 4$, work out the value of y when $x = 10$.
2. p varies as the square root of t. If $t = 16$ when $p = 12$, find the value of p when $t = 49$.
3. s is inversely proportional to the cube of r. If $r = 2$ when $s = 5$, what is the value of r when $s = 10$?

Recurring decimals and surds

Changing recurring decimals into fractions

As **recurring decimals** are rational numbers, we can change them to fractions.

Example

Change $0.\dot{1}\dot{3}$ to a fraction in its lowest terms.

Let $x = 0.131313...$ (1)

then $100x = 13.131313...$(2)

(Multiply by 10^n, where n is the length of the recurring pattern. In this example it is 2.)

Subtract equation (1) from equation (2).

$99x = 13$

This has the effect of making the recurring pattern disappear.

Divide to leave x.

$x = \frac{13}{99}$

Since 13 is a prime number, this fraction is in its lowest terms.

So $0.\dot{1}\dot{3} = \frac{13}{99}$.

'Trick' method

There is a quick method of changing recurring decimals to fractions – the fraction has the repeating unit on the top and the same number of nines on the bottom.

Examples

$0.\dot{2} = \frac{2}{9}$ $0.\dot{4}\dot{1} = \frac{41}{99}$ $0.\dot{1}\dot{2}\dot{3} = \frac{123}{999}$

$0.\dot{2}\dot{7}\dot{1}\dot{3} = \frac{2713}{9999}$

Remember to simplify if possible.

If you have a question where there is one or more numbers that are not repeated, extra care is needed.

Example

Change $0.2\dot{3}$ into a fraction.

$x = 0.2333...$ (1)

$10x = 2.3333...$ (2) multiply by 10

$100x = 23.3333...$ (3) multiply by 100

Subtract equation (2) from equation (3).

$90x = 21$ $x = \frac{21}{90} = \frac{7}{30}$

Surds

Numbers written under a square root sign are called **surds** – $\sqrt{2}$, $\sqrt{3}$, $\sqrt{13}$ etc. Surds are **irrational numbers**.

To work with surds, there are a few rules you need to learn.

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Example $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

- $(\sqrt{b})^2 = \sqrt{b} \times \sqrt{b} = b$

Example $(\sqrt{3})^2 = 3$

- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Example $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$

- $(a + \sqrt{b})^2 = (a + \sqrt{b})(a + \sqrt{b})$
  $= a^2 + 2a\sqrt{b} + (\sqrt{b})^2 = \mathbf{a^2 + 2a\sqrt{b} + b}$

- $(a + \sqrt{b})(a - \sqrt{b})$
  $= a^2 - a\sqrt{b} + a\sqrt{b} - (\sqrt{b})^2 = \mathbf{a^2 - b}$

Examples

Simplify $\sqrt{200}$.

(Hint – look for perfect square factors.)

$\sqrt{200} = \sqrt{2} \times \sqrt{100}$

$= 10 \times \sqrt{2}$

$= 10\sqrt{2}$

Simplify $(\sqrt{2} + 3)(\sqrt{2} + 3)$.

$= (\sqrt{2})^2 + 3\sqrt{2} + 3\sqrt{2} + 9$

$= 2 + 6\sqrt{2} + 9$

$= 11 + 6\sqrt{2}$

Simplify $\frac{1}{\sqrt{5}}$.

Multiply the top and bottom by $\sqrt{5}$.

$\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

This is known as 'rationalising the denominator'.

KEY TERMS

Make sure you understand these terms before moving on!

- recurring decimal
- surd
- irrational number

QUICK TEST

1. Change the following recurring decimals into fractions. Write them in their simplest form.
   a) $0.1\dot{5}$ b) $0.\dot{7}$ c) $0.28\dot{3}$
2. Simplify the following:
   a) $\sqrt{75}$ b) $\sqrt{500}$ c) $(\sqrt{2} - 3)^2$
   d) $6(\sqrt{2} + 3) - \sqrt{2}(2 + \sqrt{3})$
3. Simplify $\frac{1}{\sqrt{3}}$.

Upper and lower bounds of measurement

Upper and lower bounds for a single measurement

If a length x (cm) is given as 6.2, correct to the nearest millimetre, then by the usual conventions of rounding:

$$6.15 \leqslant x < 6.25$$

6.15 is called the **lower bound**

6.25 is called the **upper bound**

In general, the real value can be as much as half the rounded unit above and below the rounded-off value.

Examples

If $x = 6.23$ (correct to 2 d.p.) then the rounded unit is 0.01, so the real value can be anything between $6.225 \leqslant 6.23 < 6.235$

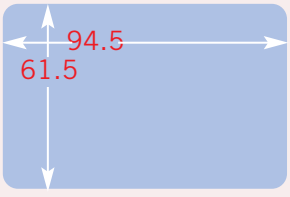
Finding the maximum and minimum possible values of a calculation

When calculations are carried out using rounded-off values, then the calculated value lies between a maximum and minimum possible value.

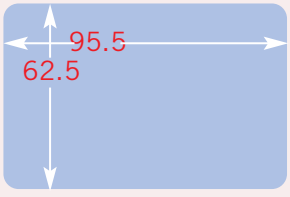
Example 1

A desk measures 62cm by 95cm. Work out the lower and upper bounds of the area of the desk.

lower bound



upper bound



lower bound of area
$= 61.5 \times 94.5 = 5811.75\text{cm}^2$

upper bound of area
$= 62.5 \times 95.5 = 5968.75\text{cm}^2$

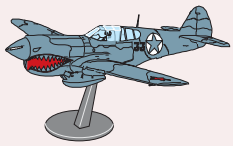
The rounded value $= 95 \times 62 = 5890\text{cm}^2$

This is a popular topic on exam papers. You must be careful on questions with division and subtraction, as it is easy to make a mistake. If in doubt, try all the different combinations (if you have time), but don't forget to cross out the ones you don't want!

Example 2

A toy has a mass of 120g and a volume of 60cm^3, both measurements are correct to the nearest 10 units.

Calculate the lower and upper bounds for the density of the toy.



The upper and lower bounds of the mass are 115g and 125g; the upper and lower bounds of the volume are 55cm^3 and 65cm^3.

Lower bound of density $= \frac{115}{65} = 1.77\text{g/cm}^3$ (2 d.p.)

Upper bound of density $= \frac{125}{55} = 2.27\text{g/cm}^3$ (2 d.p.)

The process for working out the lower and upper bounds in multiplication and addition is quite straightforward. For subtraction and division, more care is needed. Here are a few tips:

- Finding a difference

$$\text{upper bound} = \text{upper bound of larger quantity} - \text{lower bound of smaller quantity}$$

$$\text{lower bound} = \text{lower bound of larger quantity} - \text{upper bound of smaller quantity}$$

- Division

$$\text{upper bound} = \frac{\text{upper bound of quantity 1}}{\text{lower bound of quantity 2}}$$

$$\text{lower bound} = \frac{\text{lower bound of quantity 1}}{\text{upper bound of quantity 2}}$$

Maximum percentage error

This is found by using the following formula:

$$\text{Percentage error} = \frac{\text{maximum error}}{\text{original}} \times 100\%$$

The 'original' is the value that is the furthest from the rounded value.

For the desk in the example opposite:

$$\text{Percentage error} = \frac{5968.75 - 5890}{5890} \times 100\% = 1.34\%$$

Calculations are not always as straightforward as this.

KEY TERMS

Make sure you understand these terms before moving on!

- lower bound
- upper bound

QUICK TEST

1. If $x = \frac{4}{y}$ and $y = 6.2$ (1 d.p.), calculate the upper and lower bounds of x. (C)
2. If $h = ab$, $a = 2.7$ (1 d.p.) and $b = 20$ (to the nearest whole number), calculate the lower and upper bounds of h. (C)

(C) *Indicates that a calculator may be used.*

Practice questions

Use the questions to test your progress.
Check your answers on page 108.

1. Mrs Patel inherits £55 000. She divides the money between her children in the ratio 3 : 3 : 5.
How much does the child with the largest share receive?

.....

2. Work these out on your calculator, giving your answers to 3 s.f.
a) $\frac{4.2\ (3.6 + 5.1)}{2 - 1.9}$ = b) $\frac{3.8 + 4.6}{2.9 \times 4.1}$ = (C)

.....

3. Show how you would estimate the answer to this expression without using a calculator.
Work out the estimate.
$\frac{8.7 + 9.02}{0.2 \times 48}$

.....

4. Toothpaste is sold in three different-sized tubes. (C)
50 ml = £1.24 75 ml = £1.96 100 ml = £2.42
Which of the tubes of toothpaste is the best value for money?
You must show full working in order to justify your answer.

.....

5. A piece of writing paper is 0.01 cm thick. A notepad has 150 sheets of paper.
How thick is the notepad?

.....

6. The price of a CD player has been reduced by 15% in a sale. It now costs £320. (C)
What was the original price?

.....

7. A car was bought in 2004 for £9000. Each year it depreciates in value by 15%. (C)
What is the car worth two years later?

.....

8. James put £632 in a new savings account. At the end of every year interest at 4.2%
is added to the amount in his savings account at the beginning of that year. (C)
Calculate the amount in James's savings account at the end of 3 years.

.....

9. The price of a television has risen from £350 to £420.
Work out the percentage increase in the price.

.....

10. Write these numbers in standard form: (C)
a) 2 670 000 b) 4270 c) 0.03296 d) 0.027

.....

(C) *Indicates that a calculator may be used.*

11. The mass of a hair is 0.000 042 g.
    a) Write this number in standard form. .....
    b) Calculate, in standard form, the mass of 6×10^5 hairs. .....

12. Work out the answers to these questions giving your answer in standard form:
    a) $(2 \times 10^9) \times (6 \times 10^{12})$ .....
    b) $(8 \times 10^9) \div (4 \times 10^{-2})$ .....

13. y is directly proportional to x. When $x = 4$, $y = 12$. Find: C
    a) an equation connecting x and y. .....
    b) the value of x when $y = 20$. .....

14. y is inversely proportional to x^2. When $x = 2$, $y = 5$. Find: C
    a) the value of y when $x = 4$. .....
    b) the value of x when $y = 10$. .....

15. Write these recurring decimals as fractions:
    a) $0.\dot{4}$ b) $0.\dot{2}\dot{1}$ c) $0.2\dot{3}\dot{4}$ d) $0.2\dot{7}$
    .....

16. Simplify the following:
    a) $\sqrt{12}$ b) $\sqrt{75}$ c) $\sqrt{200}$ d) $\sqrt{6}(\sqrt{3} - 2)$ e) $\sqrt{3} \times \sqrt{75}$
    .....

17. A rectangular carpet is measured as 1.42 m by 1.61 m.
    Find the upper and the lower bounds for the area of the carpet. C
    .....

18. Thomas cycles 1250 m in 95 s. Assuming that the time is measured to the nearest second and the distance to the nearest 10 metres, find the upper and lower bounds of Thomas' average speed in metres per second. C
    .....

19. The area of a circle is 142 cm^2 (correct to 3 significant figures).
    Find the upper and lower bounds of the radius of the circle. C
    .....

20. Evaluate the following:
    a) $64^{\frac{1}{3}}$ b) $8^{\frac{2}{3}}$ c) 6^{-2} d) $49^{\frac{1}{2}}$ e) $25^{-\frac{1}{2}}$ f) $(4)^{-2}$ g) $(\frac{5}{7})^{-2}$

.....

How well did you do? ✗ 0–5 Try again 6–10 Getting there 11–15 Good work 16–20 Excellent! ✓

Answers

Number

Quick test answers

Page 5 Types of numbers

1. 2, 3, 5, 7, 11, 13, 17, 19
2. HCF: 12; LCM: 120
3. a) $^{+}8$ b) 6
4. a) $\frac{12}{9}$ b) $\frac{p}{x}$

Page 7 Positive and negative numbers

1. 3°C
2. a) 4 b) –16 c) –12 d) –12 e) 5 f) 6 g) 7 h) –10 i) 36

Page 9 Fractions

1. a) $\frac{1}{3}$ b) $\frac{7}{20}$ c) $\frac{6}{13}$ d) $1\frac{1}{3}$ e) $\frac{5}{21}$ f) $\frac{4}{21}$ g) $\frac{49}{121}$ h) $\frac{50}{63}$ 2. £40 3. 247 mm

Page 11 Decimals

1. a) 36.48 b) 17.679 c) 26.88 d) 12.3 e) 6 f) 52 g) 0.4 h) 0.0037 i) 4000 j) 45 000 k) 470 000 l) 32 500
2. a) 7.47 b) 12.04 c) 9.37 d) 10.04 e) 8.18

Page 13 Percentages 1

1. £210 2. 74.6% 3. 9.9 lb 4. £411.76 5. Super's; £33.33

Page 15 Percentages 2

1. £289 2. a) £1800 b) £6180 3. £100 440 4. £3496.73

Page 16 Equivalents

1. i)a) 0.2857 b) 28.57% (2 d.p.) ii)a) 0.6 b) 60% iii)a) $0.8\dot{8}$ b) $88.\dot{8}8\%$
2. 0.041, 5%, 26%, $\frac{1}{3}$, $\frac{2}{5}$, 0.42

Page 17 Using a calculator

a) 14.45 (2 d.p.) b) 769.6 (1 d.p.) c) 7.052 (3 d.p.) d) $8\frac{1}{3}$ or $8.\dot{3}$

Page 19 Approximating and checking calculations

1. a) 0.003 79 b) 27 500 c) 307 000 2. 100 3. 10 rolls 4. £4.75

Page 21 Ratio

1. a) 4 : 5 b) 1 : 2 c) 5 : 2
2. 10, 15 and 35 sweets respectively
3. £2.76
4. 36 cm

Page 23 Indices

1. a) 12^{12} b) 9^{-6} c) 1 d) 18^{8} e) 4^{10} f) 1
2. a) x^{13} b) $6x^{13}$ c) $4x^{2}$ d) $5x^{11}$ e) $2x^{12}$
3. a) 16 b) $\frac{1}{125}$ c) 12 d) $\frac{1}{6}$
4. a) $\frac{1}{16x^2}$ b) $\frac{1}{36x^4y^8}$

Page 25 Standard index form

1. a) 6.3×10^{5} b) 2.73×10^{3} c) 4.29×10^{-5} d) 6.3×10^{-7}
2. a) 6×10^{12} b) 1.22×10^{9} c) 4×10^{3} d) 3×10^{18}
3. a) 4.35×10^{10} b) 4.59×10^{16}
4. 9.76×10^{10}

Page 26 Diverse and inverse proprtion

1. $y = 1$ (equation: $y = \frac{100}{x^2}$)
2. $p = 21$ (equation: $p = 3\sqrt{t}$)
3. $r = 1.587$ (equation: $s = \frac{40}{r^3}$)

Page 27 Recurring decimals and surds

1. a) $\frac{15}{99} = \frac{5}{33}$ b) $\frac{7}{9}$ c) $\frac{281}{990}$
2. a) $5\sqrt{3}$ b) $10\sqrt{5}$ c) $11 - 6\sqrt{2}$ d) $4\sqrt{2} - \sqrt{6} + 18$
3. $\frac{\sqrt{3}}{3}$

Page 29 Upper and lower bounds of measurement

1. upper bound = 0.65 (2 d.p.) lower bound = 0.64 $\therefore$ $0.64 \leqslant x < 0.65$
2. upper bound = 56.375 lower bound = 51.675 $\therefore$ $51.675 \leqslant x < 56.375$

Pages 30–31 Answers to practice questions

1. £25 000
2. a) 365 b) 0.706
3. $\frac{9+9}{0.2 \times 50} = \frac{18}{10} = 1.8$
4. The 100 ml tube of toothpaste.
5. 1.5 cm
6. £376.47
7. £6502.50
8. £715.02
9. 20%
10. a) 2.67×10^{6} b) 4.27×10^{3} c) 3.296×10^{-2} d) 2.7×10^{-2}
11. a) 4.2×10^{-5} g b) 2.52×10^{1} g
12. a) 1.2×10^{22} b) 2×10^{11}
13. a) $y = 3x$
    b) $x = 6.\dot{6}$
14. $y = \frac{20}{x^2}$ a) $y = 1\frac{1}{4}$ b) $x = \sqrt{2} = (1.414\ldots)$
15. a) $\frac{4}{9}$ b) $\frac{21}{99} = \frac{7}{33}$ c) $\frac{234}{999} = \frac{26}{111}$ d) $\frac{25}{90} = \frac{5}{18}$
16. a) $2\sqrt{3}$ b) $5\sqrt{3}$ c) $10\sqrt{2}$ d) $3\sqrt{2} - 2\sqrt{6}$ e) 15
17. Upper bound = 2.301375 m^2
    Lower bound = 2.271075 m^2
18. Upper bound = 13.28 m/s (2 d.p.)
    Lower bound = 13.04 m/s (2 d.p.)
19. Upper bound = 6.7349 cm (4 d.p.)
    Lower bound = 6.7112 cm (4 d.p.)
20. a) 4 b) 4 c) $\frac{1}{36}$ d) 7 e) $\frac{1}{5}$ f) $\frac{1}{16}$ g) $\frac{49}{25}$

Algebra

Quick test answers

Page 33 Algebra 1

1. a) $10a$ b) $8a + b$ c) $9x + 4y$ d) $4x^2y - 5xy^2$
2. a) –19.2 b) 28.8 c) 0.94 d) 53.8

Page 35 Algebra 2

1. a) $3x + 6$ b) $2x + 2y$ c) $-6x - 12$ d) $x^2 + 5x + 6$ e) $y^2 - 7y + 12$ f) $a^2 + 4a + 4$
2. a) $3(x + 3)$ b) $5(y - 3)$ c) $6x(2x - 1)$ d) $(x - 6)(x + 1)$ e) $(x - 1)(x - 2)$ f) $(x - 4)(x + 4)$
3. $u = \pm\sqrt{v^2 - 2as}$

Page 37 Equations 1

1. $x = 6$ 2. $x = 4.5$ 3. $x = 2$ 4. $x = 4$ 5. $x = -1$ 6. $x = -1.4$

7. $x = 1, x = -5$ 8. $x = 2, x = 3$ 9. $k = 1\frac{3}{4}$

Page 39 Equations 2

1. a) $x = -4.5, y = 4$ b) $a = 2, b = 1$

2. $x = 3, y = -1$

3. 3.3

Page 41 Further algebra and equations

1. a) $x = \frac{1}{3}, x = 2$ b) $x = -\frac{1}{2}, x = -2$ c) $x = -\frac{1}{4}, x = 2$

2. $r = \frac{-(y + ps)}{(p - 1)}$ or $\frac{(y + ps)}{(1 - p)}$

3. a) $\frac{5x + 4}{(x + 2)(x - 1)}$ b) $\frac{(s + 2)(s - 1)}{6(s - 3)}$ c) $\frac{18(a + b)(a + 1)}{(a + 2)}$

Page 43 Inequalities

1. a) 13, 15 b) 25, 36 c) 4, 2

2. a) $2n + 3$ b) $3n - 1$ c) $4n + 2$ d) $10 - 2n$

Page 45 Straight line graphs

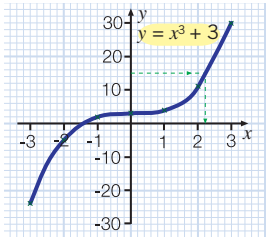
1. a) $x = 1$ b) $x = 1.5$

2. a) Gradient 2; Intercept (0, 4) b) Gradient 3; Intercept (0, –2)
   c) Gradient 3; Intercept (0, 2)

Page 47 Curved graphs

1. a)

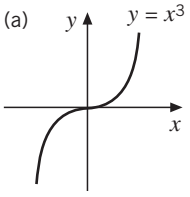
x	–3	–2	–1	0	1	2	3
y	–24	–5	2	3	4	11	30

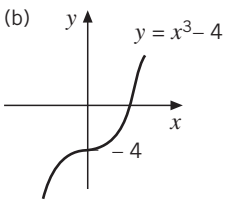


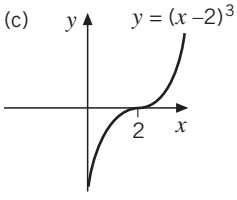
   b) The coordinates that need to be plotted are:
   (3, 30), (2, 11), (1, 3), (0, 3), (–1, 2), (–2, –5), (–3, –24)
   c) When $y = 15$, $x = 2.3$ (to 1 d.p.)

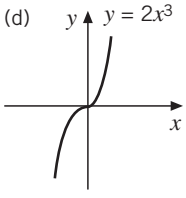
2. Graph A: $y = 3 - x^2$ Graph B: $y = \frac{2}{x}$ Graph C: $y = 5 - x$ Graph D: $y = x^3$

Page 49 Harder work on graphs

1. (a) 

(b) 

(c) 

(d) 

2. $x = -1$, $y = 1$ $x = 4$, $y = 16$

Page 51 Interpreting graphs

1. Container A graph 3; Container B graph 1; Container C graph 2

2. a) $x = -2.8, 1.8$ b) $x = -1.4, 1.4$ c) $x = 2.7, -0.7$

Pages 52–53 Answers to practice questions

1. a) $x = 3$ b) $x = 4$ c) $x = 5$ d) $x = 2$ e) $x = 1$

2. $2n^2 + 1$

3. a) $12x^4$ b) $12x^5y^3$
   c) $4y^3$ d) $9y^4$

4. $a = 3, b = -2$

5. $P = 54.3$ (3 s.f.) b) $y = \frac{P^2 + 3x^2}{5x}$

6. $x = 1.8$

7. $1 \leqslant n \leqslant 3$

8. A is $(y = 3 - 4x)$ B is $(y = 2x + 1)$
   C is $(xy = 6)$ D is $(y = x^2 - 4)$

9. a) $\frac{5x + 8}{(x + 4)(x - 2)}$ b) $\frac{4x + 30}{(x - 3)(x + 4)}$ c) $\frac{1}{2}$

10. $(x - 2)(2x - 9) = 0$ $\therefore x = 2$ $x = \frac{9}{2}$

11. a) $x = 1.721, x = -0.387$
    b) $x = 0.8385, x = -0.2385$

12. a) $(x - 2)^2 - 3 = 0$ b) $(x - 4)^2 - 6 = 0$
    $x = 3.732$ $x = 6.449$
    $x = 0.268$ $x = 1.551$

13. i) $(2x + 4)(x - 1) = 50$
    $2x^2 + 2x - 4 = 50$
    $2x^2 + 2x - 54 = 0$
    $x^2 + x - 27 = 0$
    ii) $x = 4.72$
    Length = 13.44 cm

14. $x = 0$ $y = 4$
    $x = -4$ $y = 0$
    The line $y = x + 4$ intersects the circle $x^2 + y^2 = 16$ at (0, 4) and (–4, 0).

15. $p = \frac{4x - 2r}{6 + x}$

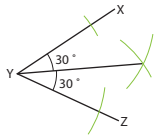
16. a) Move graph 2 units to left.
    b) Move graph 3 units down the y-axis.
    c) Reflect the graph in the x axis.
    d) Multiply all x values by $\frac{1}{2}$.

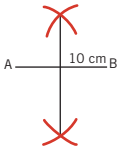
17. a = 6 b = 1.414

Shape, space and measures

Quick test answers

Page 55 Constructions





Page 57 Angles

1. a) $a = 150^o$ b) $b = 70^o$, $c = 110^o$, $d = 70^o$
   c) $a = 50^o$, $b = 50^o$, $c = 130^o$, $d = 50^o$

2. a) 72^o b) 108^o

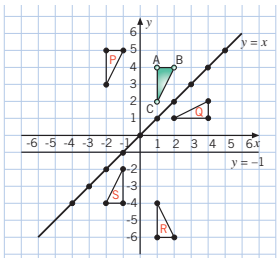
Page 59 Bearings and scale drawings

1. a) 072^o b) 305^o c) 145^o

2. a) 252^o b) 125^o c) 325^o

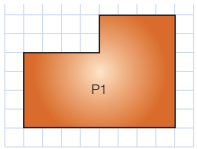
3. 7 km

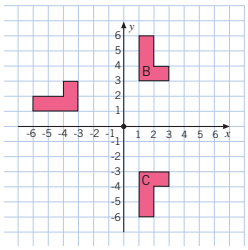
Page 61 Transformations 1

1. 

2. Move 2 to the left and 3 upwards

Page 63 Transformations 2

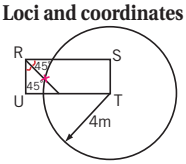
1. 

2. 

Page 65 Similarity and congruency

1. a) 7.8 cm b) 9.1 cm
2. 15.9 cm
3. Yes, because SAS, i.e. 2 sides and included angle are equal.
4. Area = 90 cm^2

Page 66 Loci and coordinates in 3D



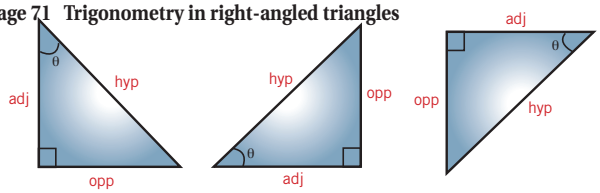
Page 67 Angle properties of circles

$a = 84°$; $b = 20°$; $c = 94°$; $d = 86°$

Page 69 Pythogoras' theorem

1. a) 17.2 cm b) 20.0 cm
2. 17 cm
3. 94.3 km

Page 71 Trigonometry in right-angled triangles

1. 

2. a) 7.5 cm b) 10.1 cm c) 18 cm 3. a) 25.6° b) 62.3° c) 36.9°

Page 73 Application of trigonometry

1. 35.8 m (3 s.f.)
2. a) 20 cm b) 68.0 cm c) 17.1°
3. a) HF = 25 cm b) Angle CHF = 11.3°

Page 75 Further trigonometry

1. a) 8.08 cm b) 28.4° c) 28.0° d) 68.83 cm 2. 41.8°, 138.2°

Page 77 Measures and measurement

1. 3.5 kg 2. 6.6 lb 3. 10 $\frac{1}{2}$ pints 4. $9.15 \leqslant 9.2 < 9.25$
5. $57.5 \leqslant 58 < 58.5$ 6. 2.25 m.p.h.
7. 8.57 hours (or 8 hours 34 minutes) 8. 2.2 g/cm^3

Page 79 Area of 2D shapes

1. a) 68.0 cm^2 (3 s.f.) b) 63.6 cm^2 (3 s.f.) c) 63.6 cm^2 (3 s.f.)
   d) 208 cm^2 (3 s.f.)
2. 21.5 cm^2 (3 s.f.)

Page 81 Volume of 3D shapes

1. a) 1750 cm^3 (3 s.f.) b) 60 100 cm^3 (3 s.f.) 2. 20.3 cm (3 s.f.)
3. a) Perimeter b) Volume c) Volume d) Area

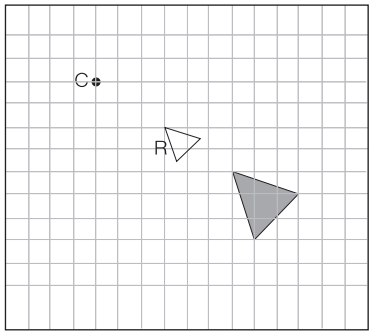
Page 83 Further length, area and volume

1. Arc length = 13.96 cm. Sector area = 139.6 cm^2 2. 123 π cm^3

Page 85 Vectors

1. a) Since $\vec{AC} = \begin{pmatrix}6\\8\end{pmatrix}$ and $\vec{PQ} = \begin{pmatrix}3\\4\end{pmatrix}$ then $2\vec{PQ} = \vec{AC}$ so they are parallel.
   b) Ratio 2:1
2. a) $\vec{PR} = \mathbf{p} + \mathbf{q}$ b) $\vec{MQ} = \frac{1}{2}(\mathbf{p} - \mathbf{q})$

Pages 86–87 Answers to practice questions

1. 

2. a) 122.5 m^2
   b) 63.6 cm^2
3. 52.5 mph
4. 5.2 m
5. 1.94 m^3
6. 9.2 m
7. a) 68°
   b) 3.65 cm
   c) 11.74 cm
8. $l\sqrt{r^2 + t^2}$; πr^2; $2\pi rl$
9. Arc length = 4.80 cm
   Sector area = 12.0 cm
10. Volume = 697.43 cm^3
11. Radius = 3.99 cm (2 d.p.)
12. Surface area = 1920 cm^2
13. Volume = 113.1 cm^3
14. a) $-\mathbf{r} + \mathbf{t}$ b) $-\mathbf{r} + 2\mathbf{t}$ c) $-3\mathbf{r} + 2\mathbf{t}$
15. −300 degrees, −60 degrees, 60 degrees, 300 degrees
16. a) 12.32 m (2 d.p.)
    b) $x = 147.5$ degrees (1 d.p.)
    c) $x = 45.2$ degrees (1 d.p.)
    d) $x = 7.11$ cm (2 d.p.)
17. height = 81.4 m (1 d.p.)
18. (1) $a = 55°$ $b = 35°$
    (2) $a = 75°$ $b = 58°$
    (3) $a = 85°$ $b = 62°$
    (4) $a = 55°$ $b = 70°$

Handling data

Quick test answers

Page 89 Collecting data

1. Many possible answers
2. Yr 7 – 24 Yr 8 – 30 Yr 9 – 46

Page 91 Representing data

1. The angles for the pie chart are: Brown 120°, Auburn 60°, Blonde 90°, Black 90°
2. a) The frequencies for the heights are: $140 \leqslant h < 145 = 6$, $145 \leqslant h < 150 = 10$, $150 \leqslant h < 155 = 11$, $155 \leqslant h < 160 = 5$, $160 < h \leqslant 165 = 2$
   b) 34 people were in the survey
   c) The frequency polygon should be plotted at the midpoints of the bars.

Page 93 Scatter diagrams and correlation

1. a) Positive b) Zero c) Negative d) Negative

Page 95 Averages 1

1. mean = 5; median = 4; mode = 4; range = 7
2. a) mean = 1.32 mins b) median = 1 c) mode = 0 d) range = 4

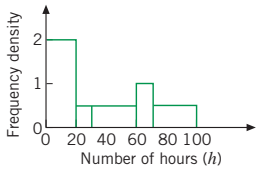
Page 97 Averages 2

1. a) 151.56 b) $150 \leqslant h < 155$

Page 99 Cumulative frequency graphs

1. A cumulative frequency graph with the following points should be plotted: (5,15) (10,75) (15,142) (20,172) (25,194) (30,200).
2. a) median approx. 12 miles b) interquartile range = 9 miles (approx.)

Page 101 Histograms



Page 103 Probability 1

1. Answers could be: I will get a 7 when I throw a die; or I will get a 4 when I throw a coin. 2. a) $\frac{1}{3}$ b) $\frac{4}{9}$ c) 0 3. $\frac{7}{9}$ 4. 240 5. $\frac{47}{200}$

Page 105 Probability 2

1. 0.36
2. a)

		Die 1					
		1	2	3	4	5	6
Die 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

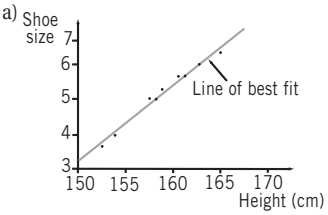
   b) $\frac{4}{36} = \frac{1}{9}$ c) 0
3. $\frac{24}{49}$

Pages 106–107 Answers to practice questions

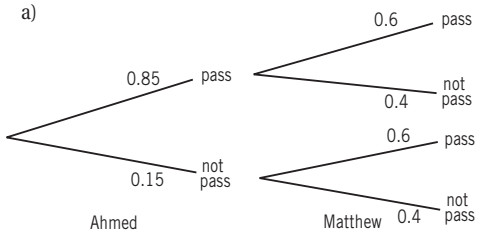
1. 0.18
2.

	Under 13 yrs	13 yrs+	Totals
Boys	15	27	42
Girls	12	21	33
Total	27	48	75

3. 140 people
4. 42 races
5. 3.5
6. a)



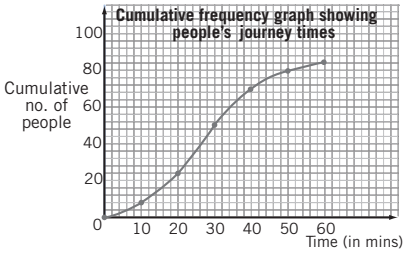
   b) Positive correlation
   c) See scatter diagram
   d) 155.5 cm
7. a) 50.7 kg b) $50 \leqslant W < 55$
8. a)



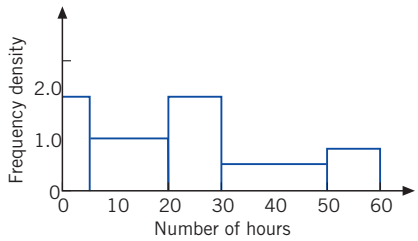
   b) 0.51
   c) 0.43
9. a)

Time (t mins)	Frequency	Cumulative frequency
$0 \leqslant t < 10$	5	5
$10 \leqslant t < 20$	20	25
$20 \leqslant t < 30$	26	51
$30 \leqslant t < 40$	18	69
$40 \leqslant t < 50$	10	79
$50 \leqslant t < 60$	4	83

   b)



   c) Approx 26 min
   d) 18 min
   e) 8 people
10.



11. a) $\frac{16}{49}$ b) $\frac{24}{49}$
12. Yr 7 30
    Yr 8 44
    Yr 9 41
    Yr 10 35
    Yr 11 50