C2 JAN 09

1) $(a+b)^5 = a^5 + 5a^4b + \binom{5}{2}a^3b^2$

$\Rightarrow (3-2x)^5 = 3^5 - 5(3)^4(2x) + 10(3)^3(2x)^2$

$= 243 - 810x + 1080x^2$

2) $\int_{-1}^{4} y\,dx = \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x\right]_{-1}^{4} = \frac{56}{3} - \left(-\frac{13}{6}\right) = \frac{125}{6}$

$-x^2+3x+4$

3) $1.8 \to 3.84$
$2.2 \to 4.14$
$3 \to 4.58$

$\int_1^3 \sqrt{10x-x^2} \simeq \frac{1}{2}(0.4)\left(3+4.58+2(3.47+3.84\ldots)\right)$

$= 7.852 \quad = \underline{7.9 \text{ (2sf)}}.$

4) $\log_5(4-x) - \log_5 x^2 = 1 \Rightarrow \log_5\left(\frac{4-x}{x^2}\right) = 1$

$\frac{4-x}{x^2} = 5^1 \Rightarrow 4-x = 5x^2 \Rightarrow 5x^2+x-4=0$

$\Rightarrow (5x-4)(x+1) = 0 \Rightarrow x = \underline{\frac{4}{5}}$

5) PR diameter $\Rightarrow \angle PQR = 90^\circ \Rightarrow$ PQ is perp to QR

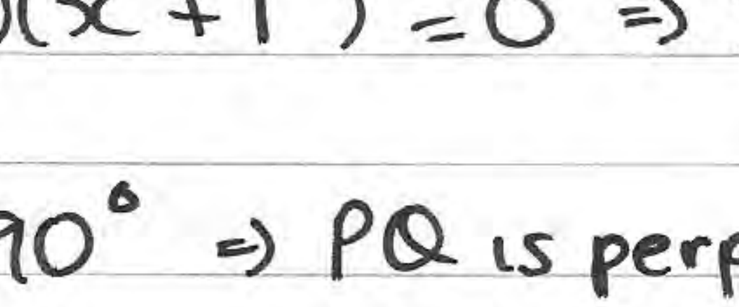
$M_{PQ} = \frac{8}{12} = \frac{2}{3} \qquad M_{QR} = \frac{-6}{a-9}$

perp $\Rightarrow \frac{2}{3} \times \frac{-6}{a-9} = -1 \Rightarrow \frac{-12}{3a-27} = -1$

$\Rightarrow 12 = 3a - 27 \Rightarrow 3a = 39 \Rightarrow \underline{a = 13}$ #

b) Midpoint PR = (5,3)

$(x-5)^2+(y-3)^2 = 65$



$r^2 = 8^2+1^2$
$= 65$

6) $f(2) = 16+40+2a+b = 1-5-a+b = f(-1)$

$\Rightarrow 2a+56 = -a-4 \Rightarrow 3a = -60 \Rightarrow a = -20$

b) $f(-3) = 81-135+60+b = 0 \quad 6+b=0 \Rightarrow \underline{b=-6}$

7) $\text{area} = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2(2.2) = \underline{39.6}$

b) $\angle DAC = \frac{2\pi - 2.2}{2} = \pi - 1.1 = \underline{2.04^c}$

c) $\triangle ADC = \frac{1}{2}ab\sin C = \frac{1}{2}(4)(6)\sin 2.04^c = 10.69...$

$\text{Area} = 39.6 + 2\times 10.69.. = \underline{61 cm^2 \text{ (2sf)}}$

8) $4(1-\cos^2 x)+9\cos x - 6 \Rightarrow 4 - 4\cos^2 x + 9\cos x - 6 = 0$

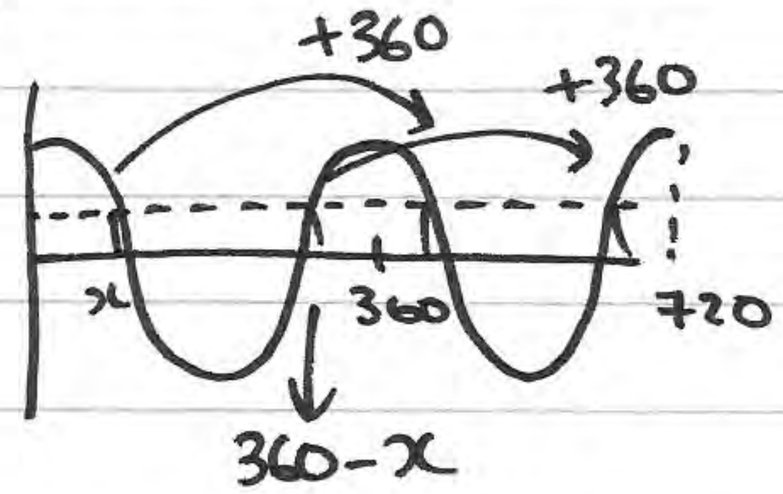
$\Rightarrow 4\cos^2 x - 9\cos x + 2 = 0$

b) $(4\cos x - 1)(\cos x - 2) = 0 \quad \cos x = \frac{1}{4} \quad \cos x = 2$

$x = \cos^{-1}\left(\frac{1}{4}\right)$ no solutions

$x = 75.5^\circ; 284.4^\circ; 435.5^\circ; 644.4^\circ$

$(360-\ ) \quad (+360) \quad (+360)$



9) $r = \frac{u}{u+4} = \frac{2u-15}{u} \Rightarrow u^2 = 2u^2 - 7u - 60$

$\Rightarrow u^2 - 7u - 60 = 0$

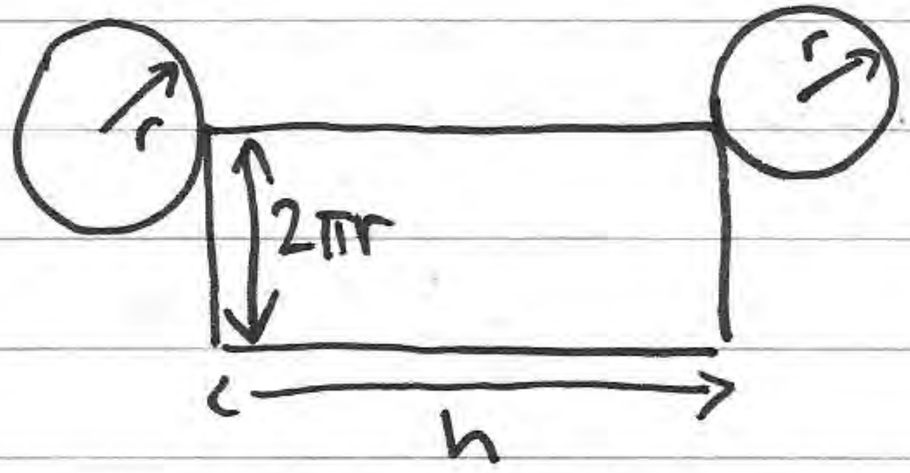
$\Rightarrow (u-12)(u+5) = 0 \quad \underline{u=12}$

c) $r = \frac{12}{16} = \frac{3}{4}$

$a = u+4 = 16$

d) $S_\infty = \frac{a}{1-r} = \frac{16}{1-\frac{3}{4}} = \frac{16}{\frac{1}{4}} = \underline{64}$

10)



$$SA = 2\pi r^2 + 2\pi rh = 800$$

$$\Rightarrow \pi r^2 + \pi rh = 400$$

$$\Rightarrow \pi rh = 400 - \pi r^2$$

$$\Rightarrow h = \frac{400 - \pi r^2}{\pi r}$$

$$V = \pi r^2 h = \pi r^2 \left(\frac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3 \quad \#$$

b) $\frac{dV}{dr} = 400 - 3\pi r^2$ at Max $\frac{dV}{dr} = 0 \Rightarrow 3\pi r^2 = 400$

$$\Rightarrow r^2 = \frac{400}{3\pi}$$

$$\Rightarrow r = 6.5147...$$

$$\frac{d^2V}{dr^2} = -6\pi r$$

$$\text{Max } V = 400(6.5147...) - \pi(6.5147...)$$

$$\text{max } V = \underline{1737 cm^3}$$

when $r = 6.5...$ $\frac{d^2V}{dr^2} = -6\pi(6.5...) < 0$ ∩ maximum