C2 June 2008

① $f(x) = 2x^3 - 3x^2 - 39x + 20$

a) $f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20$
$= 0$

$\therefore (x+4)$ is a factor of $f(x)$.

b)
$$\begin{array}{r} 2x^2 - 11x + 5 \\ x+4 \enclose{longdiv}{2x^3 - 3x^2 - 39x + 20} \\ \underline{2x^3 + 8x^2} \\ -11x^2 - 39x \\ \underline{-11x^2 - 44x} \\ 5x + 20 \\ \underline{5x + 20} \\ 0 \end{array}$$

$f(x) = (x+4)(2x^2 - 11x + 5)$
$= (x+4)(2x-1)(x-5)$

② $y = \sqrt{5^x + 2}$

i)

x	0	0.5	1	1.5	2
y	$\sqrt{3}$ ↓ 1.732	2.058	2.646	3.630	$3\sqrt{3}$ ↓ 5.196

b) $n = 4$, $h = 0.5$

$I \approx \frac{h}{2}\{y_0 + y_4 + 2(y_1 + y_2 + y_3)\}$
$\approx \frac{0.5}{2}\{\sqrt{3} + 3\sqrt{3} + 2(2.058 + 2.646 + 3.630)\}$
$\approx 0.25\{4\sqrt{3} + 16.668\}$
≈ 5.899

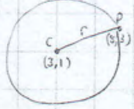
③ a) $(1+ax)^{10} = 1 + 10(ax) + \frac{10.9}{2!}(ax)^2 + \frac{10.9.8}{3!}(ax)^3 + \ldots$
$= 1 + 10ax + 45a^2x^2 + 120a^3x^3 + \ldots$

b) $120a^3 = 2 \times 45a^2 \Rightarrow 120a^3 = 90a^2$
$\frac{a^3}{a^2} = \frac{90}{120} \Rightarrow a = \frac{3}{4}$

④ a) $5^x = 7$
$x = \log_5 7 = 1.21$

b) $5^{2x} - 12(5^x) + 35 = 0$
$(5^x)^2 - 12(5^x) + 35 = 0$
Let $y = 5^x$: $y^2 - 12y + 35 = 0$
$(y-7)(y-5) = 0$
$y = 7$ or $y = 5$
$5^x = 7$ $5^x = 5$
$x = 1.21$ $x = 1$

⑤ a)



$r = \sqrt{5^2 + 2^2} = \cancel{29}\ \sqrt{29}$

C: $(x-3)^2 + (y-1)^2 = 29$

b) $m_{cp} = \frac{2}{5}$
$m_{tangent} = -\frac{5}{2}$

$y - 3 = -\frac{5}{2}(x - 8)$
$2y - 6 = 40 - 5x$
$5x + 2y - 46 = 0$

⑥ $a = 5$, $r = \frac{4}{5}$

a) $u_{20} = ar^{19} = 5 \times (\frac{4}{5})^{19} = 0.072$

b) $S_\infty = \frac{a}{1-r} = \frac{5}{1-\frac{4}{5}} = \frac{5}{\frac{1}{5}} = 25$

c) $S_k = \frac{a(1-r^k)}{1-r} = \frac{5(1-(\frac{4}{5})^k)}{\frac{1}{5}} = 25(1-(\frac{4}{5})^k) > 24.95$
$\Rightarrow 1 - (\frac{4}{5})^k > 0.998$
$\Rightarrow 0.002 > (\frac{4}{5})^k$
$\Rightarrow \log(0.002) > k\log(\frac{4}{5})$
$\Rightarrow k > \frac{\log(0.002)}{\log(0.8)}$ QED

d) $k > 27.85$
$\Rightarrow k = 28$

⑦

a) Length of arc $= r\theta = 7 \times 0.8 = 5.6\text{cm}$

b) Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 7^2 \times 0.8 = 19.6\text{ cm}^2$

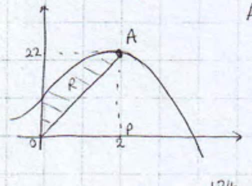
c) $AC = 7\text{cm}$ $\therefore AD = DC = 3.5\text{cm}$
$BD^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5\cos 0.8 = 27.11$
$BD = 5.207\text{cm}$
Perimeter of $R = BD + BC + DC$
$= 5.21 + 5.6 + 3.5 = 14.3\text{ cm}$

d) Area of triangle $ABD = \frac{1}{2} \times 7 \times 3.5\sin 0.8$
$= 8.788\text{ cm}^2$
Area of $R = 19.6 - 8.788 = 10.8\text{ cm}^2$

⑧ $y = 10 + 8x + x^2 - x^3$

a) $\frac{dy}{dx} = 8 + 2x - 3x^2$
Maximum at $\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 2x - 8 = 0$
$(3x+4)(x-2) = 0$
$x = -\frac{4}{3}$ or $x = 2$ QED.

b) When $x = 2$, $y = 10 + 8(2) + (2)^2 - (2)^3 = 22$.
Area of triangle $OAP = \frac{1}{2} \times 2 \times 22 = 22$.

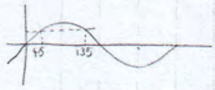


Area under curve $= \int_0^2 10 + 8x + x^2 - x^3\, dx$
$= [10x + 4x^2 + \frac{x^3}{3} - \frac{x^4}{4}]_0^2$
$= [(10(2) + 4(2)^2 + \frac{(2)^3}{3} - \frac{(2)^4}{4}) - (0)]$
$= \frac{104}{3} - 0 = \frac{104}{3}$

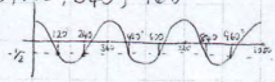
Area of $R = \frac{104}{3} - 22 = \frac{38}{3}$

⑨ $0 \leq x \leq 360 \Rightarrow -20 \leq x - 20 \leq 340$

a) $\sin(x - 20) = \frac{1}{\sqrt{2}}$
$x - 20 = 45°, 135°$
$\Rightarrow x = 65°, 155°$



b) $\cos 3x = -\frac{1}{2}$ $0 \leq 3x < 1080$
$3x = 120°, 240°, 480°, 600°, 840°, 960°$



$x = 40°, 80°, 160°, 200°, 280°, 320°$