C2 January 2008

(1) a) $x^3 - 2x^2 - 4x + 8$

i) $f(3) = (3)^3 - 2(3)^2 - 4(3) + 8 = 5$

ii) $f(-2) = (-2)^3 - 2(-2)^2 - 4(-2) + 8 = 0$

b) $x+2 \overline{)\, x^3 - 2x^2 - 4x + 8}$

$x^2 - 4x + 4$

$x^3 + 2x^2$

$-4x^2 - 4x$

$-4x^2 - 8x$

$4x + 8$

$4x + 8$

0

$x^3 - 2x^2 - 4x + 8 = (x+2)(x^2 - 4x + 4)$

$= (x+2)(x-2)^2$

Solutions are $x = -2$ and $x = 2$

(2)

a) $ar^3 = 10$, $ar^6 = 80$ $\Rightarrow r^3 = 8 \Rightarrow r = 2$

b) $ar^3 = 10 \Rightarrow 8a = 10 \Rightarrow a = \frac{10}{8} = 1.25$

c) $S_{20} = \frac{a(r^{20} - 1)}{r - 1} = \frac{1.25(2^{20} - 1)}{1} \approx 1310719$

(3) a) $(1 + \frac{x}{2})^{10} = 1 + 10(\frac{x}{2}) + \frac{10.9}{2!}(\frac{x}{2})^2 + \frac{10.9.8}{3!}(\frac{x}{2})^3 + \ldots$

$= 1 + 5x + \frac{15x^2}{4} + 15x^3 + \ldots$

b) $(1 + \frac{x}{2})^{10} = (1.005)^{10} \Rightarrow \frac{x}{2} = 0.005 \Rightarrow x = 0.01$

$(1.005)^{10} \approx 1 + 5(0.01) + \frac{5(0.01)^2}{36} + 15(0.01)^3$

(4) a) $3\sin^2\theta - 2\cos^2\theta = 1$

$3\sin^2\theta - 2(1 - \sin^2\theta) = 1$

$5\sin^2\theta - 2 = 1$

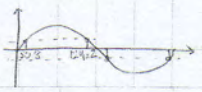
$5\sin^2\theta = 3$ QED

b) $5\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{5}$

$\sin\theta = \pm\sqrt{\frac{3}{5}} \Rightarrow$ If $\sin\theta = \sqrt{\frac{3}{5}}$, $\theta = 50.8°, 129.2°$

If $\sin\theta = -\sqrt{\frac{3}{5}}$, $\theta = -50.8, 230.8°, 309.2$



(5) Eqn. (1) $a = 3b$

Eqn. (2) $\log_3 a + \log_3 b = 2 \Rightarrow \log_3 ab = 2$

$\Rightarrow ab = 3^2$

$\Rightarrow ab = 9$

Sub. (1) into (2): $3b \times b = 9 \Rightarrow 3b^2 = 9$

$b^2 = 3$

$b = \pm\sqrt{3}$

If $b = \sqrt{3}$, $a = 3\sqrt{3}$

If $b = -\sqrt{3}$, $a = -3\sqrt{3}$

(6)

a) $BC^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15$

$= 63851.9 \Rightarrow BC = 253\,m$

b) To find angle $A\hat{B}C$: $\sin B = \frac{700 \sin 15}{253} = 0.717 \Rightarrow B = 45.8°$

Ordinarily, the bearing would be $180 - 45.8$, but clearly θ is an acute angle, so $\theta = 046°$

(7)

a) $y = 6x - x^2$ $\quad 6x - x^2 = 0 \Rightarrow x(6 - x) = 0$

$\Rightarrow x = 0$ or $x = 6$ QED

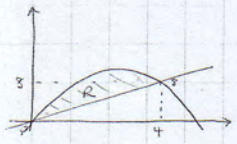
b) $2x = 6x - x^2$

$x^2 - 4x = 0 \Rightarrow x(x - 4) = 0$ so $x = 0$ or $x = 4$.

If $x = 0$, $y = 0$

If $x = 4$, $y = 8$ (using $y = 2x$).

c)



Area of triangle $= \frac{1}{2} \times 4 \times 8 = 16$

Area under curve $= \int_0^4 6x - x^2\,dx = \left[3x^2 - \frac{x^3}{3}\right]_0^4$

$= \left[\left(3(4)^2 - \frac{(4)^3}{3}\right) - (0)\right]$

$= \frac{80}{3}$

Area of $R = \frac{80}{3} - 16 = \frac{32}{3}$

(8)

a) $(x-6)^2 + (y-4)^2 = 3^2$

$(x-6)^2 + (y-4)^2 = 9$

b) $MP = \sqrt{6^2 + 2^2} = 2\sqrt{10}$

Angle $MTP = 90°$

$\cos(TMQ) = \frac{3}{2\sqrt{10}} \Rightarrow TMQ = 1.0766^c$

c) Area of triangle $MPT = \frac{1}{2} \times 3 \times 2\sqrt{10} \times \sin(1.0766)$

$= 8.352$

Area of sector $TMQ = \frac{1}{2} r^2\theta = \frac{1}{2} \times 3^2 \times 1.0766 = 4.8446$

Shaded area $= 8.352 - 4.8446 = 3.507$

(9)

a) Surface area $= 2x^2 + 3xy$

Volume $= x^2 y = 100 \Rightarrow y = \frac{100}{x^2}$.

$A = 2x^2 + 3x\left(\frac{100}{x^2}\right) = 2x^2 + \frac{300}{x}$ QED

b) $\frac{dA}{dx} = 4x - 300x^{-2} = 4x - \frac{300}{x^2}$

Stationary points at $\frac{dA}{dx} = 0$

$4x - \frac{300}{x^2} = 0 \Rightarrow 4x = \frac{300}{x^2}$

$4x^3 = 300 \Rightarrow x^3 = 75$

$x = 5\sqrt{3}$

c) $\frac{d^2A}{dx^2} = 4 + 600x^{-3} = 4 + \frac{600}{x^3}$

At $x = 5\sqrt{3}$, $\frac{d^2A}{dx^2} = 4 + \frac{600}{(5\sqrt{3})^3} = 4.92 > 0 \therefore$ minimum

d) When $x = 5\sqrt{3}$, $A = 2(5\sqrt{3})^2 + \frac{300}{5\sqrt{3}}$

$= 184.6\ m^2$

$(185\,m^2)$.