C2 Jan 2005
Model Solutions

① $(3+2x)^5 = \left[3\left(1+\frac{2x}{3}\right)\right]^5 = 3^5\left(1+\frac{2x}{3}\right)^5 = 243\left(1+\frac{2x}{3}\right)^5$

$243\left(1+\frac{2x}{3}\right)^5 = 243\left(1 + 5\left(\frac{2x}{3}\right) + \frac{5\times4}{2!}\left(\frac{2x}{3}\right)^2 + \dots\right)$

$= \underline{243\left(1 + \frac{10x}{3} + \frac{40x^2}{9} + \dots\right)} = \underline{243 + 810x + 1080x^2 + \dots}$

② a) $M = \left(\frac{5+13}{2}, \frac{-1+11}{2}\right) = \underline{(9, 5)}$

b) Radius (length) from $(5,-1)$ to $(9,5)$:

$= \sqrt{(9-5)^2 + (5--1)^2} = \sqrt{4^2+6^2} = \sqrt{52}$

Centre: $(9, 5)$

Equation: $\underline{(x-9)^2 + (y-5)^2 = 52}$

③

a) Method 1:

$3^x = 5$

$\log_3 3^x = \log_3 5$

$x = \log_3 5$

$= \underline{1.46}$

Method 2:

$3^x = 5$

$\log 3^x = \log 5$

$x \log 3 = \log 5$

$x = \frac{\log 5}{\log 3} = \underline{1.46}$

b) $\log_2(2x+1) - \log_2 x = 2$

$\log_2\left(\frac{2x+1}{x}\right) = 2$

$\frac{2x+1}{x} = 2^2 = 4 \Rightarrow 2x+1 = 4x$

$\Rightarrow \underline{x = \tfrac{1}{2}}$

④ a) $5\cos^2 x = 3(1+\sin x)$

$5(1-\sin^2 x) = 3 + 3\sin x$ (Using $\sin^2 x + \cos^2 x = 1$ $\therefore \cos^2 x = 1 - \sin^2 x$)

$5 - 5\sin^2 x = 3 + 3\sin x$

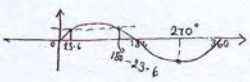
$5\sin^2 x + 3\sin x - 2 = 0$ QED

b) $(5\sin x - 2)(\sin x + 1) = 0$ $\qquad 0 \le x \le 360°$

$\Rightarrow \sin x = \tfrac{2}{5}$ or $\sin x = -1$

$x = \sin^{-1}\left(\tfrac{2}{5}\right)$ $\qquad x = -90$

$= \underline{23.6°}, \underline{156.4°}$ $\qquad = \underline{270°}$



⑤ $f(x) = x^3 - 2x^2 + ax + b$

a) $f(2) = 1 \Rightarrow (2)^3 - 2(2)^2 + 2a + b = 1$

$2a + b = 1 \Rightarrow b = 1 - 2a$

$f(-1) = 28 \Rightarrow (-1)^3 - 2(-1)^2 + a(-1) + b = 28$

$-a + b = 31 \Rightarrow b = 31 + a$

$1 - 2a = 31 + a \Rightarrow 3a = -30, \ \underline{a = -10}$

$b = 31 + -10 = \underline{21}$

$f(x) = x^3 - 2x^2 - 10x + 21.$

b) $f(3) = (3)^3 - 2(3)^2 - 10(3) + 21 = 27 - 18 - 30 + 21 = 0$ Q

⑥ a) $ar = 7.2$, $ar^3 = 5.832$ $\Big\}$ $\frac{ar^3}{ar} = r^2 = \frac{5.832}{7.2} = 0.81$

$\Rightarrow \underline{r = 0.9}$

b) $ar = 7.2 \Rightarrow a = \frac{7.2}{r} = \frac{7.2}{0.9} = \underline{8}.$

c) $S_{50} = \frac{a(1-r^{50})}{1-r} = \frac{8(1-0.9^{50})}{1-0.9} = \underline{79.588}$

d) $S_\infty = \frac{a}{1-r} = \frac{8}{1-0.9} = 80$

Difference $= 80 - 79.588 = \underline{0.412}$

⑦ a) Arc length $= r\theta = 8 \times 0.7 = \underline{5.6}$

b) Length: $BC^2 = 8^2 + 11^2 - 2\times 8 \times 11 \cos 0.7$ (Cosine Rule)

$= 64 + 121 - 176\cos 0.7$

$= 50.388$

$BC = 7.098.$

Perimeter of R = $BD + BC + DC = 5.6 + 3 + 7.098 = \underline{15.7}$

c) Area of triangle $= \tfrac{1}{2} \times 8 \times 11 \times \sin 0.7 = 28.346$

Area of sector $= \tfrac{1}{2} \times 8^2 \times 0.7 = 22.4.$

$(\tfrac{1}{2}r^2\theta)$

Area of $R = 28.346 - 22.4$

$= \underline{5.95}$

⑧

a) Intersection of $y = 3x + 20$ and $y = x^2 + 6x + 10$

$3x + 20 = x^2 + 6x + 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5, \ y = 3(-5) + 20 = 5$ $\qquad A\ \underline{(-5, 5)}$

$x = 2, \ y = 3(2) + 20 = 26$ $\qquad B\ \underline{(2, 26)}$

b) $\int_{-5}^{2} x^2 + 6x + 10 \ dx = \left[\frac{x^3}{3} + \frac{6x^2}{2} + 10x\right]_{-5}^{2}$

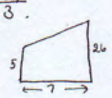
$= \left[\frac{x^3}{3} + 3x^2 + 10x\right]_{-5}^{2}$

$= \left[\left(\frac{(2)^3}{3} + 3(2)^2 + 10(2)\right) - \left(\frac{(-5)^3}{3} + 3(-5)^2 + 10(-5)\right)\right]$

$= \left(\frac{104}{3}\right) - \left(-\frac{50}{3}\right)$

$= \frac{154}{3}.$

Area trapezium:

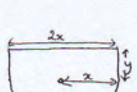


$A = \tfrac{1}{2} \times 7 \times (26 + 5)$

$= 108.5$

Area ® $= 108.5 - \frac{154}{3} = \underline{\frac{343}{6}}$

⑨ a) Perimeter $= 2x + 2y + \tfrac{1}{2}(2\pi x) = 80$



$\Rightarrow 2x + 2y + \pi x = 80$

$\Rightarrow y = \frac{80 - 2x - \pi x}{2} = 40 - x - \frac{\pi x}{2}$

Area $= 2xy + \tfrac{1}{2}(\pi x^2) = 2x\left(40 - x - \frac{\pi x}{2}\right) + \frac{\pi x^2}{2}$

$= 80x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$

$A = 80x - 2x^2 - \frac{\pi x^2}{2} = 80x - \left(2 + \frac{\pi}{2}\right)x^2$ QED

b) Stationary point at $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 80 - 2\left(2 + \frac{\pi}{2}\right)x = 0 \Rightarrow 80 = 2\left(2+\frac{\pi}{2}\right)x$

$\Rightarrow \underline{x = \frac{40}{2 + \pi/2}}$

c) $\frac{d^2A}{dx^2} = -2\left(2 + \frac{\pi}{2}\right)$. This is always < 0, so we have a $\underline{\text{maximum}}$.

d) Max $A = 80\left(\frac{40}{2+\pi/2}\right) - \left(2 + \frac{\pi}{2}\right)\left(\frac{40}{2+\pi/2}\right)^2$

$= \underline{448 \text{ m}^2}$