C2 June 2005
Model Solutions

① $y = 2x^2 - 12x$

$\frac{dy}{dx} = 4x - 12$

At stationary point, $\frac{dy}{dx} = 0 \Rightarrow 4x - 12 = 0$

$4x = 12 \Rightarrow x = 3,\ y = 2(3)^2 - 12(3) = -18$

Stationary point: $\underline{(3, -18)}$

②
a) Method 1:

$5^x = 8$

$\log_5 5^x = \log_5 8$

$x = \log_5 8$

$= \underline{1.80}$

Method 2:

$5^x = 18$

$\log 5^x = \log 18$

$x \log 5 = \log 18$

$x = \frac{\log 18}{\log 5} = \underline{1.80}$

b) $\log_2(x+1) - \log_2 x = \log_2 7$

$\log_2\left(\frac{x+1}{x}\right) = \log_2 7$

$\frac{x+1}{x} = 7 \Rightarrow x + 1 = 7x \Rightarrow 6x = 1 \Rightarrow \underline{x = 1/6}$

③ $f(x) = 2x^3 + x^2 - 25x + 12$

a) $f(-4) = 2(-4)^3 + (-4)^2 - 25(-4) + 12 = -128 + 16 + 100 + 12 = 0$ QED

b)
$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+4 \overline{)\,2x^3 + x^2 - 25x + 12} \\ \underline{2x^3 + 8x^2} \\ -7x^2 - 25x \\ \underline{-7x^2 - 28x} \\ 3x + 12 \\ \underline{3x + 12} \\ 0 \end{array}$$

$f(x) = (x+4)(2x^2 - 7x + 3)$

$= \underline{(x+4)(2x-1)(x-3)}$

④
a) $(1 + px)^{12} = 1 + 12(px) + \frac{12 \times 11}{2!}(px)^2 + \ldots$

$= \underline{1 + 12px + 66p^2x^2 + \ldots}$

b) $\left.\begin{array}{l} 12p = -q \Rightarrow q = -12p \\ 66p^2 = 11q \Rightarrow q = 6p^2 \end{array}\right\}$ $6p^2 = -12p \Rightarrow 6p^2 + 12p = 0$

$\Rightarrow p^2 + 2p = 0$

$p(p+2) = 0$

$p = 0$ or $\underline{p = -2}$

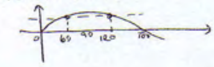
"p is a non-zero constant"

$q = -12p = 12 \times 2 = \underline{24}$

⑤
a) $\sin(x + 10) = \frac{\sqrt{3}}{2}$ $\quad 0 \le x \le 180$

Let $X = x + 10$ $\quad (x = X - 10)$ $\quad 10 \le X \le 190$

$\sin X = \frac{\sqrt{3}}{2} \Rightarrow X = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$



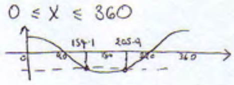
$X = 60°, 120°$

$x = \underline{50°}, \underline{110°}$

b) $\cos 2x = -0.9$ $\quad 0 \le x \le 180$

Let $X = 2x$ $\quad (x = \frac{X}{2})$ $\quad 0 \le X \le 360$

$\cos X = -0.9$



$X = \cos^{-1}(-0.9) = 154.1, 205.9$

$x = \underline{77.1°}, \underline{103.0°}$

⑥ $y = \frac{1}{10} x \sqrt{20 - x}$ $\quad 0 \le x \le 20$

a)

x	0	4	8	12	16	20
y	0	1.6	2.771	3.394	3.2	0

b) $n = 5,\ h = 4$

$A \approx \frac{1}{2} \times 4 \times \{0 + 0 + 2(1.6 + 2.771 + 3.394 + 3.2)\}$

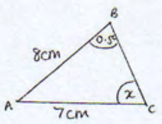
$\approx 2 \times 21.93$

$\approx \underline{43.86\ m^2}$

c) $2\,ms^{-1} = 120\,m\,min^{-1}$

$V \approx 43.86 \times 120 \approx \underline{5263.2\ m^3}$

⑦



a) $\frac{\sin x}{8} = \frac{\sin 0.5}{7}$

$\Rightarrow \sin x = \frac{8 \sin 0.5}{7} = \underline{0.548}$

b) $x = \sin^{-1}(0.548) = \underline{0.58^c}$

or $x = \pi - 0.58 = \underline{2.56^c}$

⑧ $x^2 + y^2 - 10x + 9 = 0$

a) $x^2 - 10x + y^2 = 9$ (Complete the square for x and y)

$(x-5)^2 - 25 + (y-0)^2 - 0 = 9$

$\underline{(x-5)^2 + y^2 = 36}$ So $\underline{A = (5, 0)}$

b) $r = \sqrt{36} = \underline{6}$

c) $m_1 = 7/2 \Rightarrow m_{AT} = -2/7$

AT: $y - 0 = -2/7(x - 5)$

$\underline{7y = -2x + 10}$

⑨ a) $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$

$S_n - rS_n = (a + ar + ar^2 + \ldots + ar^{n-1}) - (ar + ar^2 + \ldots + ar^n)$

$= a - ar^n$

$S_n(1 - r) = a(1 - r^n) \Rightarrow \underline{S_n = \frac{a(1-r^n)}{1-r}}$ QED

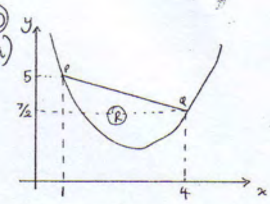
b) $a = \overset{(2005)}{35{,}000}$, $ar = \overset{(2006)}{35{,}000 \times 1.04}$, $ar^2 = \overset{(2007)}{35{,}000 \times 1.04^2}$, ...

In 2008, salary $= ar^3 = 35{,}000 \times 1.04^3 = £\underline{39{,}400}$

c) 1st term: 2005, 2nd term: 2006, ... 20th term: 2024

$S_{20} = \frac{a(1 - r^{20})}{1 - r} = \frac{35{,}000(1 - 1.04^{20})}{1 - 1.04} = £\underline{1{,}042{,}000}$

⑩
a)



When $x = 1$, $y = 2(1) + \frac{8}{(1)^2} - 5$

$= 5$

When $x = 4$, $y = 2(4) + \frac{8}{(4)^2} - 5$

$= 7/2$

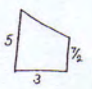
$\int_1^4 2x + \frac{8}{x^2} - 5\ dx = \int_1^4 2x + 8x^{-2} - 5\ dx$

$= \left[\frac{2x^2}{2} + \frac{8x^{-1}}{-1} - 5x\right]_1^4 = \left[x^2 - \frac{8}{x} - 5x\right]_1^4$

$= \left[(4^2 - 8/4 - 5 \times 4) - (1^2 - 8/1 - 5 \times 1)\right]$

$= (-6) - (-12) = 6$

Area trapezium:



$A = \frac{1}{2} \times 3 \times (5 + 7/2)$

$= 12.75$

Area ® $= 12.75 - 6 = \underline{6.75}$

b) $y = 2x + 8x^{-2} - 5$

$\frac{dy}{dx} = 2 - 16x^{-3}$

y is increasing when $\frac{dy}{dx} > 0$

$2 - 16x^{-3} > 0$

$2 - \frac{16}{x^3} > 0 \Rightarrow 2 > \frac{16}{x^3}$

$\Rightarrow 2x^3 > 16 \Rightarrow x^3 > 8$

$\Rightarrow x > 2$ QED