C2 Jan 2006

Model Solutions

(1) $f(x) = 2x^3 + x^2 - 5x + c$

a) $f(1) = 2\times1^3 + 1^2 - 5\times1 + c = 0$

$2 + 1 - 5 + c = 0$

$\underline{c = 2}$

b)
$$\begin{array}{r} 2x^2+3x-2 \\ x-1\,\overline{)\,2x^3+x^2-5x+2} \\ \underline{2x^3-2x^2} \\ 3x^2-5x \\ \underline{3x^2-3x} \\ -2x+2 \\ \underline{-2x+2} \\ 0 \end{array}$$

$f(x) = (x-1)(2x^2+3x-2)$
$= \underline{(x-1)(2x-1)(x+2)}$

c) $f(\tfrac{3}{2}) = 2(\tfrac{3}{2})^3 + (\tfrac{3}{2})^2 - 5(\tfrac{3}{2}) + 2$
$= \frac{27}{4} + \frac{9}{4} - \frac{15}{2} + 2$
$= \underline{\tfrac{7}{2}}$

(2)
a) $(1+px)^q = 1 + q(px) + \frac{q\times8}{2!}(px)^2 + \ldots$
$= \underline{1 + 9px + 36p^2x^2 + \ldots}$

b) $9p = 36 \Rightarrow \underline{p = 4}$
$36p^2 = q \Rightarrow q = 36\times16 = \underline{576}$

(3)
a) Length $AB = \sqrt{(4-3)^2 + (5-0)^2} = \sqrt{1^2+5^2} = \underline{\sqrt{26}}$.

b) Midpoint $AB = M\left(\frac{4+3}{2}, \frac{0+5}{2}\right) = \underline{(3.5, 2.5)}$

c) $r = \tfrac{1}{2}\sqrt{26} \Rightarrow r^2 = (\tfrac{1}{2}\sqrt{26})^2 = 6.5$
Equation of circle: $(x-a)^2 + (y-b^2) = r^2$
$\Rightarrow \underline{(x-3.5)^2 + (y-2.5)^2 = 6.5^2}$

(4) $a = 120 \qquad S_\infty = \frac{a}{1-r} = 480$

a) $\frac{a}{1-r} = 480 \Rightarrow a = 480(1-r)$
$120 = 480(1-r)$
$\frac{120}{480} = 1 - r$
$\Rightarrow \underline{r = \tfrac{3}{4}}$ QED

b) 5^{th} term $= ar^4 = 120\times(\tfrac{3}{4})^4 = 37.96875$
6^{th} term $= ar^5 = 120\times(\tfrac{3}{4})^5 = 28.4765625$
Difference $(ar^4 - ar^5) = \underline{9.49}$

c) $S_7 = \frac{a(1-r^7)}{1-r} = \frac{120(1-(\frac{3}{4})^7)}{1-\frac{3}{4}} = 480\times(1-(\tfrac{3}{4})^7) = \underline{415.93}$

d) $S_n > 300 \Rightarrow \frac{120(1-r^n)}{1-r} > 300$
$\frac{120(1-(\frac{3}{4})^n)}{1-\frac{3}{4}} > 300 \Rightarrow 480(1-(\tfrac{3}{4})^n) > 300$
$\Rightarrow 1 - (\tfrac{3}{4})^n > \tfrac{5}{8} \quad \left(\tfrac{300}{480}\right)$
$\Rightarrow -(\tfrac{3}{4})^n > -\tfrac{3}{8}$

Method 1:
$-(\tfrac{3}{4})^n > -\tfrac{3}{8}$
$\tfrac{3}{8} > (\tfrac{3}{4})^n$
$\log_{\frac{3}{4}}(\tfrac{3}{8}) > n$
$n > 3.4$

Method 2:
$-(\tfrac{3}{4})^n > -\tfrac{3}{8}$
$\tfrac{3}{8} > (\tfrac{3}{4})^n$
$\log(\tfrac{3}{8}) > \log(\tfrac{3}{4})^n$
$\log\tfrac{3}{8} > n\log\tfrac{3}{4}$
$n > \frac{\log\frac{3}{8}}{\log\frac{3}{4}}$
$n > 3.4$

$\underline{n = 4}$

(5)
a) Cosine rule: $o^2 = a^2 + b^2 - 2ab\cos\hat{O}$
$6^2 = 5^2 + 5^2 - 2\times5\times5\cos\hat{O}$
$36 = 50 - 50\cos\hat{O}$
$-14 = -50\cos\hat{O}$
$\Rightarrow \cos\hat{O} = \frac{14}{50} = \frac{7}{25}$ QED

b) $A\hat{O}B = \cos^{-1}(\tfrac{7}{25}) = \underline{1.287^c}$

c) Area sector $= \tfrac{1}{2}r^2\theta = \tfrac{1}{2}\times5^2\times1.287 = \underline{16.088\ m^2}$

d) Area segment $= \tfrac{1}{2}r^2(\theta - \sin\theta) = \tfrac{1}{2}\times5^2(1.287 - \sin1.287)$
$= \underline{4.088\ m^2}$

(6) $v = \sqrt{1.2^t - 1} \qquad 0 \le t \le 30$

a)

t	0	5	10	15	20	25	30
v	0	1.22	2.28	3.80	6.11	9.72	15.37

b) $s \approx \tfrac{1}{2}h\{y_0 + y_7 + 2(y_1+y_2+y_3+y_4+y_5+y_6)\} \quad (n=6, h=5)$
$\approx \tfrac{1}{2}\times5\times\{0 + 15.37 + 2(1.22+2.28+3.80+6.11+9.72)\}$
$\approx 2.5\times\{15.37 + 46.26\}$
$\approx \underline{154.075}$

(7) $y = 2x^3 - 5x^2 - 4x + 2$

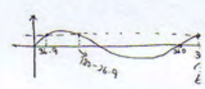
a) $\frac{dy}{dx} = \underline{6x^2 - 10x - 4}$

b) Turning points at $\frac{dy}{dx} = 0$: $6x^2 - 10x - 4 = 0$
$3x^2 - 5x - 2 = 0 \Rightarrow (3x+1)(x-2) = 0$
$\Rightarrow x = -\tfrac{1}{3}$ or $x = 2$.
If $x = -\tfrac{1}{3}$, $y = 2(-\tfrac{1}{3})^3 - 5(-\tfrac{1}{3})^2 - 4(-\tfrac{1}{3}) + 2 = \frac{73}{27}$ $\quad (-\tfrac{1}{3}, \tfrac{73}{27})$
If $x = 2$, $y = 2(2)^3 - 5(2)^2 - 4(2) + 2 = -10 \quad (2, -10)$

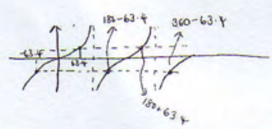
c) $\frac{d^2y}{dx^2} = \underline{12x - 10}$

d) When $x = -\tfrac{1}{3}$, $\frac{d^2y}{dx^2} = 12\times-\tfrac{1}{3} - 10 = -14 < 0 \therefore$ maximum
When $x = 2$, $\frac{d^2y}{dx^2} = 12\times2 - 10 = 14 > 0 \therefore$ minimum

(8)
a) $5\sin(\theta+30) = 3 \qquad 0 \le \theta \le 360$
$\Rightarrow \sin(\theta+30) = \tfrac{3}{5} \qquad 30 \le X \le 390$
Let $X = \theta + 30 \quad (\theta = X - 30)$
$\sin X = \tfrac{3}{5} \Rightarrow X = \sin^{-1}(\tfrac{3}{5}) = 36.9°$
$X = 36.9°, 143.1°$
$\theta = \underline{6.9}, \underline{113.1°}$

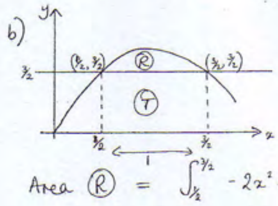


b) $\tan^2\theta = 4 \Rightarrow \tan\theta = \pm2$
If $\tan\theta = 2$, $\theta = \tan^{-1}2 = 63.4°$
If $\tan\theta = -2$, $\theta = \tan^{-1}-2 = -63.4°$
$\theta = \underline{63.4°}, \underline{116.6°}, \underline{243.4°}, \underline{296.6°}$



(9)
a) A and B at intersection of $y = -2x^2 + 4x$ and $y = \tfrac{3}{2}$
$\tfrac{3}{2} = -2x^2 + 4x \Rightarrow 3 = -4x^2 + 8x$
$\Rightarrow 4x^2 - 8x + 3 = 0$
$(2x-3)(2x-1)$
(B) $\underline{x = \tfrac{3}{2}}$ or $\underline{x = \tfrac{1}{2}}$ (A)

b)



Area (T) $= 1\times\tfrac{3}{2} = \tfrac{3}{2}$.

Area (R) $= \int_{\frac{1}{2}}^{\frac{3}{2}} -2x^2 + 4x\ dx$ − Area (T)
$= \left[-\frac{2x^3}{3} + \frac{4x^2}{2}\right]_{\frac{1}{2}}^{\frac{3}{2}} - \tfrac{3}{2}$
$= \left[-\frac{2x^3}{3} + 2x^2\right]_{\frac{1}{2}}^{\frac{3}{2}} - \tfrac{3}{2}$
$= \left[\left(-\tfrac{2}{3}(\tfrac{3}{2})^3 + 2(\tfrac{3}{2})^2\right) - \left(-\tfrac{2}{3}(\tfrac{1}{2})^3 + 2(\tfrac{1}{2})^2\right)\right] - \tfrac{3}{2}$
$= \left(-\tfrac{9}{4} + \tfrac{9}{2}\right) - \left(-\tfrac{1}{12} + \tfrac{1}{2}\right) - \tfrac{3}{2}$
$= \tfrac{9}{4} - \tfrac{5}{12} - \tfrac{3}{2}$
$= \underline{\tfrac{1}{3}}$