C2 Mock Paper

1) $f(x) = 2x^3 - x^2 + px + 6$

a) $f(1) = 0 \Rightarrow 2(1)^3 - (1)^2 + p(1) + 6 = 0$

$2 - 1 + p + 6 = 0 \Rightarrow \underline{p = -7}$

b) $f(x) = 2x^3 - x^2 - 7x + 6$

$f(-\tfrac{1}{2}) = 2(-\tfrac{1}{2})^3 - (-\tfrac{1}{2})^2 - 7(-\tfrac{1}{2}) + 6$

$= \underline{9}$

2) a) $\int (3 + 4x^3 - \tfrac{2}{x^2})\,dx = \int (3 + 4x^3 - 2x^{-2})\,dx$

$= 3x + \frac{4x^4}{4} - \frac{2x^{-1}}{-1} + C$

$= \underline{3x + x^4 + \tfrac{2}{x} + C}$

b) $\int_1^2 (3 + 4x^3 - \tfrac{2}{x^2})\,dx = \left[3x + x^4 + \tfrac{2}{x}\right]_1^2$

$= \left[(3(2) + (2)^4 + \tfrac{2}{(2)}) - (3(1) + (1)^4 + \tfrac{2}{(1)})\right]$

$= (23) - (6)$

$= \underline{17}$

3)

a) Arc length $= r\theta = 6 \times 0.4 = \underline{2.4\text{ cm}}$

b) $AB^2 = 12^2 + 6^2 - 2 \times 12 \times 6 \cos 0.4$

$= 144 + 36 - 144\cos 0.4$

$= 47.37$

$AB = \underline{6.88\text{ cm}}$

c) Perimeter $= 6 + 2.4 + 6.88 = \underline{15.3\text{cm}}$

4) $2\log_3 x - \log_3(x-2) = 2$

$\log_3 x^2 - \log_3(x-2) = 2$

$\log_3\left(\frac{x^2}{x-2}\right) = 2 \Rightarrow \frac{x^2}{x-2} = 3^2$

$x^2 = 9(x-2)$

$x^2 - 9x + 18 = 0$

$\Rightarrow (x-3)(x-6) = 0 \quad \underline{x = 3} \text{ or } \underline{x = 6}$

5) $ar = 9, \quad ar^4 = 1.125$

a) $\frac{ar^4}{ar} = \frac{1.125}{9} \Rightarrow r^3 = \frac{1.125}{9} = \frac{1}{8} \Rightarrow \underline{r = \tfrac{1}{2}}$

b) $ar = 9 \Rightarrow a = \tfrac{9}{\frac{1}{2}} = \underline{18}$

c) $S_\infty = \frac{a}{1-r} = \frac{18}{1-\frac{1}{2}} = \frac{18}{\frac{1}{2}} = \underline{36}$

6) $x^2 + y^2 - 6x + 4y - 12 = 0$

a) Complete the square for x and y:

$x^2 + y^2 - 6x + 4y - 12 = 0$

$x^2 - 6x + y^2 + 4y = 12$

$(x-3)^2 - 9 + (y+2)^2 - 4 = 12$

$(x-3)^2 + (y+2)^2 = 25$

$\Rightarrow A = \underline{(3, -2)}$

b) $r = \sqrt{25} = \underline{5}$ QED.

c) PQ must be a diameter as it is 10.

PQR is therefore a right-angled triangle so

$QR = \sqrt{10^2 - 3^2} = \sqrt{91} = \underline{9.54} = \underline{9.5}$ (1 dp)

7) a) $(1 + kx)^n = 1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2 + \frac{n(n-1)(n-2)}{3!}(kx)^3 + \ldots$

Coeff. of x^2 and x^3 are equal:

$\frac{n(n-1)}{2!}k^2 = \frac{n(n-1)(n-2)}{3!}k^3$

$\Rightarrow \frac{n(n-1)}{2}k^2 = \frac{n(n-1)(n-2)}{6}k^3$

$\Rightarrow 6k^2 = 2(n-2)k^3$

$\Rightarrow 3 = (n-2)k$ QED

b) $A = 4 \Rightarrow nk = 4$

$3 = (n-2)k$

$3 = nk - 2k$

$3 = 4 - 2k$

$2k = 1 \Rightarrow \underline{k = \tfrac{1}{2}}, \ \underline{n = 8}$

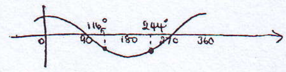
8) a) $\cos(x - 20) = -0.437 \qquad 0 \le x \le 360°$

Let $X = x - 20 \quad (x = X + 20) \qquad -20 \le X \le 340°$

$\cos X = -0.437$

$\Rightarrow X = \cos^{-1}(-0.437) = 116°, 244°$

$\Rightarrow x = \underline{136°}, \underline{264°}$



b) $3\tan\theta = 2\cos\theta \Rightarrow \frac{3\sin\theta}{\cos\theta} = 2\cos\theta \Rightarrow 3\sin\theta = 2\cos^2\theta$

(Using $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$)

$3\sin\theta = 2(1 - \sin^2\theta)$

$3\sin\theta = 2 - 2\sin^2\theta \qquad 0 \le \theta \le 360$

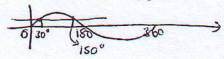
$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$\sin\theta = \tfrac{1}{2} \qquad \sin\theta = -2$ × IMPOSSIBLE

$\theta = \sin^{-1}(\tfrac{1}{2})$

$= \underline{30°}, \underline{150°}$



9)

S.A. $= 250\text{cm}^2$

$SA = 2\pi rh + \pi r^2 = 250$

$\Rightarrow 2\pi rh = 250 - \pi r^2$

$\Rightarrow h = \frac{250 - \pi r^2}{2\pi r} = \frac{250}{2\pi r} - \frac{\pi r^2}{2\pi r} = \frac{125}{\pi r} - \frac{r}{2}$.

$V = \pi r^2 h$

$= \pi r^2\left(\frac{125}{\pi r} - \frac{r}{2}\right) = \frac{125\pi r^2}{\pi r} - \frac{\pi r^3}{2} = \underline{125r - \frac{\pi r^3}{2}}$ QED

b) Stationary value at $\frac{dV}{dr} = 0$:

$\frac{dV}{dr} = 125 - \frac{3\pi r^2}{2} = 0 \Rightarrow 125 = \frac{3\pi r^2}{2}$

$250 = 3\pi r^2$

$\frac{250}{3\pi} = r^2$

$\Rightarrow \underline{r = \sqrt{\frac{250}{3\pi}}}$

c) $\frac{d^2V}{dr^2} = \frac{-6\pi r}{2} = -3\pi r$

When $r = \sqrt{\frac{250}{3\pi}}$, $\frac{d^2V}{dr^2} = -3\pi\left(\sqrt{\frac{250}{3\pi}}\right) = -48.5 < 0 \therefore$ maximum QED

d) Maximum $V = 125\left(\sqrt{\frac{250}{3\pi}}\right) - \frac{\pi}{2}\left(\sqrt{\frac{250}{3\pi}}\right)^3 = \underline{429\text{ cm}^3}$

10) a) $y = 9 - 2x - \frac{2}{\sqrt{x}}$

When $b = 4$, $y = 9 - 2(4) - \frac{2}{\sqrt{4}} = 9 - 8 - 1 = 0$. QED.

b) $y = 9 - 2x - 2x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -2 + x^{-\frac{3}{2}}$ When $x = 1$, $\frac{dy}{dx} = -2 + 1^{-\frac{3}{2}} = -1$.

$y - 5 = -1(x - 1)$

$y - 5 = 1 - x \Rightarrow y + x = 6$ QED.

c) D is when $y = 0$: $y + x = 6 \Rightarrow 0 + x = 6 \Rightarrow x = 6$.

$D = \underline{(6, 0)}$

d) Area of triangle: $\tfrac{1}{2} \times 5 \times 5 = 12.5$.

Area under curve between A and B:

$\int_1^4 (9 - 2x - 2x^{-\frac{1}{2}})\,dx = \left[9x - \frac{2x^2}{2} - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4$

$= \left[9x - x^2 - 4\sqrt{x}\right]_1^4 = \left[(9(4) - (4)^2 - 4(\sqrt{4})) - (9(1) - (1)^2 - 4(\sqrt{1}))\right]$

$= (12) - (4) = 8$.

Area of $R = 12.5 - 8 = \underline{4.5}$