C2 JUNE 2010

1.

$$y = 3^x + 2x$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65	2.35	3.13	4.01	5

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\,dx$.

(4)

$$\simeq \tfrac{1}{2}(0.2)\left(1+5+2\left(1.65+2.35+3.13+4.01\right)\right) = \underline{2.828}$$

2.

$$f(x) = 3x^3 - 5x^2 - 58x + 40$$

(a) Find the remainder when $f(x)$ is divided by $(x-3)$.

(2)

Given that $(x-5)$ is a factor of $f(x)$,

(b) find all the solutions of $f(x) = 0$.

(5)

a) $f(3) = 3(3)^3 - 5(3)^2 - 58(3) + 40 = \underline{-98}$

b)

	$3x^2$	$+10x$	-8
x	$3x^3$	$+10x^2$	$-8x$
-5	$-15x^2$	$-50x$	$+40$

$r = 0$ ✓

$$(x-5)(3x^2+10x-8) = (x-5)(3x-2)(x+4)$$

$$\underline{x = 5;\ \tfrac{2}{3};\ -4}$$

3.

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

a) $y = x^2 - kx^{\frac{1}{2}}$ $\qquad \dfrac{dy}{dx} = 2x - \frac{1}{2}k^{-\frac{1}{2}} = 2x - \dfrac{k}{2\sqrt{x}}$

b) decreasing $\Rightarrow \dfrac{dy}{dx} < 0$ when $x = 4$

$8 - \dfrac{k}{4} < 0 \Rightarrow \dfrac{k}{4} > 8 \Rightarrow \underline{k > 32}$

4. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

a) $1 + 7(ax) + \dfrac{7 \times 6}{2}(ax)^2 + \dfrac{7 \times \cancel{6} \times 5}{\cancel{6}}(ax)^3$

$1 + 7ax + 21a^2x^2 + 35a^3x^3$

b) $21a^2 = 525 \Rightarrow a^2 = 25 \Rightarrow \underline{a = 5; -5}$

5. (a) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(b) Solve, for $0 \leqslant x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)

a) $\frac{\sin\theta}{\cos\theta} = \frac{2}{5} \Rightarrow \tan\theta = 0.4$

b) $\tan 2x = 0.4 \Rightarrow 2x = \tan^{-1}(0.4)$

$2x = 21.8;\ 201.8;\ 381.8;\ 561.8$

$x = \underline{10.9};\ \underline{100.9};\ \underline{190.9};\ \underline{280.9}$

7. (a) Given that

$$2\log_3(x-5) - \log_3(2x-13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5) - \log_3(2x-13) = 1$.

(2)

a) $\log_3\left(\frac{(x-5)^2}{2x-13}\right) = 1 \Rightarrow (x-5)^2 = 3(2x-13)$

$x^2 - 10x + 25 = 6x - 39 \Rightarrow x^2 - 16x + 64 = 0$

eod

b) $(x-8)^2 = 0 \Rightarrow \underline{x = 8}$

6.

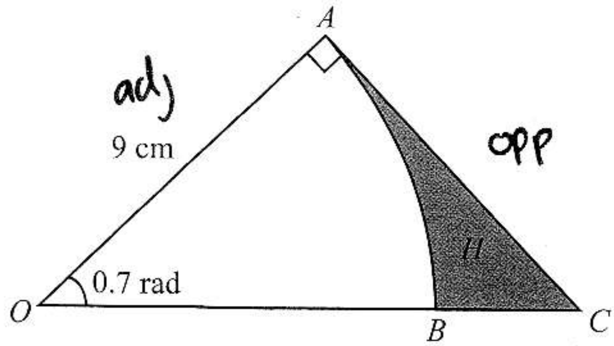


Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB. $r\theta = 9(0.7) = \underline{6.3\text{cm}}$

(2)

(b) Find the area of the sector OAB. $\frac{1}{2}r^2\theta = \frac{1}{2}(9)^2(0.7) = \underline{28.35}\text{cm}^2$

(2)

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places.

(2)

The region H is bounded by the arc AB and the lines AC and CB.

(d) Find the area of H, giving your answer to 2 decimal places.

(3)

c) $AC = 9 \times \tan(0.7^c) = \underline{7.58\text{cm}}$

d) Area $H = \frac{1}{2}(7.58...)(9) - 28.35 = \underline{5.76\text{cm}^2}$

8.

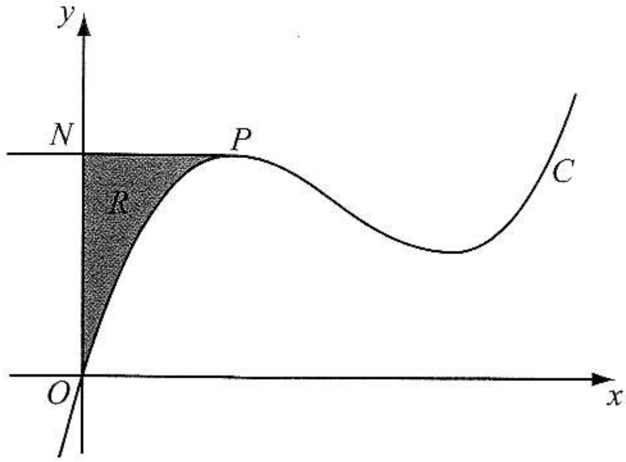


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

a) $\frac{dy}{dx} = 3x^2 - 20x + k \; (= 0 \text{ when } x = 2)$

$12 - 40 + k = 0 \Rightarrow \underline{k = 28}$

b) $y = 2^3 - 10(2)^2 + 28(2) = 24 \quad P(2,24)$

$R = \boxed{}\,24 - \int_0^2 x^3 - 10x^2 + 28x \, dx$

2

$R = 48 - \left[\frac{1}{4}x^4 - \frac{10}{3}x^3 + 14x^2\right]_0^2 = 48 - \left(\left(\frac{100}{3}\right) - (0)\right)$

$R = \frac{44}{3}$ units^2

9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750. **(1)**

(b) Write down the common ratio of the geometric sequence. **(1)**

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of N. **(2)**

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000. **(3)**

a) $3\% = \frac{3}{100} \times 25000 = £750 \quad +3\% = 25000 + 750$
$= £\underline{25750}$

b) $r = 1.03$

c) $U_N = ar^{N-1} > 40000 \Rightarrow 25000(1.03)^{N-1} > 40000$

$\Rightarrow 1.03^{N-1} > 1.6 \Rightarrow \log 1.03^{N-1} > \log 1.6$

$\Rightarrow (N-1)\log 1.03 > \log 1.6$ and.

e) $S_{10} = \frac{25000(1.03^{10}-1)}{1.03-1} = 286596.98\ldots$

$\underline{£287000}$

d) Since $\log 1.03 > 0 \Rightarrow N-1 > \dfrac{\log 1.6}{\log 1.03}$

$N-1 > 15.9 \Rightarrow N > 16.9 \Rightarrow \underline{N = 17}$

10. The circle C has centre $A(2,1)$ and passes through the point $B(10,7)$.

(a) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1.

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB.

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

a) radius $= \overrightarrow{AB}$ $\quad AB^2 = (10-2)^2 + (7-1)^2 = 8^2 + 6^2 \Rightarrow \underline{r = 10}$

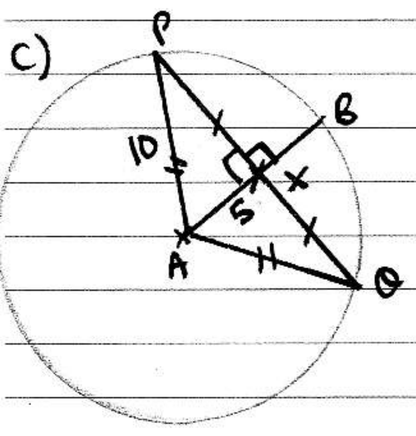
$(x-2)^2 + (y-1)^2 = 100$

b) l_1 is perp to $\overrightarrow{AB}$ $\quad m_{AB} = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$ $\quad m_{l_1} = -\frac{4}{3}$

$y - 7 = -\frac{4}{3}(x-10) \Rightarrow 3y - 21 = -4x + 40$

$\underline{4x + 3y - 61 = 0}$

c)



$MP_{AB} = \left(\frac{10+2}{2}, \frac{1+7}{2}\right) = (6,4) = X$

$PX^2 = 10^2 - 5^2 \quad PQ = \sqrt{75} = 5\sqrt{3}$

$\Rightarrow PQ = 2 \times 5\sqrt{3} \quad = \underline{10\sqrt{3}}$