C2 JAN 2010

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1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3-x)^6$$

and simplify each term.

(4)

$= 3^6\left(1-\frac{x}{3}\right)^6 = 3^6\left(1+6\left(-\frac{x}{3}\right)+\frac{6\times 5}{2}\left(-\frac{x}{3}\right)^2\right)$

$= 3^6\left(1-2x+\frac{5}{3}x^2\right) = \underline{729-1458x+1215x^2}$

2. (a) Show that the equation

$$5\sin x = 1 + 2\cos^2 x$$

can be written in the form

$$2\sin^2 x + 5\sin x - 3 = 0$$

(2)

(b) Solve, for $0 \leqslant x < 360°$,

$$2\sin^2 x + 5\sin x - 3 = 0$$

(4)

(a) $5\sin x = 1 + 2(1 - \sin^2 x)$

$5\sin x = 1 + 2 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + 5\sin x - 3 = 0$

(b) $(2\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{2}$ $\quad\quad$ $\sin x = -3$ (no solutions)

$x = 30°$

$x = 150°$

3. $$f(x) = 2x^3 + ax^2 + bx - 6$$

where a and b are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is -5.

When $f(x)$ is divided by $(x + 2)$ there is no remainder.

(a) Find the value of a and the value of b.

(6)

(b) Factorise $f(x)$ completely.

(3)

$f(\frac{1}{2}) = -5 \Rightarrow \frac{1}{4} + \frac{1}{4}a + \frac{1}{2}b - 6 = -5$

$\times 4 \Rightarrow 1 + a + 2b - 24 = -20$

$\Rightarrow a + 2b = 3$

$f(-2) = 0 \Rightarrow -16 + 4a - 2b - 6 = 0$

$\div 2 \Rightarrow -8 + 2a - b - 3 = 0$

$\Rightarrow 2a - b = 11$

$a + 2b = 3$ $\quad \times 1 \quad a + 2b = 3$ $+$

$2a - b = 11$ $\quad \times 2 \quad 4a - 2b = 22$

$5a = 25$ $\qquad \underline{a = 5}$

$5 + 2b = 3 \Rightarrow 2b = -2 \Rightarrow \underline{b = -1}$

(b) $f(x) = 2x^3 + 5x^2 - x - 6$

$\times$	$2x^2$	$+x$	-3
x	$2x^3$	x^2	$-3x$
$+2$	$4x^2$	$2x$	-6

$r = 0$ ✓

$(x+2)(2x^2+x-3) = \underline{(x+2)(2x+3)(x-1)}$

4.

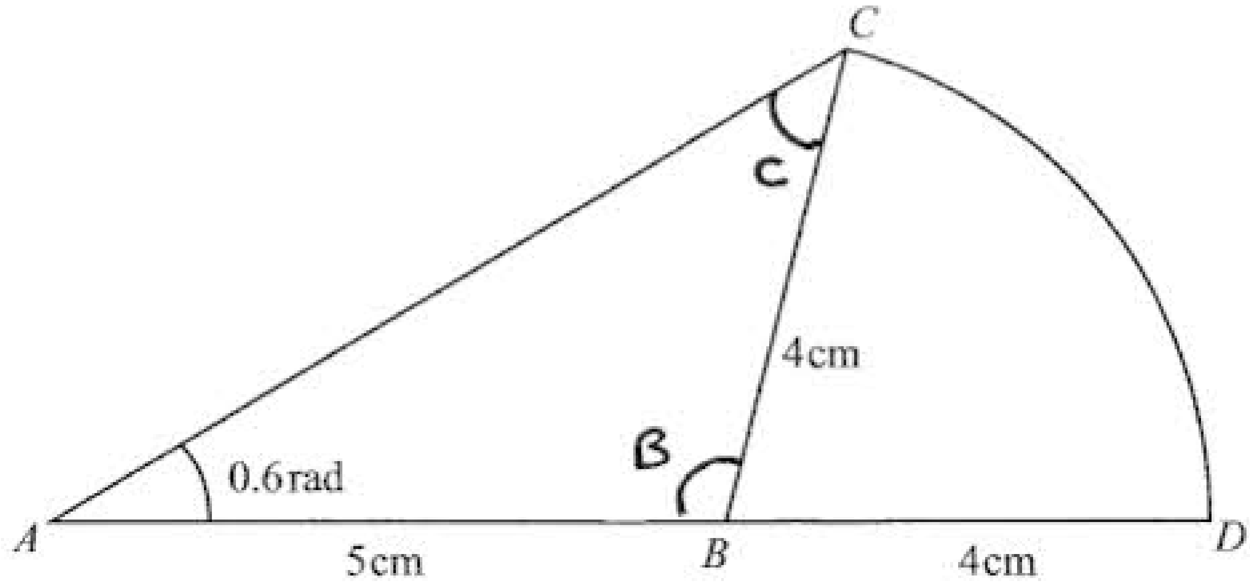


Figure 1

An emblem, as shown in Figure 1, consists of a triangle ABC joined to a sector CBD of a circle with radius 4 cm and centre B. The points A, B and D lie on a straight line with $AB = 5$ cm and $BD = 4$ cm. Angle $BAC = 0.6$ radians and AC is the longest side of the triangle ABC.

(a) Show that angle $ABC = 1.76$ radians, correct to 3 significant figures.

(4)

(b) Find the area of the emblem.

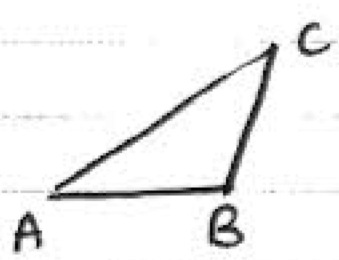
(3)

(a) $\frac{\sin C}{5} = \frac{\sin 0.6^c}{4} \Rightarrow \sin C = \frac{5\sin 0.6^c}{4}$

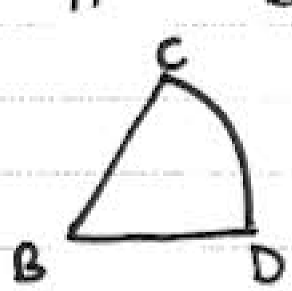
$\Rightarrow C = 0.783 \ldots^c$

$\Rightarrow B = \pi - 0.6 - 0.783 \ldots \quad B = \underline{1.76^c} \text{ (3sf)}$

(b)



$\text{Area} = \frac{1}{2}(5)(4)\sin 1.76 = \underline{9.83}\ldots$



$\text{Angle } C\hat{B}D = \pi - 1.76 = 1.38$

$\text{Area} = \frac{1}{2}(4)^2 \times 1.38 = \underline{11.04}$

$\text{Total Area} = 9.83 + 11.04 = \underline{20.9\text{cm}^2}$

5. (a) Find the positive value of x such that

$$\log_x 64 = 2$$

(2)

(b) Solve for x

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3$$

(6)

(a) $64 = x^2 \Rightarrow \underline{x = 8}$

(b) $\log_2(11-6x) - 2\log_2(x-1) = 3$

$\log_2(11-6x) - \log_2(x-1)^2 = 3$

$\log_2\left(\frac{11-6x}{(x-1)^2}\right) = 3$

$\frac{11-6x}{(x-1)^2} = 2^3 = 8$

$11-6x = 8(x-1)^2$

$11-6x = 8(x^2-2x+1)$

$11-6x = 8x^2-16x+8$

$\Rightarrow 8x^2-10x-3 = 0$

$(4x+1)(2x-3) = 0$

$x = -\frac{1}{4} \quad x = \frac{3}{2}$

6. A car was purchased for £18 000 on 1st January.
On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.
(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.
(3)

An insurance company has a scheme to cover the maintenance of the car.
The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.
(2)

(d) Find the total cost of the insurance scheme for the first 15 years.
(3)

(a) $a = 18000 \quad r = 0.8$

$u_1 = 18000$

$u_2 \to$ (after 1 year) $\quad u_3 \to$ (after 2 years) $\quad$ need u_4

$u_4 = ar^3 = 18000(0.8)^3 = \underline{£9216}$

(b)

$u_n = ar^{n-1} < 1000 \qquad 18000 \times 0.8^{n-1} < 1000$

$\Rightarrow 0.8^{n-1} < \frac{1}{18} \Rightarrow \log 0.8^{n-1} < \log\left(\frac{1}{18}\right)$

$\Rightarrow (n-1)\log 0.8 < \log\left(\frac{1}{18}\right)$

$n-1 > \dfrac{\log\left(\frac{1}{18}\right)}{\log(0.8)}$ sign reverses as $\log(0.8) < 0$

$n-1 > 12.95 \Rightarrow n > 13.95 \quad u_{14}$

So after 13 years, car is worth < £1000

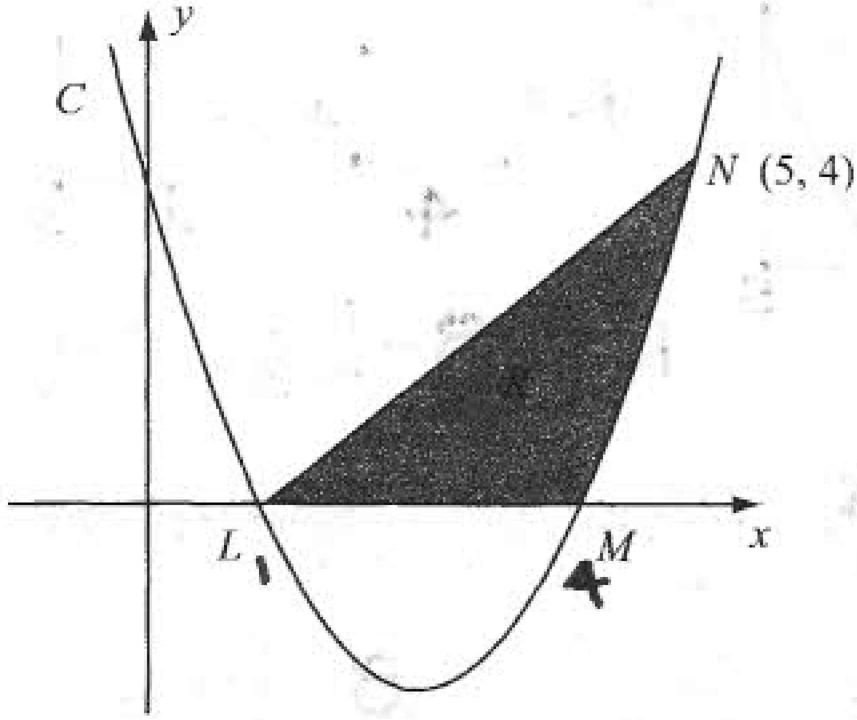
6c) $a = 200 \quad r = 1.12$

$U_5 = ar^4 = 200(1.12)^4 = \underline{£314.70}$

d) $S_n = \frac{a(r^n - 1)}{r - 1} \qquad S_{15} = \frac{200(1.12^{15} - 1)}{1.12 - 1}$

$S_{15} = \underline{£7455.94}$

7.



$y = (x-1)(x-4)$

1 4

Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M. **(2)**

$L(1,0)$
$M(4,0)$

(b) Show that the point N (5, 4) lies on C. **(1)**

(c) Find $\int (x^2 - 5x + 4)\,dx$. **(2)**

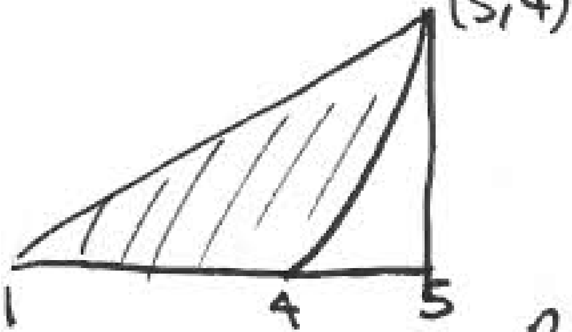
The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

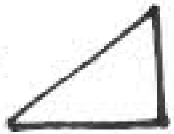
(d) Use your answer to part (c) to find the exact value of the area of R. **(5)**

(b) $x = 5 \quad y = 5^2 - 5(5) + 4 = 4 \quad (5,4)$ ✓

(c) $\int x^2 - 5x + 4\,dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + C$

(d)



$R =$ 

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$$R = 8 - \left[\tfrac{1}{3}x^3 - \tfrac{5}{2}x^2 + 4x\right]_4^5$$

$$R = 8 - \left(\left(\frac{125}{3} - \frac{125}{2} + 20\right) - \left(\frac{64}{3} - 40 + 16\right)\right)$$

$$R = 8 - \left(\left(-\frac{5}{6}\right) - \left(-\frac{8}{3}\right)\right) = 8 - \frac{11}{6}$$

$$= \frac{37}{6} = \underline{6\tfrac{1}{6}}$$

8.

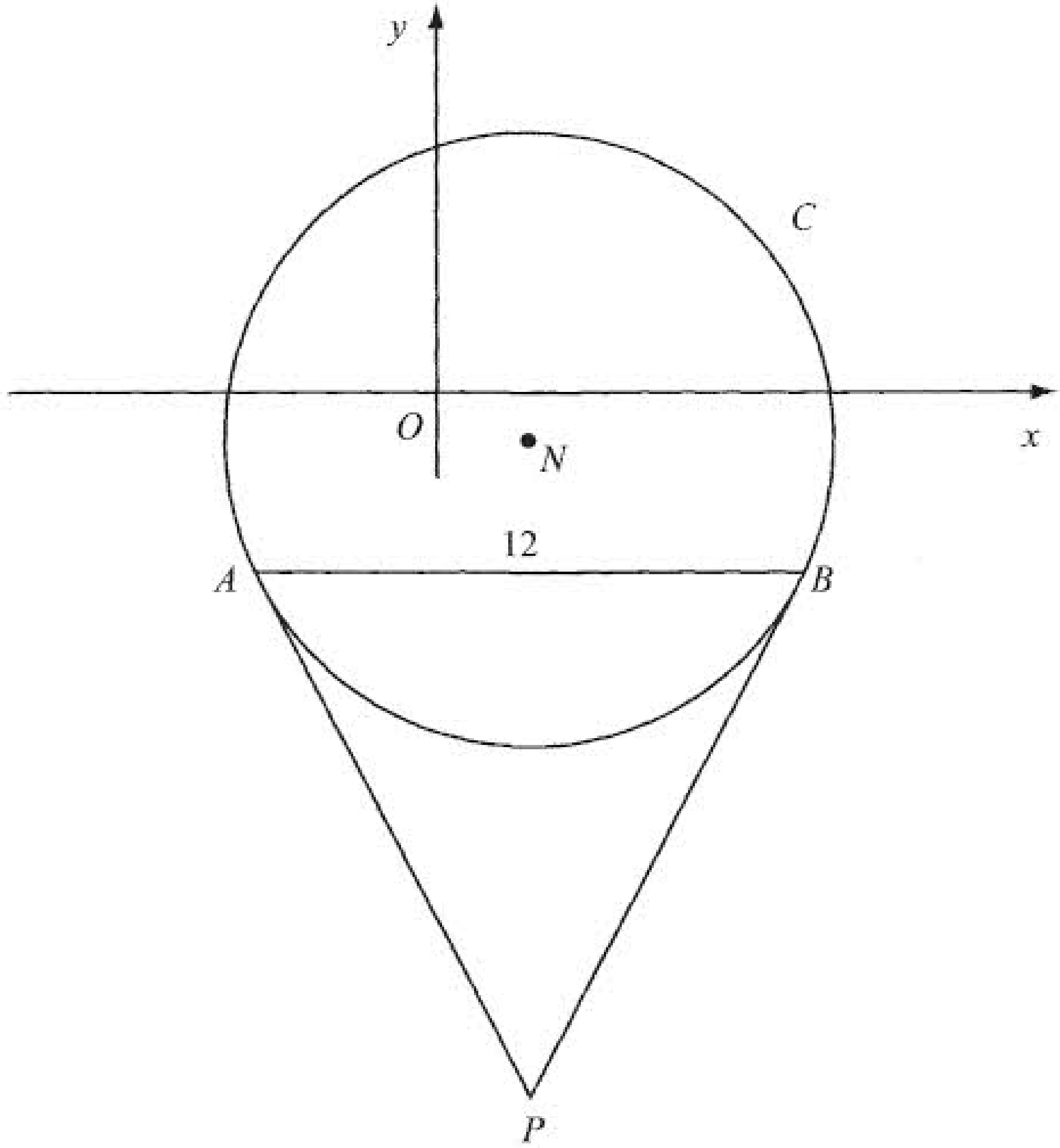


Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x-2)^2 + (y+1)^2 = \frac{169}{4}$$

(a) Write down the coordinates of N. **(2)**

$N(2,-1)$

(b) Find the radius of C. **(1)**

radius $= \sqrt{\frac{169}{4}} = \frac{13}{2}$

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in Figure 3.

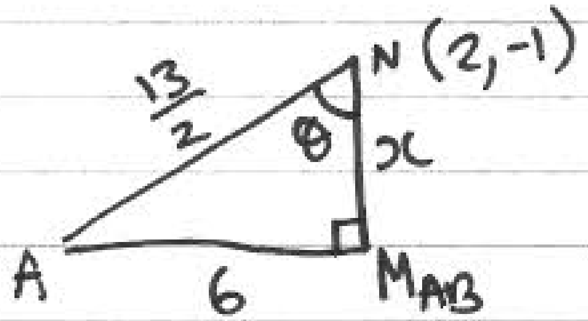
(c) Find the coordinates of A and the coordinates of B. **(5)**

(d) Show that angle $ANB = 134.8°$, to the nearest 0.1 of a degree. **(2)**

The tangents to C at the points A and B meet at the point P.

(e) Find the length AP, giving your answer to 3 significant figures. **(2)**

(c)



$x^2 = \left(\frac{13}{2}\right)^2 - 6^2 \qquad x = 2.5$

$M_{AB}\,(2, -3.5)$

y-coordinate of A and B is -3.5

$(x-2)^2 + (-2.5)^2 = \frac{169}{4}$

$(x-2)^2 = 36 \Rightarrow x-2 = 6 \text{ or } -6$

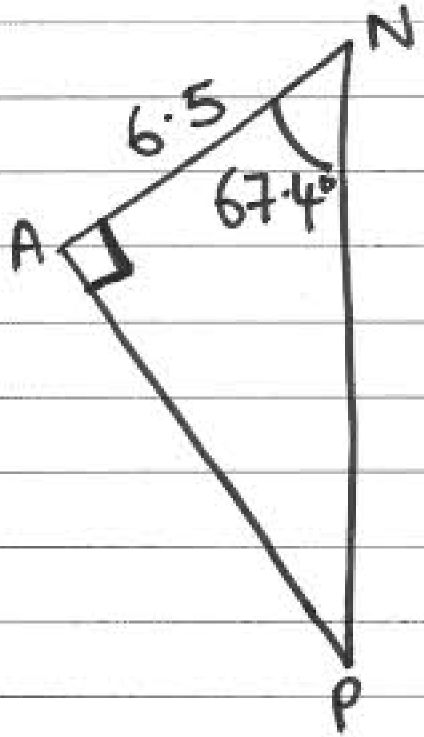
$x = 8 \text{ or } -4$

$A(-4, -3.5) \quad B(8, -3.5)$

(d) $\sin\theta = \frac{6}{6.5} \qquad \theta = 67.38°\ldots$

$ANB = 2\theta = \underline{134.8°}$

(e)



$\tan 67.38\ldots = \frac{AP}{6.5}$

$AP = 6.5 \tan 67.38\ldots$

$\underline{AP = 15.6}$

9. The curve C has equation $\quad y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \qquad x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{d^2y}{dx^2}$.

(2)

(c) State the nature of the turning point.

(1)

(a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10$

$\dfrac{dy}{dx} = 6x^{-\frac{1}{2}} - \dfrac{3}{2}x^{\frac{1}{2}}$

TP $\dfrac{dy}{dx} = 0 \qquad \dfrac{6}{\sqrt{x}} = \dfrac{3\sqrt{x}}{2} \Rightarrow 12 = 3x$

$x = 4$

$y = 12\sqrt{4} - 4^{\frac{3}{2}} - 10 = 24 - 8 - 10 = 6$

TP $\underline{(4,6)}$

(b) $\dfrac{d^2y}{dx^2} = -3x^{-\frac{3}{2}} - \dfrac{3}{4}x^{-\frac{1}{2}}$

when $x = 4 \quad \dfrac{d^2y}{dx^2} = -3 \times 4^{-\frac{3}{2}} - \dfrac{3}{4} \times 4^{-\frac{1}{2}}$

$\dfrac{d^2y}{dx^2} = -\dfrac{3}{4} < 0$

$\underline{\text{maxima}}$