

4. A researcher measured the foot lengths of a random sample of 120 ten-year-old children. The lengths are summarised in the table below.

no gaps →

Foot length, l , (cm)	Number of children	cf
10 ≤ l < 12	5	5
12 ≤ l < 14	53	58
14 ≤ l < 16	29	87
16 ≤ l < 18	15	102
18 ≤ l < 20	11	113
20 ≤ l < 22	7	120

← 60th

- (a) Use interpolation to estimate the median of this distribution. (2)
- (b) Calculate estimates for the mean and the standard deviation of these data. (6)

One measure of skewness is given by

$$\text{Coefficient of skewness} = \frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$$

- (c) Evaluate this coefficient and comment on the skewness of these data. (3)

Greg suggests that a normal distribution is a suitable model for the foot lengths of ten-year-old children.

- (d) Using the value found in part (c), comment on Greg's suggestion, giving a reason for your answer. (2)

a) $\frac{1}{2}n = 60$ (17-19)

$Q_2 = 17 + \frac{60 - 58}{29} \times 2 = 17.14$

b) $\sum fx = 2055.5$
 $\sum fx^2 = 36500.25$

$\bar{x} = \frac{\sum fx}{n} = \frac{2055.5}{120} = 17.13$
 $s^2 = \frac{\sum fx^2}{n} - \bar{x}^2 = \frac{36500.25}{120} - 17.13^2 = 3.28$

c) $\text{Skew} = \frac{3(17.13 - 17.14)}{3.28} = -0.009$

Symmetrical skew

- d) yes normal distribution seems appropriate
 Since symmetrical skew mean = median
 95% of data should lie within $\bar{x} \pm 2s$
 17.13 ± 6.56 (10.6 - 23.7) which seems to be the case.

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5. The weight, w grams, and the length, l mm, of 10 randomly selected newborn turtles are given in the table below.

l	49.0	52.0	53.0	54.5	54.1	53.4	50.0	51.6	49.5	51.2
w	29	32	34	39	38	35	30	31	29	30

(You may use $S_{ll} = 33.381$ $S_{wl} = 59.99$ $S_{ww} = 120.1$)

- (a) Find the equation of the regression line of w on l in the form $w = a + bl$. (5)
- (b) Use your regression line to estimate the weight of a newborn turtle of length 60 mm. (2)
- (c) Comment on the reliability of your estimate giving a reason for your answer. (2)

$w = a + bl$ $w = y$ $l = x$

$b = \frac{S_{wy}}{S_{xx}} = \frac{S_{wl}}{S_{ll}} = \frac{59.99}{33.381} = 1.797$

$a = \bar{y} - b\bar{x} \Rightarrow a = \bar{w} - b\bar{l}$

$\bar{w} = \frac{\sum w}{n} = \frac{318.3}{10} = 31.83$

$\bar{l} = \frac{\sum l}{n} = \frac{518.3}{10} = 51.83$

$a = 31.83 - 1.797 \times 51.83 = -60.445$

$w = -60.445 + 1.797l$

b) $l = 60$ $w = 47.375g$

c) Unreliable, no evidence to support this since max length in our data was 54.5mm

6. The discrete random variable X has probability function

$$P(X=x) = \begin{cases} a(3-x) & x=0,1,2 \\ b & x=3 \end{cases}$$

- (a) Find $P(X=2)$ and complete the table below.

x	0	1	2	3
$P(X=x)$	$3a$	$2a$	a	b

$$E(X) = 0 + 2a + 2a + 3b$$

Given that $E(X) = 1.6$

$$\Rightarrow 4a + 3b = 1.6$$

- (b) Find the value of a and the value of b .

Find

- (c) $P(0.5 < X < 3)$,

- (d) $E(3X - 2)$.

- (e) Show that the $\text{Var}(X) = 1.64$

- (f) Calculate $\text{Var}(3X - 2)$.

b) $18a + 3b = 3$
 $4a + 3b = 1.6$

$14a = 1.4 \Rightarrow a = 0.1 \Rightarrow b = 0.4$

c) $P(0.5 < X < 3) = P(1) + P(2) = 2a = 0.2$

d) $E(3X - 2) = 3E(X) - 2 = 3 \times 1.6 - 2 = 2.8$

e) $V(X) = E(X^2) - E(X)^2 = 4.2 - 1.6^2 = 1.64$

f) $V(3X - 2) = 3^2 V(X) = 14.76$

Leave blank

7. (a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

- (i) A and B are mutually exclusive,
(ii) A and B are independent.

(2)

Two events R and Q are such that

$$P(R \cap Q) = 0.15, \quad P(Q) = 0.35 \text{ and } P(R|Q) = 0.1$$

Find the value of

(b) $P(R \cup Q)$,

(1)

(c) $P(R \cap Q)$,

(2)

(d) $P(R)$.

(2)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

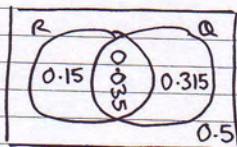
i) Mutually exclusive $\Rightarrow P(A \cap B) = 0$

$$P(A \cup B) = a + b$$

ii) Independent $\Rightarrow P(A \cap B) = P(A)P(B) = ab$

$$P(A \cup B) = a + b - ab$$

ii) $P(R|Q) = \frac{P(R \cap Q)}{P(Q)} \Rightarrow 0.1 = \frac{P(R \cap Q)}{0.35} \Rightarrow P(R \cap Q) = 0.035$



b) $P(R \cup Q) = 0.15 + 0.035 + 0.315 = 0.5$

c) $P(R \cap Q) = 0.035$

d) $P(R) = 0.15 + 0.035 = 0.185$

Leave blank

8. The lifetimes of bulbs used in a lamp are normally distributed. A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

- (a) Find the probability of a bulb, from company X , having a lifetime of less than 830 hours.

(3)

- (b) In a box of 500 bulbs, from company X , find the expected number having a lifetime of less than 830 hours.

(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

- (c) Find the standard deviation of the lifetimes of bulbs from company Y .

(4)

Both companies sell the bulbs for the same price.

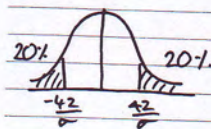
- (d) State which company you would recommend. Give reasons for your answer.

(2)

a) $P(X < 830) \Rightarrow P\left(Z < \frac{830 - 850}{50}\right) = P(Z < -0.4)$
 $= 1 - \Phi(0.4) = 0.3446$

b) $0.3446 \times 500 = 172.3$ 172 bulbs

c) $P(X < 818) = 0.2 \Rightarrow P\left(Z < \frac{818 - 860}{\sigma}\right) = 0.2$



$P(Z > \frac{42}{\sigma}) = 0.2$ % points

$\frac{42}{\sigma} = 0.8416 \Rightarrow \sigma = 49.9$

- d) Y since it has a higher mean and the standard deviation is almost identical

