C2 JAN 2011

1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$. $f(1) = 1+1+2+a+b = 7 \Rightarrow a+b=3$ #

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8.

(b) Find the value of a and the value of b.

(5)

b) $f(-2) = 16-8+8-2a+b = -8$

$2a-b=24$

$a+b=3$ +

$3a=27$ $\underline{a=9}$

$\underline{b=-6}$

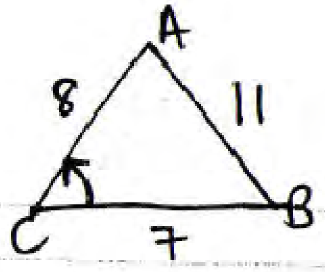
2. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C, giving your answer in radians to 3 significant figures.

(3)

(b) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.

(3)



$C = \cos^{-1}\left(\frac{8^2+7^2-11^2}{2(8)(7)}\right) = \underline{1.64^c}$ (3sf)

b) Area $= \frac{1}{2}(8)(7)\sin 1.64^c$ $= \underline{27.9\text{cm}^2}$ (3sf)

3. The second and fifth terms of a geometric series are 750 and -6 respectively.

Find

(a) the common ratio of the series, **(3)**

(b) the first term of the series, **(2)**

(c) the sum to infinity of the series. **(2)**

$$\frac{ar^4}{ar} = \frac{-6}{750}$$

$$r^3 = \frac{-1}{125} \Rightarrow r = -\frac{1}{5}$$

b) $a \times \left(-\frac{1}{5}\right) = 750 \Rightarrow a = \underline{-3750}$

c) $S_\infty = \frac{a}{1-r} = \frac{-3750}{1.2} = \underline{-3125}$

5. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b. $b = 36$ **(1)**

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$. $\frac{40!}{5!35!} \div \frac{40!}{4!36!} = \underline{7.2}$ **(3)**

4.

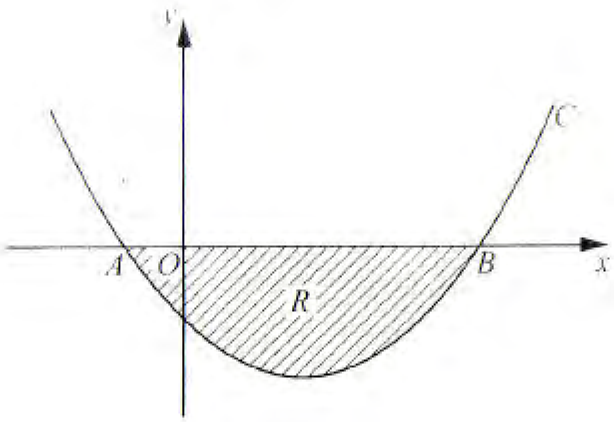


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5) \quad = x^2 - 4x - 5$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B. $A(-1,0)$ $B(5,0)$

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

$$\int_{-1}^{5} x^2 - 4x - 5\,dx = \left[\tfrac{1}{3}x^3 - 2x^2 - 5x\right]_{-1}^{5}$$

$$= \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36 \Rightarrow \text{Area } R = \underline{36}$$

6. $$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38	0.30	0.24	0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2-2}\,dx$.

(4)

$\simeq \frac{1}{2}(0.25)\left(0.5+2(0.38+0.3+0.24) + 0.2\right)$

$= 0.3175$

0.32 (2dp)

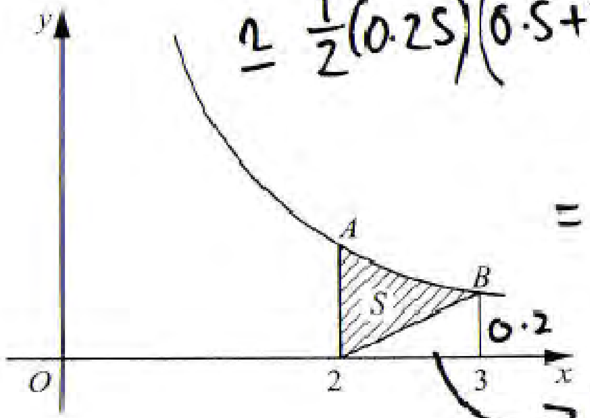


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2-2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S.

(3)

$\simeq 0.3175 - 0.1 \quad = 0.2175 \qquad 0.22$ (2dp)

7. (a) Show that the equation

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

can be written in the form

$$4\sin^2 x + 7\sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

a) $3\sin^2 x + 7\sin x = (1 - \sin^2 x) - 4$

$3\sin^2 x + 7\sin x = -3 - \sin^2 x$

$4\sin^2 x + 7\sin x + 3 = 0$ #

b) $(4\sin x + 3)(\sin x + 1) = 0$

$\sin x = -\frac{3}{4} \Rightarrow x = \underline{228.6°}; \underline{311.4°}$

$\sin x = -1 \Rightarrow x = \underline{270°}$

8. (a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

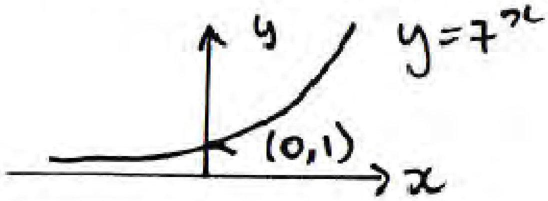
(2)

(b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate.

(6)



$y = 7^x$

b) $(7^x)^2 - 4(7^x) + 3 = 0$

$(7^x - 3)(7^x - 1) = 0$

$7^x = 3 \Rightarrow x = \log_7 3$

$x = \underline{0.56}$ (2dp)

$7^x = 1 \Rightarrow \underline{x = 0}$

9. The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$. **(1)**

Mid Point AB = Centre

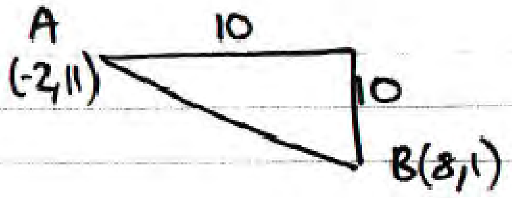
$$= \left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = (3,6)$$

(b) find an equation for C. **(4)**

(c) Verify that the point $(10, 7)$ lies on C. **(1)**

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants. **(4)**

b)



diameter $= \sqrt{10^2+10^2}$

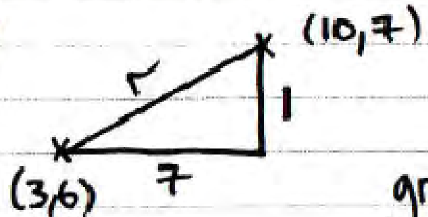
$= 10\sqrt{2}$

$\Rightarrow$ radius $= 5\sqrt{2}$

$$(x-3)^2+(y-6)^2 = 50$$

c) $x=10, y=7 \Rightarrow (10-3)^2+(7-6)^2 = 7^2+1^2 = 50$ ✓

d)



$M = \frac{1}{7}$ $M_{perp} = -7$

gradient is perp to radius.

$(y-7) = -7(x-10) \Rightarrow y-7 = -7x+70 \Rightarrow \underline{y = -7x+77}$

10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

$V = 4x(25 - 10x + x^2)$

$V = 100x - 40x^2 + 4x^3$

(a) Find $\frac{dV}{dx}$ $= 100 - 80x + 12x^2$ **(4)**

(b) Hence find the maximum volume of the box. **(4)**

(c) Use calculus to justify that the volume that you found in part (b) is a maximum. **(2)**

b) at Max $\frac{dV}{dx} = 0 \Rightarrow (\div 4) \quad 3x^2 - 20x + 25 = 0$

$(3x - 5)(x - 5) = 0 \quad \cancel{x = 5} \quad x = \frac{5}{3}$

$\max V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2 = \underline{74.1 \text{ cm}^3} \ (3sf)$

c) $\frac{d^2V}{dx^2} = -80 + 24x \quad x = \frac{5}{3} \quad \frac{d^2V}{dx^2} = -40$

$\frac{d^2V}{dx^2} < 0$ at max $\frown \Rightarrow$ maximum value