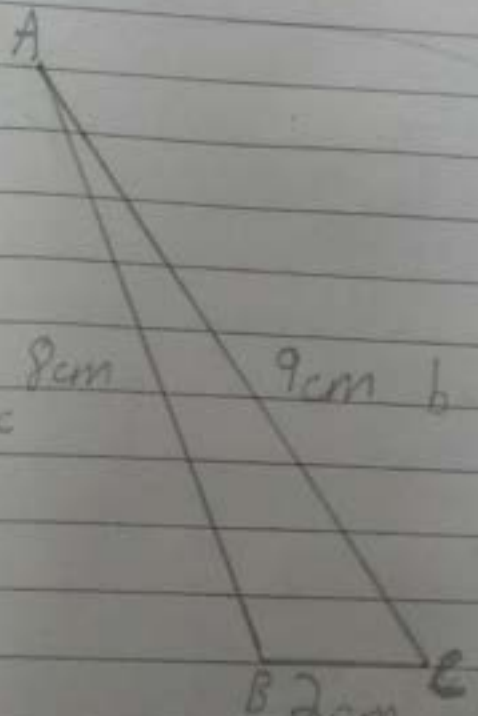
Sine and cosine rules (Hard)

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Question 1

A triangle has sides of length 2cm, 8cm and 9cm. Calculate the value of the largest angle in this triangle.




Size of angle B = ?

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$\angle B = 114.0°$

Cosine rule: $b^2 = a^2 + c^2 - 2ac\cos B$

$$b^2 - a^2 - c^2 = -2ac\cos B$$

$$\frac{b^2 - a^2 - c^2}{-2ac} = \cos B$$

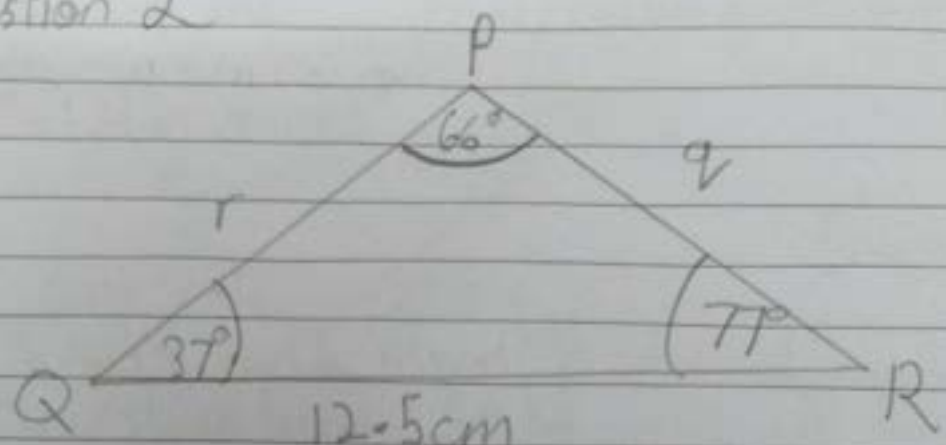
$$B = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right) = \cos^{-1}\left(\frac{9^2 - 2^2 - 8^2}{-2 \times 2 \times 8}\right)$$

$$= 113.969 = 113.97 = 114.0°$$

Cosine rule: $q^2 = r^2 + p^2 - 2rp\cos Q$

Sine rule: $\frac{p}{\sin P} = \frac{r}{\sin R} = \frac{q}{\sin Q}$

Question 2




Calculate PR.

Sine rule: $\frac{p}{\sin P} = \frac{r}{\sin R} = \frac{q}{\sin Q}$ $\quad q = PR$

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{12.5}{\sin 66^\circ} = \frac{q}{\sin 37^\circ}$$

$$q = \frac{12.5}{\sin 66^\circ} \times \sin 37^\circ$$

$q = 8.234606966\text{ cm}$

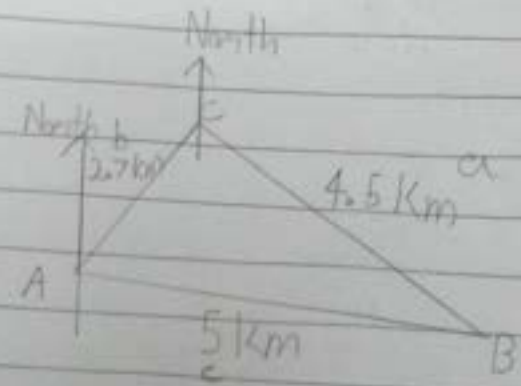
$q = 8.24\text{ cm}$

$PR = q = 8.2\text{ cm}$

Question 3

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$




The diagram shows 3 ships A, B and C at sea.

AB = 5 km, BC = 4.5 km and AC = 2.7 km.

a) Calculate angle ACB. $\angle ACB = A$

Show all your working.

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\frac{a^2 - b^2 - c^2}{-2bc} = \cos A$$

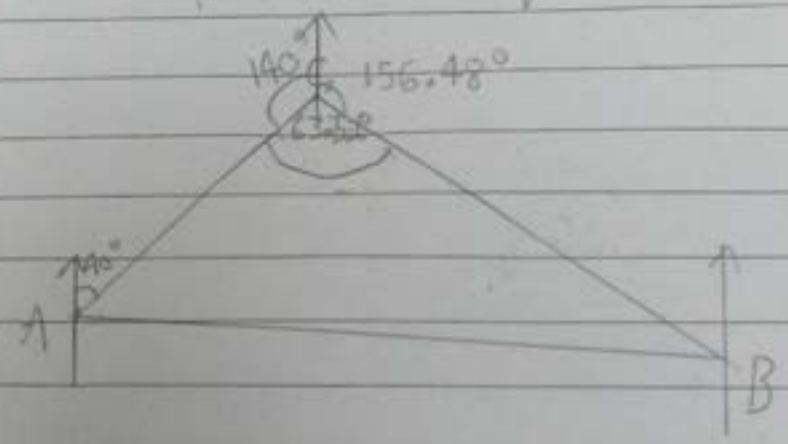
$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

$$A = \cos^{-1}\left(\frac{4.5^2 - 2.7^2 - 5^2}{-2 \times 2.7 \times 5}\right)$$

$$A = \cos^{-1}\left(\frac{301}{675}\right)$$

$$A = 63.517 = 63.5^\circ$$

b) The bearing of A from C is 220°.
Calculate the bearing of B from C.



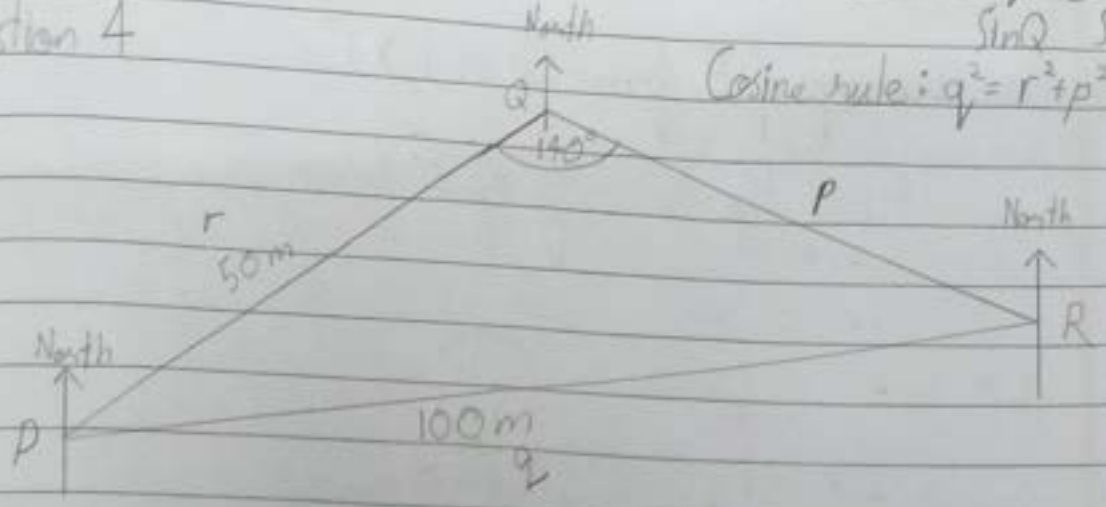

$360 - 140 - 63.52 = 156.48^\circ = 156.5^\circ$

The bearing of B from C is 156.5°

Sine rule : $\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{p}{\sin P}$

Cosine rule : $q^2 = r^2 + p^2 - 2rp\cos Q$

Question 4




The diagram shows three points P, Q and R on horizontal ground.

PQ = 50m, PR = 100m and angle PQR = 140°

a) Calculate angle PRQ.

Sine rule : $\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{p}{\sin P}$

R = ∠PRQ

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

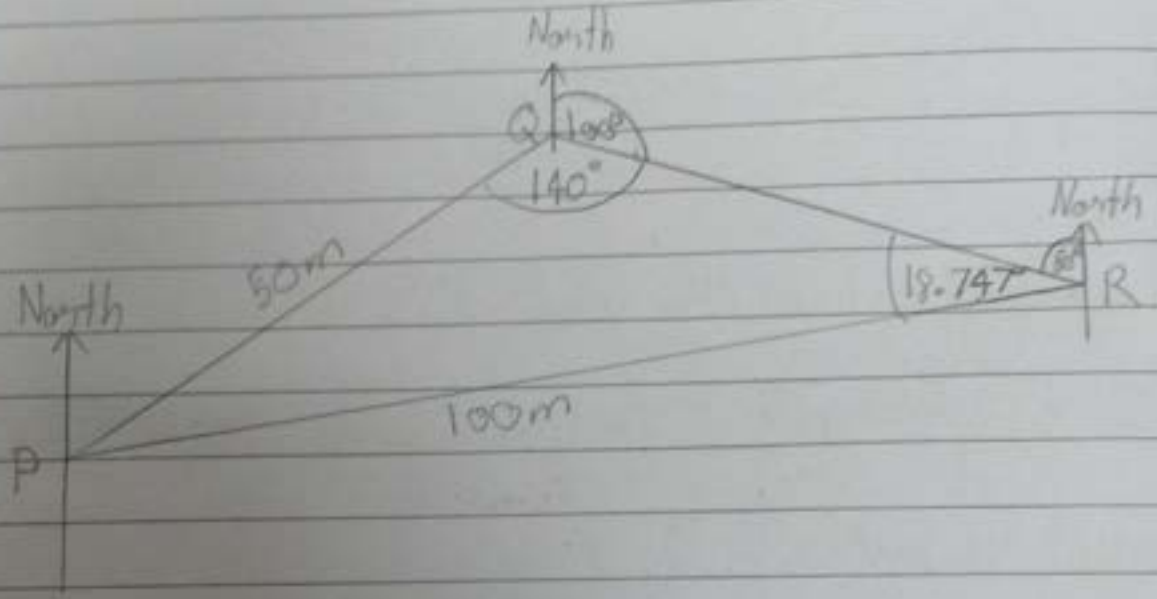
$$\frac{100}{\sin 140} = \frac{50}{\sin R}$$

$$\frac{\sin R}{50} = \frac{\sin 140}{100}$$

$$\sin R = \frac{\sin 140 \times 50}{100}$$

$$R = \sin^{-1}\left(\frac{\sin 140 \times 50}{100}\right) = 18.747 = 18.8°$$

b) The bearing of R from Q is 100°.
Find the bearing of P from R.




$180° - 100° = 80°$

$360 - (80 + 18.747) = 261.253°$

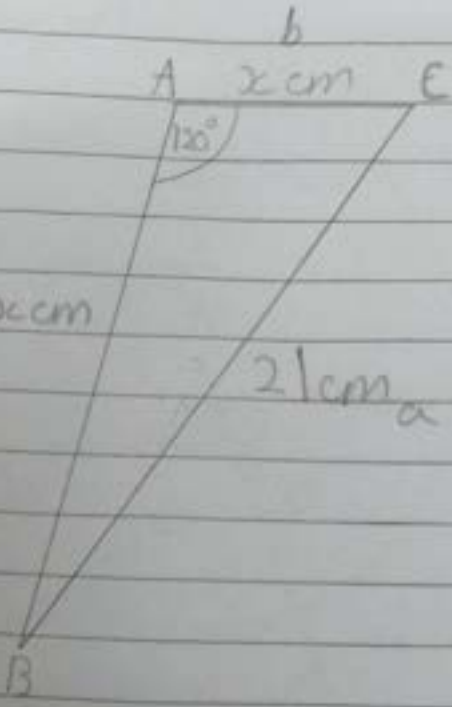
$= 261.3°$

The bearing of P from R is 261.3°.

Question 5

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$




In triangle ABC, AB = 2x cm, BC = 21cm and angle BAC = 120°.

Calculate the value of x.

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

$21^2 = x^2 + (2x)^2 - 2(x)(2x)\cos(120)$

$441 = x^2 + 4x^2 - 4x^2\cos(120)$

$441 = x^2(1 + 4 - 4\cos(120))$

$441 = x^2(7)$

$x^2 = \frac{441}{7}$

$x = \sqrt{63}$

$x = 3\sqrt{7}$

$x = 7.93725$

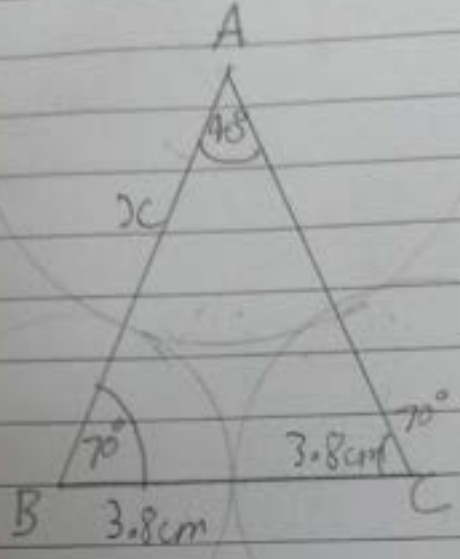
$x = 7.94$

$x = 7.9\text{cm}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Question 6




The diagram shows three touching circles.
A is the centre of a circle of radius x centimetres.
B and C are the centres of circles of radius 3.8 centimetres.
Angle ABC = 70°
Find the value of x.

Ans) Since the smaller circles have the same radius, the triangle must be an isosceles triangle. So $\angle BAC$ must be:

$$180° - (70° + 70°) = 40°$$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Sine rule: $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{7.6}{\sin(40)} = \frac{c}{\sin(70)}$

$c = \frac{7.6}{\sin(40)} \times \sin(70)$

$c = 11.11045672$

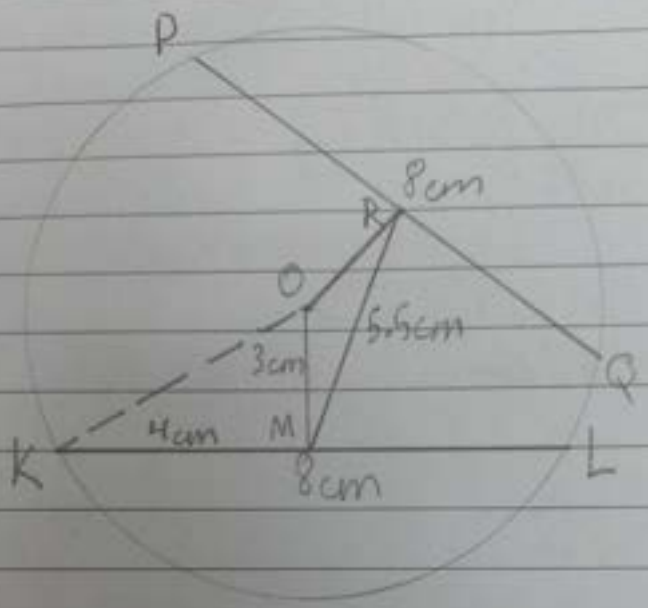
$c = 11.11\text{ cm}$

$x = c - 3.8$

$x = 11.11 - 3.80$

$x = 7.31\text{ cm}$

Question 7




In the circle, centre O, the chords KL and PQ are each of length 8cm. M is the mid-point of KL and R is the mid point of PQ. OM = 3cm

a) Calculate the length of OK.

Ans) Since OM is a perpendicular bisector, that means KM is $\frac{8}{2} = 4$cm

So OK is: $a^2 + b^2 = c^2$

$3^2 + 4^2 = c^2$

$25 = c^2$

$c = \sqrt{25}$

$c = 5$cm

b) RM has a length of 5.5cm. Calculate angle ROM.

(use) Cosine rule: $o^2 = r^2 + m^2 - 2rm\cos(O)$

$o^2 - r^2 - m^2 = -2rm\cos(O)$

$O = \cos^{-1}\left(\frac{o^2 - r^2 - m^2}{-2rm}\right)$

$O = \cos^{-1}\left(\frac{5.5^2 - 3^2 - 3^2}{-2(3)(3)}\right)$

$O = \cos^{-1}\left(\frac{-49}{72}\right)$

$O = 132.8871$

$O = 132.9°$

$\angle ROM = 132.9°$

OR = 3cm because is perpendicular bisects PQ which has the same length as KL

$O = \angle ROM$