



**General Certificate of Education (A-level)**  
**January 2011**

**Mathematics (Pilot)**

**XMCA2**

**(Specification 6360)**

**Core A2**

***Mark Scheme***

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### Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
– <i>x</i> EE	deduct <i>x</i> marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

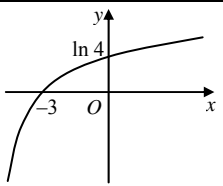
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## XMCA2

Q	Solution	Marks	Total	Comments												
1	$1 + \frac{1}{2}(4x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)(4x)^2}{2!} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(4x)^3}{3!}$ $(1+4x)^{\frac{1}{2}} = 1 + 2x - 2x^2 + 4x^3 \dots; \quad a = 2; \quad b = -2, \\ c = 4$	B1;B1 B1	3	Condone if simplified terms left in form of expansion. <b>SC</b> if 0/3 award B1 for two correct 'x'-terms unsimplified												
Total			3													
2	$x+1=3x \Rightarrow x=\frac{1}{2}$ $x+1=-3x$ $x=-\frac{1}{4}$	B1 M1  A1	3	PI or method to solve $8x^2-2x-1=0$ OE OE												
Total			3													
3(a)	<table><tr><td><math>x</math></td><td><math>y</math></td></tr><tr><td>0</td><td><math>\ln 3 = 1.0986\dots</math></td></tr><tr><td>0.5</td><td><math>\ln 3.25 = 1.1786\dots</math></td></tr><tr><td>1</td><td><math>\ln 4 = 1.3862\dots</math></td></tr><tr><td>1.5</td><td><math>\ln 5.25 = 1.6582\dots</math></td></tr><tr><td>2</td><td><math>\ln 7 = 1.9459\dots</math></td></tr></table> $\frac{1}{3} \times 0.5 \{y(0) + y(2) + 4[y(0.5) + y(1.5)] + 2y(1)\}$ $\frac{1}{3} \times 0.5 \times 17.1646\dots = 2.86 \text{ (to 2 d.p.)}$	$x$	$y$	0	$\ln 3 = 1.0986\dots$	0.5	$\ln 3.25 = 1.1786\dots$	1	$\ln 4 = 1.3862\dots$	1.5	$\ln 5.25 = 1.6582\dots$	2	$\ln 7 = 1.9459\dots$	B1  B1  M1  A1	4	$x$ values PI  At least 4 correct $y$ values PI; 'exact' or rounded or truncated to 2dp.  Use of Simpson's rule. Must be for odd number of $x$ -vals from 0 CAO Must be 2.86
$x$	$y$															
0	$\ln 3 = 1.0986\dots$															
0.5	$\ln 3.25 = 1.1786\dots$															
1	$\ln 4 = 1.3862\dots$															
1.5	$\ln 5.25 = 1.6582\dots$															
2	$\ln 7 = 1.9459\dots$															
(b)	$\int_0^2 \ln \sqrt{x^2+3} \, dx = \frac{1}{2} \times \int_0^2 \ln(x^2+3) \, dx$ $\approx \frac{1}{2} \times 2.86\dots = 1.43\dots$	M1  A1F	2	PI  Ft on $0.5 \times c$ 's "2.86" to 2dp or better even for NMS SC If <b>full</b> SR then B1 for 1.43												
Total			6													
4(a)	$1 = A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -\frac{1}{2} \quad x = -1 \quad x^2 \text{ terms } 0 = A+2B$ $A = 4 \quad C = -1 \quad B = -2$ $\int \left( \frac{4}{2x+1} - \frac{2}{x+1} - \frac{1}{(x+1)^2} \right) dx$ $= 2\ln(2x+1) - 2\ln(x+1) + \frac{1}{x+1} \quad (+c)$	M1 m1  A3;2,1F  M1  A1F A1F A1F	5  4	PI Subst values of $x$ or comparing coefficients PI Ft value of previous incorrect constant if used to find other(s) Use of (a) Ft on $c$ 's $A/2$ ie $0.5A \ln(2x+1)$ Ft on $c$ 's $B$ ie $B \ln(x+1)$ Ft on $c$ 's $-C$ ie $\frac{-C}{x+1}$												
Total			9													

## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
5(a)		B1		Correct shape with graph <b>crossing</b> negative x-axis and positive y-axis only
	(-3, 0) and (0, ln 4)	B1;B1	3	Condone just values of intercepts marked on axes instead of coords. OE for ln4 eg 2ln2 Condone 1.38(6..) in place of ln4
(b)	$\frac{dy}{dx} = \frac{1}{x+4}$ Gradient of given line = $\frac{1}{2}$ At A, $\frac{1}{x_A + 4} = \frac{1}{2}$ x-coord of A is -2 So coords of A are (-2, ln2) Eqn of tangent at A: $y - \ln 2 = \frac{1}{2}(x + 2)$	M1 B1 M1 A1 A1		Or $\frac{dy}{dx} = \frac{1}{2}$ Equating c's $\frac{dy}{dx}$ to c's const grad of line PI $x_A = -2$
	<b>Total</b>		<b>8</b>	
6(a)	$R = \sqrt{6^2 + (-8)^2} (= 10)$ $\cos \alpha = \frac{6}{R}$ , or $\sin \alpha = \frac{8}{R}$ or $\tan \alpha = \frac{8}{6}$ $\alpha = 0.927\dots$ and $R = 10$	B1 M1 A1	3	OE
(b)	$\cos(\theta + \alpha) = \frac{5}{R}$ $\cos(\theta + 0.927\dots) = 0.5 \Rightarrow \theta + 0.927\dots = \pm 1.047\dots$ $\theta = -1.9(74\dots)$ ; $\theta = -2.0$ (to 2sf) $\theta = 0.11(99\dots) = 0.12$ (to 2sf)	M1 m1 A1F A1F	4	Using (a) $\theta + \alpha = \pm \cos^{-1}(5/R)$ Condone > 2sf. Ft on -1.047- c's $\alpha$ Condone > 2sf. Ft on 1.047- c's $\alpha$ Penalise extras inside the given interval
(c)	$\frac{15}{6\cos x - 8\sin x} = \frac{15}{R\cos(x + \alpha)} \quad \{= \frac{15}{R}\sec(x + \alpha)\}$ Translation $\begin{bmatrix} -0.927\dots \\ 0 \end{bmatrix}$ Stretch, (I) parallel to y-axis, (II) scale factor 1.5 OE	M1 E1 B1F m1 A1F	5	Using (a). PI Condone $\theta$ for x Translation-transl.. or better but dep on no variables in the vector Ft on c's $\alpha$ . Accept $\begin{bmatrix} -\alpha \\ 0 \end{bmatrix}$ ‘Stretch’ <u>with</u> either (I) or (II) but dep on M. Accept ‘15/R’ for ‘1.5’ Complete description. Ft on c's R If >2 transformations, deduct 2 from any EBm and A marks awarded to min of 0
	<b>Total</b>		<b>12</b>	

## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dN}{dt} = kN$	M1	6	$\frac{dN}{dt}$ seen
		A1		ACF
	$\int N^{-1} dN = \int k dt$	m1		Separating variables with intention to then integrate
	$\ln N = kt + c$	A1		Condone absence of $+c$ for this mark
	$\ln(1.6 \times 10^6) = k \times 0 + c \Rightarrow c = \ln(1.6 \times 10^6)$	m1		Substituting $N = 1.6 \times 10^6$ (condone $N=1.6$ ) and $t = 0$ in an attempt to find $c$ <b>and</b> $\ln p = q \Rightarrow p = e^q$ applied at some stage in soln to (a)
	$\ln\left(\frac{N}{1.6 \times 10^6}\right) = kt \Rightarrow \frac{N}{1.6 \times 10^6} = e^{kt}$	A1		CSO AG
	so $N = 1.6 \times 10^6 e^{kt}$			
	(b) $\ln(4 \times 10^6) = 2.5k + \ln(1.6 \times 10^6)$	M1	5	Substituting $N = 4 \times 10^6$ and $t = 2.5$ in an attempt to find $k$ .
	$k = \frac{1}{2.5} \ln\left(\frac{4}{1.6}\right) = \frac{\ln 2.5}{2.5} (= 0.366516...)$	A1		ACF
	$\ln(13 \times 10^6) = \frac{\ln 2.5}{2.5} \times t + \ln(1.6 \times 10^6)$	m1		Substituting $N = 13 \times 10^6$ to reach a linear eqn with $t$ as the only unknown
	$kt = \ln\left(\frac{13}{1.6}\right) = \ln 8.125 = 2.09(494...)$	A1		A correct numerical expression or value for $kt$ (PI)
	$t = \frac{2.5}{\ln 2.5} \times \ln\left(\frac{13}{1.6}\right) = \frac{2.09494...}{k}$			
	$t = 5.7158... = 5.72$ (to 3 sf)	A1		Condone $> 3$ sf. Accept 5.71
Total			11	SC Substitution of $N=4$ and $N=13$ deduct maximum of 1 mark for MR.

## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\frac{dx}{d\theta} = -\sin \theta + \cos \theta, \quad \frac{dy}{d\theta} = -4\sin \theta \cos \theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ $\frac{dy}{dx} = \frac{-4\sin \theta \cos \theta}{-\sin \theta + \cos \theta}$	M1 A1 M1  A1	   4	Both derivatives OE attempted PI At least one correct Chain rule with some substitution  ACF in terms of $\theta$ .
(b)(i)	$x^2 = \cos^2 \theta + \sin^2 \theta + 2\sin \theta \cos \theta$ $x^2 = 1 + \sin 2\theta$	M1 A1	 2	Correct expn. and at least one correct identity used Condone slipping into different letter for $\theta$
(ii)	Greatest positive value of $x = \sqrt{2}$	B1F	1	Ft on $x^2 = p + q \sin 2\theta$ ie $x_{\max} = \sqrt{p+q}$ or ('or otherwise') award for $\sqrt{2}$ . Condone " $x \leq \sqrt{2}$ "
(c)	$y = \cos 2\theta; \quad y^2 = \cos^2 2\theta = 1 - \sin^2 2\theta$ $y^2 = 1 - (x^2 - 1)^2$ <b>ALT.</b> $y = x(\cos \theta - \sin \theta);$ $\frac{y^2}{x^2} = \cos^2 \theta + \sin^2 \theta - 2\sin \theta \cos \theta$ $\frac{y^2}{x^2} + x^2 = 2(\cos^2 \theta + \sin^2 \theta) = 2$ $y^2 = x^2(2 - x^2)$	B1  M1 A1	   3	$y = \cos 2\theta$ or $y = x(\cos \theta - \sin \theta)$ PI or a correct eqn involving $y^2, x^2$ (and/or $x^4$ ) and $\theta$ , with simplification. Attempt to eliminate $\theta$ completely with use of suitable trig identities ACF of $g(x)$ in $y^2 = g(x)$
<b>Total</b>			<b>10</b>	
Condone use of other letter for $\theta$ in 1 <sup>st</sup> three marks of (a)				
9	$\frac{dy}{dx} = -2e^{-2x}(2x^2 + 2x + k) + e^{-2x}(4x + 2)$ $\frac{dy}{dx} = 0 \Rightarrow -2e^{-2x}(2x^2 + k - 1) = 0$ $e^{-2x} > 0$ so $(2x^2 + k - 1) = 0 \Rightarrow x = \pm \sqrt{\frac{1-k}{2}}$ For exactly one point (one value of $x$ ), $k = 1$	M1 A1  m1 E1  m1 A1	     6	Product rule applied ACF  Reaching $pe^{-2x}(ax^2 + bx + c) = 0$ PI OE eg $e^{-2x} \neq 0$  OE eg considers equal roots of quadratic CSO $k = 1$ with full correct reasoning.
<b>Total</b>			<b>6</b>	

## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
10(a)(i)	$\vec{AB} = \vec{OB} - \vec{OA} = \begin{bmatrix} 3-1 \\ 0-2 \\ 3-2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$	M1		$\pm(\vec{OB} - \vec{OA})$ implied by 2 'correct' components
(ii)	$AB = \sqrt{2^2 + (-2)^2 + 1^2} = 3$	A1 B1	2 1	If incorrect allow ft on (a)(i)
(iii)	$\mathbf{r} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$	B1F	1	OE Ft on (a)(i) $\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = required
(iv)	$\vec{OA} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = 2 - 4 + 2$	M1		Scalar product with correct (ft) vectors; allow one component error
	$= 0 \text{ (so } \vec{OA} \text{ is perpendicular (to } l_1 \text{))}$	A1	2	CSO with conclusion.
(b)	Putting $\mathbf{r}$ 's equal	M1		Or any two components
	$1 + 2\lambda = 10\mu$			
	$2 - 2\lambda = -4\mu$			
	$2 + \lambda = \mu$			
	Solving two equations	m1		To find $\lambda$ or $\mu$
	1 <sup>st</sup> and 2 <sup>nd</sup> $\lambda = 2, \mu = 0.5$ ;			
	[2 <sup>nd</sup> and 3 <sup>rd</sup> $\lambda = -5, \mu = -3$ ]			
	[1 <sup>st</sup> and 3 <sup>rd</sup> $\lambda = -19/8, \mu = -3/8$ ]			
	Check in remaining equation	m1		Attempt to use c's solutions to check remaining equation or attempt to solve another relevant pair of equations and to compare solution(s). Not dependent on previous m
	(Since equation is not satisfied, lines do) not intersect	A1		CSO Fully correct with valid conclusion
	$\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \neq k \begin{bmatrix} 10 \\ -4 \\ 1 \end{bmatrix} \text{ so (lines } l_1 \text{ and } l_2 \text{) not parallel. (*)}$			
	Lines do not intersect and are not parallel so $l_1$ and $l_2$ are skew lines. (**)	E2,1,0	6	E1 if either just (*) or just (**)
	<b>Total</b>		<b>12</b>	
	For conclusions accept equivalent preambles			



## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
11(a)	$u = x \text{ and } \frac{dv}{dx} = e^{-3x}$ $\frac{du}{dx} = 1 \text{ and } v = -\frac{1}{3} e^{-3x}$ $\dots = -x \frac{1}{3} e^{-3x} - \int -\frac{1}{3} e^{-3x} dx$ $= -\frac{1}{3} x e^{-3x} - \frac{1}{9} e^{-3x} (+c)$	M1 A1 A1F A1	4	Attempt to use parts formula in the 'correct direction' PI ft on wrong integration of $e^{-3x}$ provided $v$ is of the form $ke^{-3x}$ ( $k \neq \pm 1$ ) CSO (Condone absence of $+c$ ) A relevant single substitution used
(b)	Let $u = x^2 + 9$ $\frac{du}{dx} = 2x$ $\int \frac{x^3}{\sqrt{x^2 + 9}} dx = \int \frac{1}{2} \times \frac{(u-9)}{\sqrt{u}} du$ $I = \frac{1}{2} \int (u^{0.5} - 9u^{-0.5}) du$ When $x=4$ , $u=25$ and when $x=0$ , $u=9$ $I = \frac{1}{2} \left[ \frac{u^{1.5}}{1.5} - \frac{9u^{0.5}}{0.5} \right]_9^{25}$ $= \frac{1}{2} \left\{ \left( \frac{125}{1.5} - \frac{45}{0.5} \right) - \left( \frac{27}{1.5} - \frac{27}{0.5} \right) \right\}$ $= \frac{1}{2} \left( -6\frac{2}{3} + 36 \right) = 14\frac{2}{3}$	M1 m1 m1 m1 m1 m1 A1	6	'du = 2x dx' OE In terms of $u$ only, must deal with all parts integrand and dx; dep on prev 2 mks Condone no $du$ provided not just dx missing Integrand 'correct' form which can be integrated directly, dep on prev 3mks Following a <b>correct integration</b> , dealing correctly with correct limits, either for $u$ or (after substituting back for $x$ ) for $x$ Accept 14.6 or 14.7 or better provided full subst method seen and no obvious errors seen. <b>Altn substitution:</b> $u^2 = x^2 + 9$ so ' $2udu = 2x dx$ ' OE (M1m1) leading to $\int \frac{(u^2 - 9)u}{u} du$ (m1) leading to $\int (u^2 - 9) du$ (m1)
	<b>Total</b>		<b>10</b>	
12	$V = \pi \int_0^{\frac{\pi}{4}} \tan^2 x dx$ $= (\pi) \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) dx$ $= (\pi) \left[ \tan x - x \right]_0^{\frac{\pi}{4}}$ $= \pi \left( \tan \frac{\pi}{4} - \frac{\pi}{4} \right) = \pi \left( 1 - \frac{\pi}{4} \right)$	B1 M1 B1 A1	4	Must be completely correct including dx seen on this line or next line Explicitly seen using the identity $1 + \tan^2(x) = \sec^2(x)$ to simplify integrand Terms inside [ ] AG Be convinced
	<b>Total</b>		<b>4</b>	

## XMCA2 (cont)

Q	Solution	Marks	Total	Comments
<b>13(a)(i)</b>	$\frac{x+1}{x-1} = \sqrt{x}$ ; $\frac{(x+1)^2}{(x-1)^2} = x$ ; $(x+1)^2 = x(x-1)^2$ $x^2 + 2x + 1 = x^3 - 2x^2 + x$ ; $x^3 = 3x^2 + x + 1$ Dividing each term by $x^2$ gives $x = 3 + \frac{1}{x} + \frac{1}{x^2}$ So $\alpha$ satisfies the equation $x = 3 + \frac{1}{x} + \frac{1}{x^2}$	M1 m1 m1 A1	4	$f(x)=g(x)$ and either elim of $\sqrt{\quad}$ or elim of fraction or $(x-1)=(x+1)/\sqrt{x}$ Elim of both $\sqrt{\quad}$ and fraction or $(x-1)^2=x+2+(1/x)$ Valid methods to penultimate stage CSO AG
<b>(ii)</b>	$x_2 = 3.444$ $x_3 = 3.375$	B1 B1	2	AWR T 3.444. Condone exact value eg 31/9 CAO
<b>(iii)</b>	Let $p(x) = 3 + \frac{1}{x} + \frac{1}{x^2} - x$ $p(3.3825) = 0.0005(4..)$ $p(3.3835) = -0.0005(9..)$ Since change of sign (and p is continuous close to $\alpha$ ), $\alpha$ is 3.383 correct to 3 dp	M1 A1	2	Both p(3.3825) and p(3.3835) attempted OE <b>or</b> subst of these vals in given equation (a)(i) OE comparing sides of eqn. in (i) for the two values of $x$ oe with a valid conclusion with explicit reference to 3.383
<b>(b)(i)</b>	(Range of $f^{-1}$ is) $f^{-1}(x) > 1$	B1	1	Allow $y$ for $f^{-1}(x)$
<b>(ii)</b>	$y = f^{-1}(x) \Rightarrow f(y) = x$ $\Rightarrow \frac{y+1}{y-1} = x$ $y+1 = xy - x \Rightarrow y(x-1) = x+1$	M1 m1		$x \leftrightarrow y$ at any stage Into a form where just one step is required
<b>(c)(i)</b>	$0 < 2\theta < \frac{\pi}{2}$ so $0 < \cos 2\theta < 1$ so $\frac{1}{\cos 2\theta} > 1$ ie $\sec 2\theta > 1$	E2,1,0	2	Graphical approach also valid within explanation
<b>(ii)</b>	$gf(\sec 2\theta) = g\left(\frac{\sec 2\theta + 1}{\sec 2\theta - 1}\right) = \sqrt{\frac{\sec 2\theta + 1}{\sec 2\theta - 1}}$ $\sec 2\theta = \frac{1}{\cos 2\theta}$ ; $\cos 2\theta = 2\cos^2 \theta - 1$ and $\cos 2\theta = 1 - 2\sin^2 \theta$ $\frac{\sec 2\theta + 1}{\sec 2\theta - 1} = \frac{1 + \cos 2\theta}{1 - \cos 2\theta} = \frac{1 + 2\cos^2 \theta - 1}{1 - (1 - 2\sin^2 \theta)} = \frac{\cos^2 \theta}{\sin^2 \theta}$ $gf(\sec 2\theta) = \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta}} = \cot \theta.$	B1 M1 A1 A1	4	PI All three identities attempted- condone sign slips CSO AG
<b>Total</b>			<b>18</b>	

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