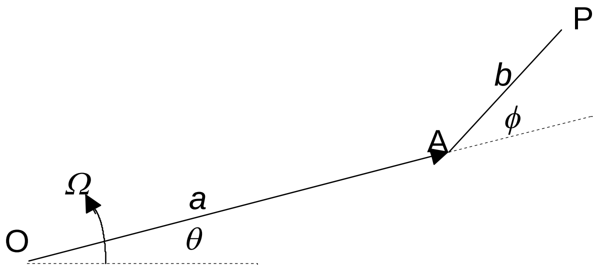
12.



$\dot{\theta} = \Omega$

Acceleration of A perpendicular to AP is $a\Omega^2 \sin\phi$

Acceleration of P perpendicular to AP and relartive to A is $b\ddot{\phi}$

So acceleration of P perpendicular to AP is $a\Omega^2 \sin\phi + b\ddot{\phi}$

Since pivot at A is smooth there is no force perpendicular to AP

so $a\Omega^2 \sin\phi + b\ddot{\phi} = 0 \Rightarrow \ddot{\phi} = -\frac{a}{b}\Omega^2 \sin\phi \Rightarrow 2\ddot{\phi}\dot{\phi} = -2\left(\frac{a}{b}\Omega^2 \sin\phi\right)\dot{\phi}$

integrating we then have $\left(\frac{d\phi}{dt}\right)^2 = \frac{2a}{b}\Omega^2 \cos\phi + C$

Initially, $\frac{d\phi}{dt} = \frac{1}{b}\left\{\left(a + 2\sqrt{ab}\right)\Omega - a\Omega\right\} = 2\sqrt{\frac{a}{b}}\,\Omega > 0$

Hence, $C = \frac{2a}{b}\Omega^2$ and so $\frac{d\phi}{dt} = \sqrt{\frac{2a}{b}\Omega^2(\cos\phi + 1)} = 2\sqrt{\frac{a}{b}}\cos\left(\frac{1}{2}\phi\right)$

Integrating $\int \sec\left(\frac{1}{2}\phi\right)d\phi = \int 2\sqrt{\frac{a}{b}}\,dt \Rightarrow \frac{1}{2}\ln\left(\sec\left(\frac{1}{2}\phi\right) + \tan\left(\frac{1}{2}\phi\right)\right) = 2\sqrt{\frac{a}{b}}\,t + C$

$\phi = 0$ when $t = 0$ so $C = 0$ and $\sec\left(\frac{1}{2}\phi\right) + \tan\left(\frac{1}{2}\phi\right) = e^{4\sqrt{\frac{a}{b}}\,t}$

ϕ can never be equal to π so AP cannot cross OP .
