1. The time in minutes that Elaine takes to checkout at her local supermarket follows a continuous uniform distribution defined over the interval [3, 9].

Find

(a) Elaine's expected checkout time,
(1)

(b) the variance of the time taken to checkout at the supermarket,
(2)

(c) the probability that Elaine will take more than 7 minutes to checkout.
(2)

Given that Elaine has already spent 4 minutes at the checkout,

(d) find the probability that she will take a total of less than 6 minutes to checkout.
(3)

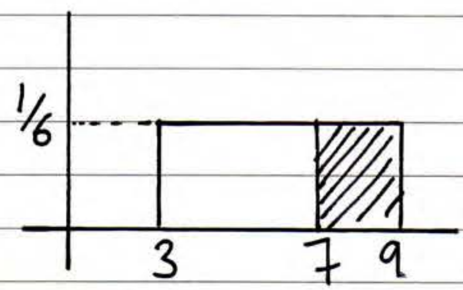
X = time taken to checkout at the supermarket

$X \sim U[3,9]$

a) $E(X) = \frac{(a+b)}{2} = \frac{3+9}{2} = \underline{\underline{6 \text{ mins}}}$

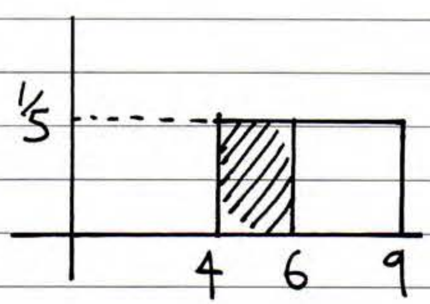
b) $Var(X) = \frac{(b-a)^2}{12} = \frac{(9-3)^2}{12} = \frac{36}{12} = \underline{\underline{3 \text{mins}}}$

c)



$P(X>7) = \underline{\underline{2/6}}$

b)



$P(X<6 \mid X>4) = \underline{\underline{2/5}}$

2. David claims that the weather forecasts produced by local radio are no better than those achieved by tossing a fair coin and predicting rain if a head is obtained or no rain if a tail is obtained. He records the weather for 30 randomly selected days. The local radio forecast is correct on 21 of these days.

Test David's claim at the 5% level of significance.

State your hypotheses clearly.

(7)

X = no. of times the forcast is correct

$H_0 : p = 0.5 \quad H_1 : p > 0.5$

under H_0 $X \sim B(30, 0.5)$

$P(X \geq 21) = 1 - P(X \leq 20)$

$= 1 - 0.9786$

$= 0.0214$

$\therefore$ since $2.14\% < 5\%$ reject H_0.

Therefore there is evidence to say that the weather forcasts are better than those produced by tossing a coin.

3. The probability of a telesales representative making a sale on a customer call is 0.15

Find the probability that

(a) no sales are made in 10 calls,
(2)

(b) more than 3 sales are made in 20 calls.
(2)

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement.
(2)

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95
(3)

X = no. of sales made

a) $X \sim B(10, 0.15)$

$P(X=0) = 0.85^{10} = \underline{\underline{0.1969}}$ (4dp)

b) $X \sim B(20, 0.15)$

$P(X>3) = 1 - P(X \leq 3)$

$= 1 - 0.6477 = \underline{\underline{0.3523}}$

c) $np = 5 \quad \therefore \quad 0.15n = 5$

$n = 33.\dot{3} \Rightarrow \underline{\underline{34 \text{ calls}}}$

d) $P(X \geq 1) = 0.95$

$P(X=0) = 0.05$

$0.85^n = 0.05$

$n = \log 0.05 / \log 0.85 = 18.43$

$= \underline{\underline{19 \text{ calls}}}$

4. A website receives hits at a rate of 300 per hour.

(a) State a distribution that is suitable to model the number of hits obtained during a 1 minute interval.
(1)

(b) State two reasons for your answer to part (a).
(2)

Find the probability of

(c) 10 hits in a given minute,
(3)

(d) at least 15 hits in 2 minutes.
(3)

The website will go down if there are more than 70 hits in 10 minutes.

(e) Using a suitable approximation, find the probability that the website will go down in a particular 10 minute interval.
(7)

a) Poisson distribution

X = no. of hits $X \sim Po(5)$

b) • hits occur at a constant rate
• hits occur independently
• hits occur singly in time etc.

c) $P(X=10) = \frac{e^{-5}5^{10}}{10!} = \underline{\underline{0.0181}}$ (4dp)

$X \sim Po(10)$

d) $P(X \geq 15) = 1 - P(X \leq 14)$

$= 1 - 0.9165 = \underline{\underline{0.0835}}$

Question 4 continued

e) $X \sim Po(50)$

$X \approx Y \sim N(50, \sqrt{50}^2)$

$P(X>70) = P(X \geqslant 71) \approx P(Y \geqslant 70.5)$

$= P\left(Z \geqslant \frac{70.5-50}{\sqrt{50}}\right)$

$= P(Z \geqslant 2.899)$

$= 1 - \Phi(2.90)$

$= 1 - 0.9981$

$= \underline{\underline{0.0019}}$