5. The probability of an electrical component being defective is 0.075
The component is supplied in boxes of 120

(a) Using a suitable approximation, estimate the probability that there are more than 3 defective components in a box.

(5)

A retailer buys 2 boxes of components.

(b) Estimate the probability that there are at least 4 defective components in each box.

(2)

a) X = no. of defective components

$X \sim B(120, 0.075)$

$X \approx \sim Po(120 \times 0.075) = Po(9)$

$P(X>3) = P(X\geq 4)$

$= 1 - P(X\leq 3)$

$= 1 - 0.0212$

$= \underline{\underline{0.9788}}$

b) $0.9788^2 = \underline{\underline{0.9580}}$ (4dp)

6. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leqslant x < 1 \\ x - \frac{1}{2} & 1 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

(a) Sketch the graph of $f(x)$.
(2)

(b) Show that $k = \frac{1}{2}(1+\sqrt{5})$.
(4)

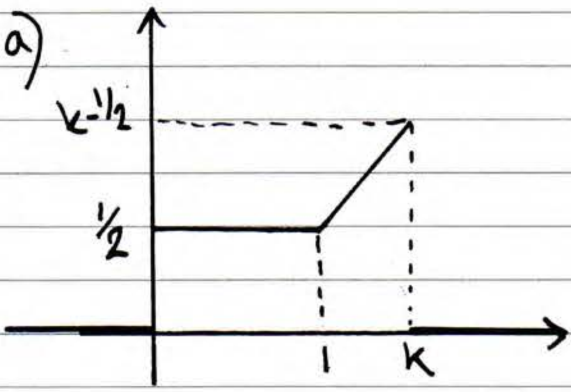
(c) Define fully the cumulative distribution function $F(x)$.
(6)

(d) Find $P(0.5 < X < 1.5)$.
(2)

(e) Write down the median of X and the mode of X.
(2)

(f) Describe the skewness of the distribution of X. Give a reason for your answer.
(2)

a)



b) Area = 1

Rectangle + Trapezium = 1

$\frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} + k - \frac{1}{2}\right)(k-1) = 1$

Question 6 continued

$\frac{1}{2} + \frac{1}{2}k(k-1) = 1$

$1 + k^2 - k = 2$

$k^2 - k - 1 = 0$

$k = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$ $\quad \underline{\underline{k = \frac{1}{2}(1+\sqrt{5})}}$ #

(k is positive)

c)

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 0 \le x < 1 \\ \frac{1}{2} + \frac{1}{2}x(x-1) & 1 \le x \le k \\ 1 & x > k \end{cases}$$

d) $P(0.5 < X < 1.5) = F(1.5) - F(0.5)$

$= \frac{1}{2} + \frac{1}{2}(1.5)(1.5-1) - 0.5 \times 0.5$

$= \underline{\underline{5/8}}$

e) median $= \underline{\underline{1}}$

mode $= \underline{\underline{k}}\ (= 1.618)$

F) negative skew since mode > median

$(k = 1.618)$.

7. (a) Explain briefly what you understand by

(i) a critical region of a test statistic,

(ii) the level of significance of a hypothesis test.

(2)

(b) An estate agent has been selling houses at a rate of 8 per month. She believes that the rate of sales will decrease in the next month.

(i) Using a 5% level of significance, find the critical region for a one tailed test of the hypothesis that the rate of sales will decrease from 8 per month.

(ii) Write down the actual significance level of the test in part (b)(i).

(3)

The estate agent is surprised to find that she actually sold 13 houses in the next month. She now claims that this is evidence of an increase in the rate of sales per month.

(c) Test the estate agent's claim at the 5% level of significance. State your hypotheses clearly.

(5)

a) I) The critical region is where any observation falling inside that region results in the rejection of H_0.

II) The significance level is the probability of incorrectly rejecting H_0.

b) X = no. of houses sold in a month

$H_0: \lambda = 8 \qquad H_1: \lambda < 8$

under H_0 $X \sim Po(8)$

I) $P(X \leq 4) = 0.0996$

$P(X \leq 3) = 0.0424$

$\therefore$ Critical region $\underline{\underline{X \leq 3}}$

Question 7 continued

II) $\underline{\underline{4.24\%}}$

c) $H_0 : \lambda = 8 \qquad H_1 : \lambda > 8$

under H_0 $X \sim Po(8)$

$P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - 0.9658$

$= 0.0342$

Since $3.42\% < 5\%$ reject H_0

There is evidence that the rate of sales per month has increased.