

QUESTION 4

Let $S = \log_2 e - \log_4 e + \log_{16} e - \dots + (-1)^r \log_{2^{2^r}} e + \dots$

Further let $a_r = \log_{2^{2^r}} e \Rightarrow \left(2^{2^r}\right)^{a_r} = e \Rightarrow a_r \ln 2^{2^r} = 1$

$$\Rightarrow a_r 2^r \ln 2 = 1 \Rightarrow a_r = \frac{1}{2^r \ln 2}$$

$$\text{Then } S = \sum_{r=0}^{\infty} (-1)^r \frac{1}{2^r \ln 2} = \frac{1}{\ln 2} \sum_{r=0}^{\infty} (-1)^r \frac{1}{2^r}$$

$$= \frac{1}{\ln 2} \times \frac{1}{1 - \left(-\frac{1}{2}\right)} \text{ (Geometric Progression)}$$

$$= \frac{1}{\frac{3}{2} \ln 2} = \frac{1}{\ln(2\sqrt{2})}$$
