AQA C2 Jan12

1 (a) $A = \frac{1}{2}r^2\theta$

$21.6 = \frac{1}{2} \times 6^2 \times \theta$

$21.6 = 18\theta$ [1] $\theta = 1.2$ [1]

(b) $AB = r\theta = 6 \times 1.2$ [1]

$= 7.2$ cm [1]

2 (a) $h = \frac{4-0}{4} = 1$

$x_0 = 0 \quad y_0 = 1/1 = 1$

$x_1 = 1 \quad y_1 = 2/2 = 1$

$x_2 = 2 \quad y_2 = 2^2/3 = 4/3$ [1]

$x_3 = 3 \quad y_3 = 2^3/4 = 2$

$x_4 = 4 \quad y_4 = 2^4/5 = 16/5$ [1]

$\Rightarrow \frac{1}{2}\left[1 + \frac{16}{5} + 2\left(1 + \frac{4}{3} + 2\right)\right]$ [1]

$= \frac{193}{30}$ or 6.43 [1]

(b) More ordinates (or strips) [1]

3 (a) $x^{3/4}$ [1]

(b) $\frac{1 - x^2}{x^{3/4}} = (1 - x^2)x^{-3/4}$ [1]

$= x^{-3/4} - x^{+5/4}$ [1]

4 (a) $A = \frac{1}{2}ab\sin C$

$40 = \frac{1}{2} \times 10 \times b \times \sin 150$ [1]

$\frac{40}{5\sin 150} = b \Rightarrow b = 16$ m [1]

(b) $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 10^2 + 16^2 - 2 \times 10 \times 16 \times \cos 150$ [1]

$= 633.128\ldots$

$a = 25.16$ m [1]

(c) Smallest angle = ACB (opposite smallest side) [1]

$\frac{\sin C}{c} = \frac{\sin A}{a} \Rightarrow \frac{\sin C}{10} = \frac{\sin 150}{25.16}$ [1]

$C = 11.462\ldots$

$= 11.5°$ [1]

5 (a) (i) Stretch scale factor $\frac{1}{6}$ in x direction [2]

(ii) $y = \left(1 + \frac{x-3}{3}\right)^6 = \left(\frac{x}{3}\right)^6$

$= \frac{x^6}{729}$ [1]

(b) $x \Rightarrow \frac{x}{3} \quad n \Rightarrow 6$

$= 1 + (6)\left(\frac{x}{3}\right) + \frac{(6)(5)}{2}\left(\frac{x}{3}\right)^2$ [1]

$+ \frac{(6)(5)(4)}{6}\left(\frac{x}{3}\right)^3$ [1]

$= 1 + 2x + \frac{5}{3}x^2$ [1] $+ \frac{20}{27}x^3$ [1]

$a = 2 \quad b = \frac{5}{3} \quad c = \frac{20}{27}$

6 (a) $S_{25} = 3500 = \frac{25}{2}(2a + 24d)$ [1]

$7000 = 50a + 600d$ [1]

$a + 12d = 140$ [1]

(b) $u_5 = a + 4d = 100$ [1]

Subtract $8d = 40 \Rightarrow d = 5$ [1]

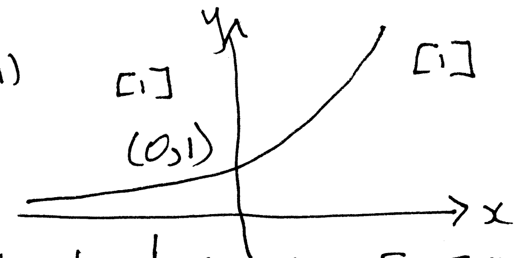
$\therefore a + 20 = 100$ [1] $a = 80$ [1]

(c) $33(3500 - S_k) = 67 \times S_k$

$3500 \times 33 = 100S_k$ [1]

$S_k = 1155$ [1]

7 (a)



(b) $\log \frac{1}{2^x} = \log \frac{5}{4}$ [1]

$\log 2^{-x} = \log \frac{5}{4}$ [1]

$-x\log 2 = \log \frac{5}{4}$

$x = \frac{\log 5/4}{-\log 2} = -0.322$ [1]

7 (c) $\log b^2 + 3\log y = 3\log_a a + 2\log \frac{y}{a}$ [1]

$\log b^2 + \log y^3 = \log a^3 + \log \frac{y^2}{a^2}$ [1]

$\log b^2 y^3 = \log \frac{a^3 y^2}{a^2}$ [1]

$b^2 y^3 = \frac{a^3 y^2}{a^2}$ [1]

$\frac{b^2 y^3}{y^2} = a \Rightarrow y = \frac{a}{b^2}$ [1]

8 (a) $\sin\theta = \frac{7}{2}\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{7}{2}$ [1]

$\tan\theta = \frac{7}{2}$ [1]

(b) (i) $\sin^2 x = 1 - \cos^2 x$

$\therefore 6(1-\cos^2 x) = 4 + \cos x$ [1]

$6 - 6\cos^2 x = 4 + \cos x$

$6\cos^2 x + \cos x - 2 = 0$ [1]

(ii) $(3\cos x + 2)(2\cos x - 1) = 0$ [1]

$\cos x = -\frac{2}{3}$ or $\frac{1}{2}$

$x = 131.8$ or 60° [1]

Also $360 - 131.8$ or $360 - 60$ [1]

$= 228.2$ or 300

So $x = 132^\circ, 228^\circ, 60^\circ, 300^\circ$ [1]

9 (a) $12 - 5x^{2/3}$ [2]

(b) (i) when $x = 0$

$\frac{dy}{dx} = 12$ [1]

$y - 0 = 12(x - 0)$

$y = 12x$ [1]

(ii) when $x = 8$

$\frac{dy}{dx} = 12 - 5(8)^{2/3} = -8$ [1]

$y - 0 = -8(x - 8)$ [1]

$y = -8x + 64$

$y + 8x = 64$ [1]

(c) $6x^2 - \frac{9}{8}x^{8/3} + c$ [1]

[1] [1]

(d) Intersection of tangents

$12x = -8x + 64$ [1]

$20x = 64 \Rightarrow x = \frac{16}{5}$ [1]

$\therefore y = 2 \times \frac{16}{5} = \frac{32}{5}$ [1]

$P = \left(\frac{16}{5}, \frac{32}{5}\right)$ [1]

Area of triangle $= \frac{1}{2} \times 8 \times \frac{32}{5}$

$= \frac{128}{5}$ [1]

Area under curve

$= \left[6x^2 - \frac{9}{8}x^{8/3}\right]_0^8$

$= \left[6(8)^2 - \frac{9}{8}(8)^{8/3}\right] - [0]$ [1]

$= 96$ [1]

So shaded area

$= 96 - \frac{128}{5}$

$= \frac{352}{5} = \underline{\underline{70.4}}$ [1]