OCR
RECOGNISING ACHIEVEMENT

Friday 13 January 2012 – Morning

A2 GCE MATHEMATICS

4726 Further Pure Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4726
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Given that $f(x) = \ln(\cos 3x)$, find $f'(0)$ and $f''(0)$. Hence show that the first term in the Maclaurin series for $f(x)$ is ax^2, where the value of a is to be found. **[4]**

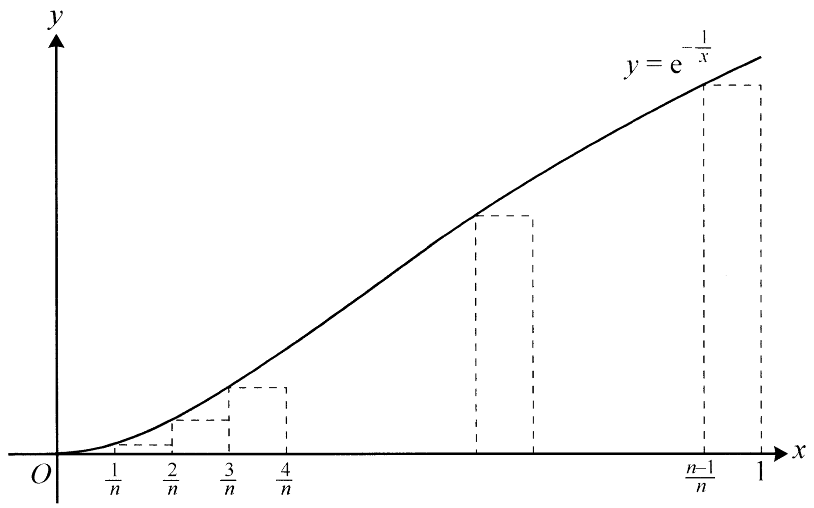
2 By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5}\,dx.$$

[5]

3 Express $\dfrac{2x^3 + x + 12}{(2x-1)(x^2+4)}$ in partial fractions. **[7]**

4



The diagram shows the curve $y = e^{-\frac{1}{x}}$ for $0 < x \leqslant 1$. A set of $(n-1)$ rectangles is drawn under the curve as shown.

(i) Explain why a lower bound for $\int_0^1 e^{-\frac{1}{x}}\,dx$ can be expressed as

$$\frac{1}{n}\left(e^{-n} + e^{-\frac{n}{2}} + e^{-\frac{n}{3}} + \ldots + e^{-\frac{n}{n-1}}\right).$$

[2]

(ii) Using a set of n rectangles, write down a similar expression for an upper bound for $\int_0^1 e^{-\frac{1}{x}}\,dx$. **[2]**

(iii) Evaluate these bounds in the case $n = 4$, giving your answers correct to 3 significant figures. **[2]**

(iv) When $n \geqslant N$, the difference between the upper and lower bounds is less than 0.001. By expressing this difference in terms of n, find the least possible value of N. **[3]**

5 It is given that $f(x) = x^3 - k$, where $k > 0$, and that α is the real root of the equation $f(x) = 0$. Successive approximations to α, using the Newton-Raphson method, are denoted by $x_1, x_2, \ldots, x_n, \ldots$.

(i) Show that $x_{n+1} = \dfrac{2x_n^3 + k}{3x_n^2}$. **[2]**

(ii) Sketch the graph of $y = f(x)$, giving the coordinates of the intercepts with the axes. Show on your sketch how it is possible for $|\alpha - x_2|$ to be greater than $|\alpha - x_1|$. **[3]**

It is now given that $k = 100$ and $x_1 = 5$.

(iii) Write down the exact value of α and find x_2 and x_3 correct to 5 decimal places. **[3]**

(iv) The error e_n is defined by $e_n = \alpha - x_n$. By finding e_1, e_2 and e_3, verify that $e_3 \approx \dfrac{e_2^3}{e_1^2}$. **[3]**

6 **(i)** Prove that the derivative of $\cos^{-1}x$ is $-\dfrac{1}{\sqrt{1-x^2}}$. **[3]**

A curve has equation $y = \cos^{-1}(1 - x^2)$, for $0 < x < \sqrt{2}$.

(ii) Find and simplify $\dfrac{dy}{dx}$, and hence show that

$$\left(2 - x^2\right)\frac{d^2y}{dx^2} = x\frac{dy}{dx}.$$

[5]

7 **(i)** Given that $y = \sinh^{-1}x$, prove that $y = \ln\left(x + \sqrt{x^2+1}\right)$. **[3]**

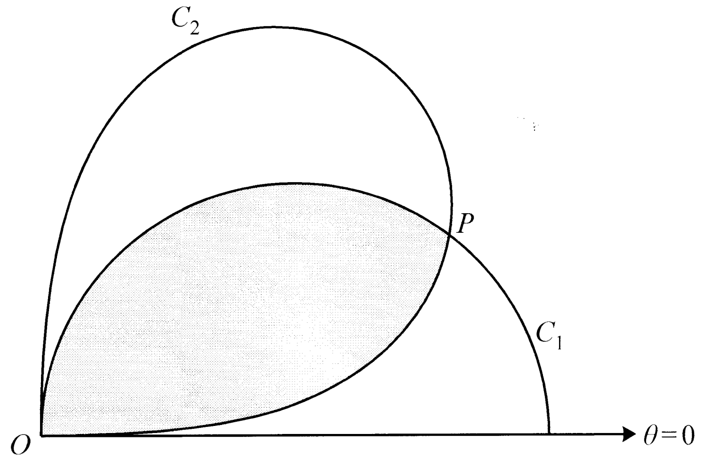
(ii) It is given that x satisfies the equation $\sinh^{-1}x - \cosh^{-1}x = \ln 2$. Use the logarithmic forms for $\sinh^{-1}x$ and $\cosh^{-1}x$ to show that

$$\sqrt{x^2+1} - 2\sqrt{x^2-1} = x.$$

Hence, by squaring this equation, find the exact value of x. **[5]**

[Questions 8 and 9 are printed overleaf.]

8



The diagram shows two curves, C_1 and C_2, which intersect at the pole O and at the point P. The polar equation of C_1 is $r=\sqrt{2}\cos\theta$ and the polar equation of C_2 is $r=\sqrt{2\sin 2\theta}$. For both curves, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. The value of θ at P is α.

(i) Show that $\tan\alpha = \frac{1}{2}$. **[2]**

(ii) Show that the area of the region common to C_1 and C_2, shaded in the diagram, is $\frac{1}{4}\pi - \frac{1}{2}\alpha$. **[7]**

9 **(i)** Show that $\tanh(\ln n) = \dfrac{n^2-1}{n^2+1}$. **[2]**

It is given that, for non-negative integers n, $I_n = \displaystyle\int_0^{\ln 2} \tanh^n u \, du$.

(ii) Show that $I_n - I_{n-2} = -\dfrac{1}{n-1}\left(\dfrac{3}{5}\right)^{n-1}$, for $n \geqslant 2$. **[3]**

(iii) Find the value of I_3, giving your answer in the form $a + \ln b$, where a and b are constants. **[4]**

(iv) Use the method of differences on the result of part **(ii)** to find the sum of the infinite series

$$\frac{1}{2}\left(\frac{3}{5}\right)^2 + \frac{1}{4}\left(\frac{3}{5}\right)^4 + \frac{1}{6}\left(\frac{3}{5}\right)^6 + \dots .$$

[2]