

# Friday 13 January 2012 - Morning

# A2 GCE MATHEMATICS

4726 Further Pure Mathematics 2

# QUESTION PAPER

Candidates answer on the Printed Answer Book.

#### OCR supplied materials:

- Printed Answer Book 4726
- List of Formulae (MF1)

#### Other materials required:

Scientific or graphical calculator

Duration: 1 hour 30 minutes

# INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer **Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### **INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part guestion on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

# INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 Given that  $f(x) = \ln(\cos 3x)$ , find f'(0) and f''(0). Hence show that the first term in the Maclaurin series for f(x) is  $ax^2$ , where the value of *a* is to be found. [4]
- 2 By first completing the square in the denominator, find the exact value of

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{4x^2 - 4x + 5} \, \mathrm{d}x \, .$$

[5]

[7]

3 Express  $\frac{2x^3 + x + 12}{(2x-1)(x^2+4)}$  in partial fractions.

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The diagram shows the curve  $y = e^{-\frac{1}{x}}$  for  $0 < x \le 1$ . A set of (n-1) rectangles is drawn under the curve as shown.

- (i) Explain why a lower bound for  $\int_0^1 e^{-\frac{1}{x}} dx$  can be expressed as  $\frac{1}{n} \left( e^{-n} + e^{-\frac{n}{2}} + e^{-\frac{n}{3}} + \dots + e^{-\frac{n}{n-1}} \right).$ [2]
- (ii) Using a set of *n* rectangles, write down a similar expression for an upper bound for  $\int_0^1 e^{-\frac{1}{x}} dx$ . [2]
- (iii) Evaluate these bounds in the case n = 4, giving your answers correct to 3 significant figures. [2]
- (iv) When  $n \ge N$ , the difference between the upper and lower bounds is less than 0.001. By expressing this difference in terms of *n*, find the least possible value of *N*. [3]

- 5 It is given that  $f(x) = x^3 k$ , where k > 0, and that  $\alpha$  is the real root of the equation f(x) = 0. Successive approximations to  $\alpha$ , using the Newton-Raphson method, are denoted by  $x_1, x_2, \dots, x_n, \dots$ .
  - (i) Show that  $x_{n+1} = \frac{2x_n^3 + k}{3x^2}$ . [2]
  - (ii) Sketch the graph of y = f(x), giving the coordinates of the intercepts with the axes. Show on your sketch how it is possible for  $|\alpha x_2|$  to be greater than  $|\alpha x_1|$ . [3]

It is now given that k = 100 and  $x_1 = 5$ .

- (iii) Write down the exact value of  $\alpha$  and find  $x_2$  and  $x_3$  correct to 5 decimal places. [3]
- (iv) The error  $e_n$  is defined by  $e_n = \alpha x_n$ . By finding  $e_1, e_2$  and  $e_3$ , verify that  $e_3 \approx \frac{e_2^2}{e_1^2}$ . [3]
- 6 (i) Prove that the derivative of  $\cos^{-1}x$  is  $-\frac{1}{\sqrt{1-x^2}}$ . [3]

A curve has equation  $y = \cos^{-1}(1 - x^2)$ , for  $0 < x < \sqrt{2}$ .

(ii) Find and simplify  $\frac{dy}{dx}$ , and hence show that

$$\left(2-x^2\right)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = x\frac{\mathrm{d}y}{\mathrm{d}x} \ .$$

[5]

- 7 (i) Given that  $y = \sinh^{-1} x$ , prove that  $y = \ln \left( x + \sqrt{x^2 + 1} \right)$ . [3]
  - (ii) It is given that x satisfies the equation  $\sinh^{-1} x \cosh^{-1} x = \ln 2$ . Use the logarithmic forms for  $\sinh^{-1} x$  and  $\cosh^{-1} x$  to show that

$$\sqrt{x^2 + 1} - 2\sqrt{x^2 - 1} = x \; .$$

Hence, by squaring this equation, find the exact value of x.

[5]

#### [Questions 8 and 9 are printed overleaf.]



The diagram shows two curves,  $C_1$  and  $C_2$ , which intersect at the pole *O* and at the point *P*. The polar equation of  $C_1$  is  $r = \sqrt{2}\cos\theta$  and the polar equation of  $C_2$  is  $r = \sqrt{2}\sin 2\theta$ . For both curves,  $0 \le \theta \le \frac{1}{2}\pi$ . The value of  $\theta$  at *P* is  $\alpha$ .

(i) Show that 
$$\tan \alpha = \frac{1}{2}$$
. [2]

(ii) Show that the area of the region common to  $C_1$  and  $C_2$ , shaded in the diagram, is  $\frac{1}{4}\pi - \frac{1}{2}\alpha$ . [7]

(i) Show that 
$$\tanh(\ln n) = \frac{n^2 - 1}{n^2 + 1}$$
. [2]

It is given that, for non-negative integers n,  $I_n = \int_0^{\ln 2} \tanh^n u \, du$ .

(ii) Show that 
$$I_n - I_{n-2} = -\frac{1}{n-1} \left(\frac{3}{5}\right)^{n-1}$$
, for  $n \ge 2$ . [3]

- (iii) Find the value of  $I_3$ , giving your answer in the form  $a + \ln b$ , where a and b are constants. [4]
- (iv) Use the method of differences on the result of part (ii) to find the sum of the infinite series

$$\frac{1}{2}\left(\frac{3}{5}\right)^2 + \frac{1}{4}\left(\frac{3}{5}\right)^4 + \frac{1}{6}\left(\frac{3}{5}\right)^6 + \dots$$
 [2]



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