Marks

Answer all the questions

1. (*a*) Given $f(x) = \dfrac{3x+1}{x^2+1}$, obtain $f'(x)$. **3**

   (*b*) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer. **4**

2. The first and fourth terms of a geometric series are 2048 and 256 respectively. Calculate the value of the common ratio. **2**

   Given that the sum of the first n terms is 4088, find the value of n. **3**

3. Given that $(-1 + 2i)$ is a root of the equation

$$z^3 + 5z^2 + 11z + 15 = 0,$$

   obtain all the roots. **4**

   Plot all the roots on an Argand diagram. **2**

4. Write down and simplify the general term in the expansion of $\left(2x - \dfrac{1}{x^2}\right)^9$. **3**

   Hence, or otherwise, obtain the term independent of x. **2**

5. Obtain an equation for the plane passing through the points $P(-2, 1, -1)$, $Q(1, 2, 3)$ and $R(3, 0, 1)$. **5**

6. Write down the Maclaurin expansion of e^x as far as the term in x^3. **1**

   Hence, or otherwise, obtain the Maclaurin expansion of $(1 + e^x)^2$ as far as the term in x^3. **4**

7. A function is defined by $f(x) = |x + 2|$ for all x.

   (*a*) Sketch the graph of the function for $-3 \le x \le 3$. **2**

   (*b*) On a separate diagram, sketch the graph of $f'(x)$. **2**

8. Use the substitution $x = 4 \sin \theta$ to evaluate $\displaystyle\int_0^2 \sqrt{16 - x^2}\, dx$. **6**

Marks

9. A non-singular $n \times n$ matrix A satisfies the equation $A + A^{-1} = I$, where I is the $n \times n$ identity matrix. Show that $A^3 = kI$ and state the value of k. **4**

10. Use the division algorithm to express 1234_{10} in base 7. **3**

11. (*a*) Write down the derivative of $\sin^{-1}x$. **1**

(*b*) Use integration by parts to obtain $\int \sin^{-1}x . \frac{x}{\sqrt{1-x^2}} dx$. **4**

12. The radius of a cylindrical column of liquid is decreasing at the rate of $0{\cdot}02$ m s^{-1}, while the height is increasing at the rate of $0{\cdot}01$ m s^{-1}.

Find the rate of change of the volume when the radius is $0{\cdot}6$ metres and the height is 2 metres. **5**

[*Recall that the volume of a cylinder is given by* $V = \pi r^2 h$.]

13. A curve is defined parametrically, for all t, by the equations

$$x = 2t + \frac{1}{2}t^2, \qquad y = \frac{1}{3}t^3 - 3t.$$

Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t. **5**

Find the values of t at which the curve has stationary points and determine their nature. **3**

Show that the curve has exactly two points of inflexion. **2**

14. (*a*) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ.

$$4x + 6z = 1$$
$$2x - 2y + 4z = -1$$
$$-x + y + \lambda z = 2$$ **5**

(*b*) Describe what happens when $\lambda = -2$. **1**

(*c*) When $\lambda = -1{\cdot}9$ the solution is $x = -22{\cdot}25$, $y = 8{\cdot}25$, $z = 15$.

Find the solution when $\lambda = -2{\cdot}1$. **2**

Comment on these solutions. **1**

15. (*a*) Express $\dfrac{1}{(x-1)(x+2)^2}$ in partial fractions. 4

(*b*) Obtain the general solution of the differential equation 7

$$(x-1)\frac{dy}{dx} - y = \frac{x-1}{(x+2)^2},$$

expressing your answer in the form $y = f(x)$.

16. (*a*) Prove by induction that 6

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

for all integers $n \geq 1$.

(*b*) Show that the real part of $\dfrac{\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^{11}}{\left(\cos\frac{\pi}{36} + i\sin\frac{\pi}{36}\right)^{4}}$ is zero. 4

[*END OF QUESTION PAPER*]