Algebraic functions

09 February 2012
13:43

- Only identical factors can be cancelled

Algebraic long division

Example:
Divide $x^3+2x^2-17x+6$ by $(x-3)$

this is the answer

$$
\begin{array}{r|l}
 & x^2+5x-2 \\
\hline
x-3 & x^3+2x^2-17x+6 \\
 & -\ x^3-3x^2 \\
\hline
 & 5x^2-17x \\
 & -\ 5x^2-15x \\
\hline
 & -2x+6 \\
 & -\ -2x+6 \\
\hline
 & 0+0
\end{array}
$$

- Algebraic long division can be used to
  - Find factors
  - Find if something is a factor
    - This can be done easier by putting the factor into the equation and seeing if it equals 0 or not

The sine and cosine rule

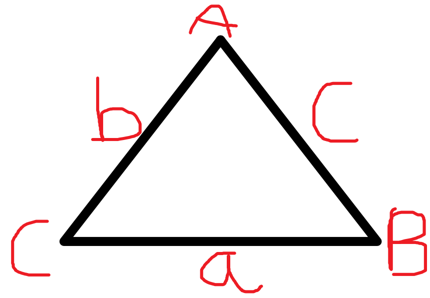
27 January 2012
10:51

The sine rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

or

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



The cosine rule

$$a^2=b^2+c^2-2bc\cos A$$

or

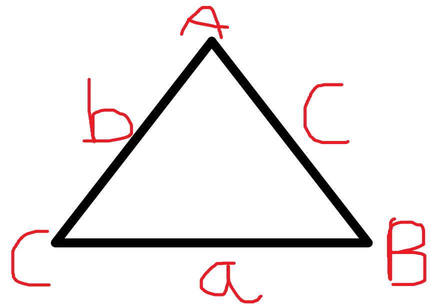
$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

or

$$\cos B=\frac{b^2+c^2-a^2}{2ac}$$

or

$$\cos C=\frac{b^2+c^2-a^2}{2ab}$$



Logs and exponentials

25 January 2012
14:25

Power

$$\log_a b = c$$

base answer

$\log_a b = c$ can be written as $a^c = b$

Rules

$$\log_a b + \log_a c = \log_a (bc)$$

$$\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$$

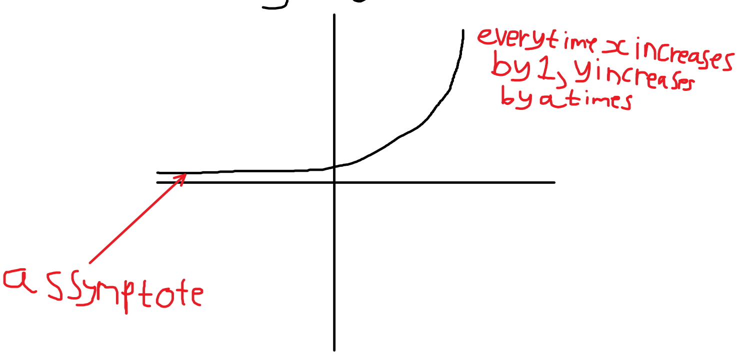
$$\log_a (b)^c = c \log_a b$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

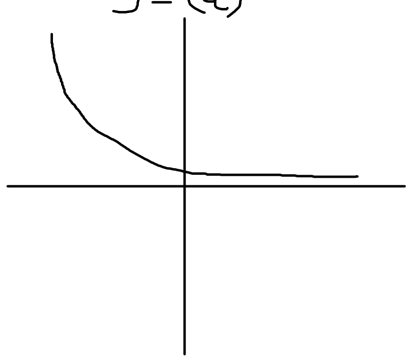
$$\log_a a = 1$$

Graphs

$y = a^x$



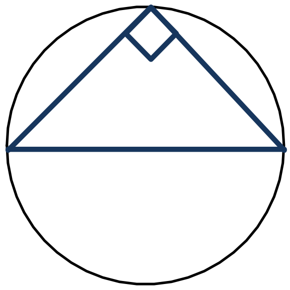
$y = \left(\frac{1}{a}\right)^x$

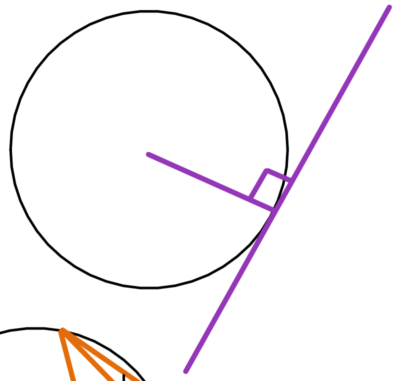


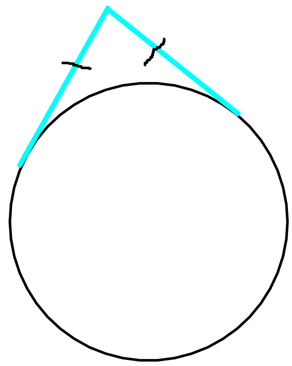
Circles

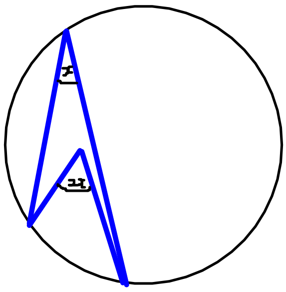
11 March 2012
09:24

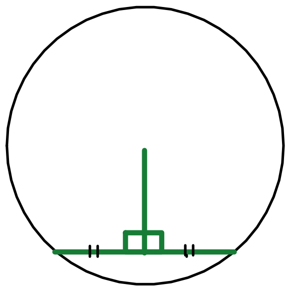
- $(y-a)^2+(x-b)^2=r^2$
  - Origin of circle: (b, a)
  - Radius of circle: r

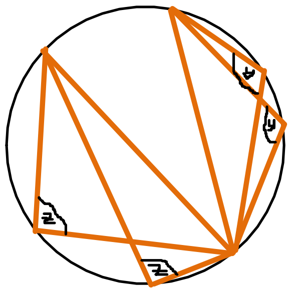


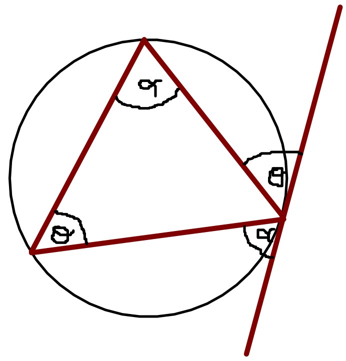


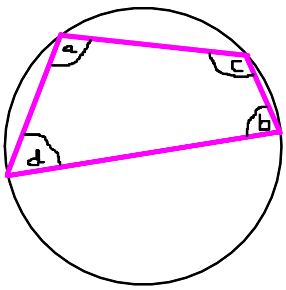












$a+b=180$
$c+d=180$

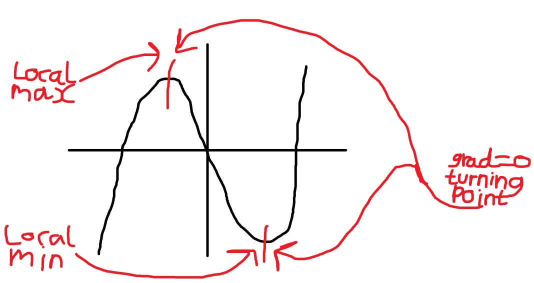
Binomial expansion

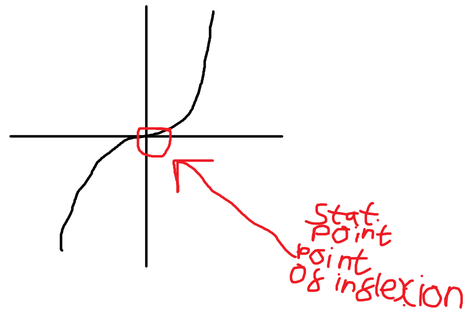
11 March 2012
10:19

- $(a+b)^n = (\_)a^nb^0 + (\_)a^{n-1}b^1 + (\_)a^{n-2}b^2 + (\_)a^{n-3}b^3 ......(\_)a^0b^n$
    - The number in front of each term can be obtained by using the ^nC_r button
        - The n is the term on the bracket, and the r is the power on the b term
- So therefore, $(a+b)^6$ expanded would be:
    - $(1)(a)^6(b)^0 + (6)(a)^5(b)^1 + (15)(a)^4(b)^2 + (20)(a)^3(b)^3 + (15)a^2+b^4 + (6)(a)^1(b)^5 + (1)(a)^0(b)^6$
- If a question said 'Find the first four terms of $(1-\frac{x}{4})^{10}$ and use your result to find an approximate value to $(0.975)^{10}$. Find the degree of accuracy of your approximation' you would have to do a series of steps
    - First you would have to multiply out the bracket to get the first four terms
        - $1-\frac{5x}{2}+\frac{45x^2}{16}-\frac{15x^3}{8}$
    - Next, you would have to find out what value of the x to use, by solving $1-\frac{x}{4}=0.975$
        - $x=0.1$
    - Then you would have to substitute in the value for x into the expansion and find the answer
        - 0.77625
    - To find the degree of accuracy, you put $(0.975)^{10}$ into the calculator and get 0.77632962 and see how many decimal places your answer is accurate to
        - 0.77625 is accurate to 4 d.p.
- $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\frac{n(n-1)(n-2)(n-3)}{4!}x^4 ...$

Differentiation

10 February 2012
13:31





$\frac{dy}{dx}=0 \quad \frac{d^2y}{dx^2}<0$ maxima

$\frac{dy}{dx}=0 \quad \frac{d^2y}{dx^2}>0$ minima

$\frac{dy}{dx}=0 \quad \frac{d^2y}{dx^2}\neq 0$ POI

Trigonomic ratios of common angles

24 March 2012

13:38

	0	30	45	60	90
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tangent	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X

Trigonomic identities

24 March 2012
13:47

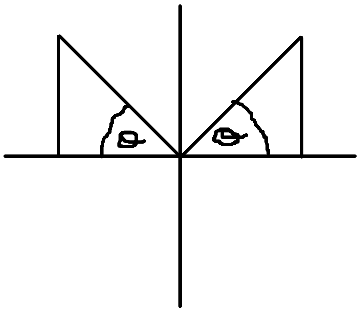
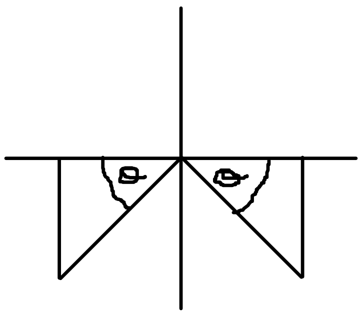
$\cos^2\theta + \sin^2\theta = 1$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

- The trigonomic identities on the right can be used in proof or to rearrange an equation into a useable form
- An example is shown below:
  - Show that $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$
  - $\tan x + \frac{1}{\tan x} = \frac{\sin x}{\cos x} + \frac{1}{\tan x} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{(\sin x)^2}{\cos x \sin x} + \frac{(\cos x)^2}{\sin x \cos x} = \frac{(\cos x)^2 + (\sin x)^2}{\sin x \cos x} = \frac{1}{\sin x \cos x}$

Solving trigonomic equations

24 March 2012
14:09

- Trigonomic equations in the form sin/cos/tan θ=k can be solved using the cast diagram
- The positivness or negativness of the k value decides which quadrant it is in
- For sinθ = K, the diagram would look like
  - 
  - The answers for 0<θ<360 would be: θ, 180-θ
  - The answers for -360<θ<360 would be: -180-θ, -360+θ, θ, 180-θ
- For sinθ = -k, the diagram would look like
  - |
    
  - The answers for 0<θ<360 would be: θ+180, 360-θ
  - The answers for -360<θ<360 would be: -θ, -180+θ, θ+180, 360-θ

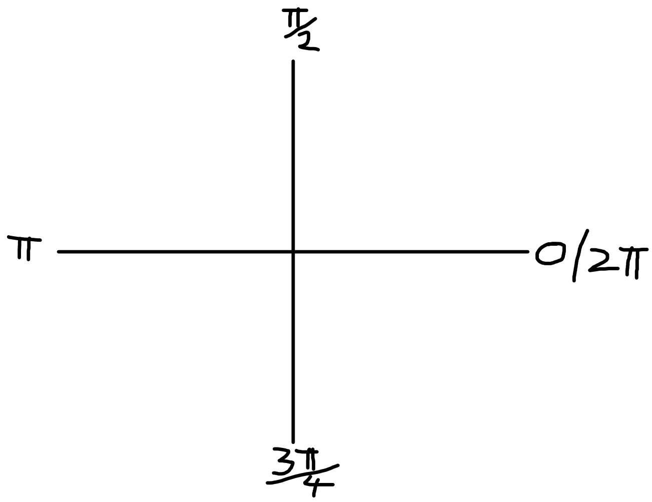
Geometric sequences

24 March 2012
14:23

- Geometric sequences increase by a common ratio called r
- r=$\frac{U_{n+1}}{U_n}$
- U_n = ar^{n-1}
- S_n = $\frac{a(1-r^n)}{1-r}$
- The sum to infinity exists as long as |r|<1
- $S_\infty = \frac{a}{1-r}$

Radians

24 March 2012
15:35



- This formula will allows degrees to be converted in radians, and radians into degrees
  - $\frac{\theta^{\circ}}{360} = \frac{\theta^{c}}{2\pi}$
- The length of an arc = $r \times \theta^{c}$
- Area of a sector = $\frac{r^2\theta^c}{2}$
- Area of a segment = $\frac{r^2}{2}(\theta - \sin\theta)$

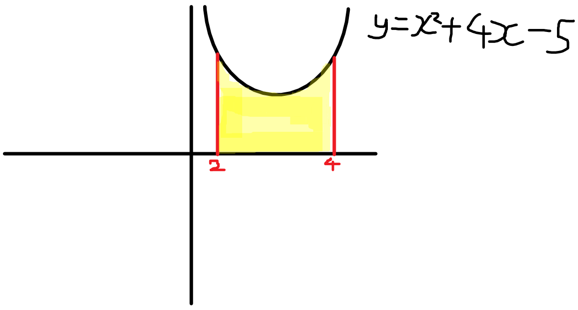
Integration

18 April 2012
13:29

- Trapezium rule
  - $\frac{h}{2}(y_0 + y_n + 2(y_1 \dots y_{n-1})$

Examples:

- Find the area under the curve (y=x^2+4x-5) and above the X-axis between x=2 and x=4
  - Using integration



  - $\int_2^4 (x^2 + 4x - 5)dx = \left|\frac{x^3}{3} + 2x^2 - 5x\right|_2^4 = \frac{4^3}{3} + 2 \times 4^2 - 5 \times 4 - \left(\frac{2^3}{3} + 2 \times 2^2 - 5 \times 2\right) = 32\frac{2}{3}$ Units^2

  - Trapezium rule

X	2	2.5	3	3.5	4
y	7	11.25	16	21.25	27

  - $\frac{0.5}{2}(7 + 27 + 2(11.25 + 16 + 21.25) = 32.75$

- To find out the area between a curve and line, simply integrate the equation of the curve minus the equation of the line