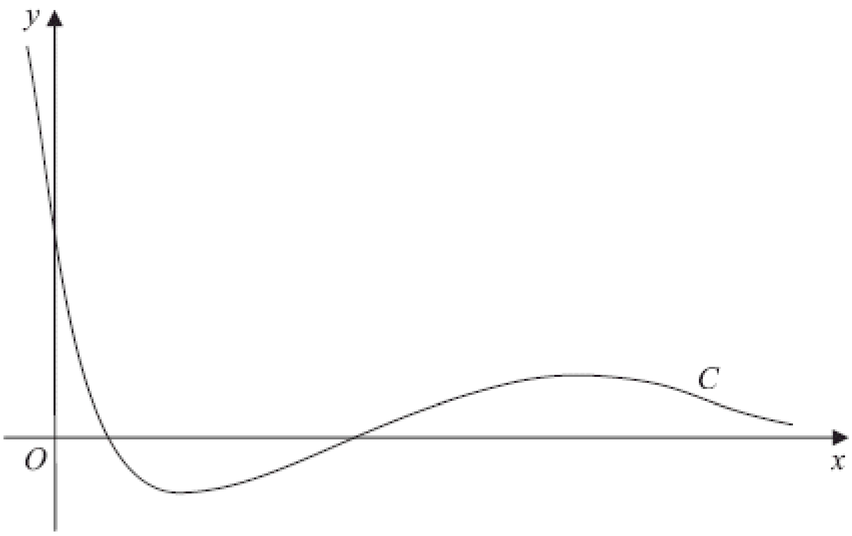
1. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(Total 7 marks)

2.



The diagram above shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.

(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(c) Find $\dfrac{dy}{dx}$.

(3)

(d) Hence find the exact coordinates of the turning points of C.

(5)
(Total 12 marks)

3. (i) Given that $y = \frac{\ln(x^2 + 1)}{x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(ii) Given that $x = \tan y$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1 + x^2}$.

(5)
(Total 9 marks)

4. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)
(Total 11 marks)

5. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{1n(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)},\ x > -\frac{1}{4},\ y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)
(Total 13 marks)

6. The function f is defined by

$$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)},\quad x \in \mathbb{R}, x \neq -4, x \neq 2$$

(a) Show that $\mathrm{f}(x) = \dfrac{x-3}{x-2}$

(5)

The function g is defined by

$$g(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2},\qquad x \in \mathbb{R}, x \neq 1n\ 2$$

(b) Differentiate g(x) to show that $\mathrm{g}'(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$,

(3)

(c) Find the exact values of x for which $g'(x) = 1$

(4)
(Total 12 marks)

7. (a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x-1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)
(Total 10 marks)

8.

$$\mathrm{f}(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express f (x) as a single fraction in its simplest form.

(4)

(b) Hence show that $\mathrm{f}'(x) = \frac{2}{(x-3)^2}$

(3)
(Total 7 marks)

9. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(Total 6 marks)

10. $$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

(Total 11 marks)

11. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)
(Total 14 marks)

12. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)
(Total 8 marks)

13. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when t = 22 920.

(2)

(d) In the space provided on page 13, sketch the graph of R against t.

(2)
(Total 9 marks)

14. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \quad -\pi \leq x \leq \pi.$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places.

(4)
(Total 13 marks)

15. A curve C has equation

$$y = x^2e^x.$$

(a) Find $\dfrac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve *C*.

(2)
(Total 10 marks)

16. The curve *C* has equation

$$x = 2 \sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on *C*.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ = at *P*.

(4)

(c) Find an equation of the normal to *C* at *P*. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)
(Total 9 marks)

17. (i) The curve *C* has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of *C*.

(6)

(ii) Given that

$$y = \left(1 + e^{2x}\right)^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)
(Total 11 marks)

18. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(b) $(5 + x^2)^{\frac{3}{2}}$

(3)
(Total 6 marks)

19. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\, e^{-0.05t} + 25,\ t \geq 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

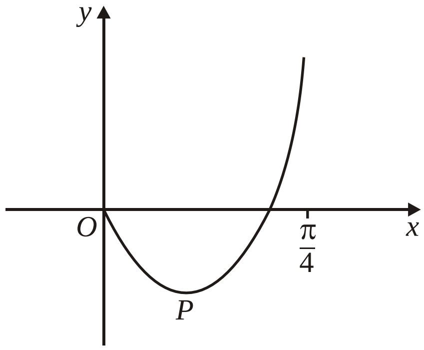
(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)
(Total 9 marks)

20.



The figure above shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leq x < \frac{\pi}{4}$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \qquad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

(Total 11 marks)

21. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(Total 5 marks)

22. The functions f and g are defined by

$$\mathrm{f} : x \rightarrow 2x + \ln 2, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \rightarrow \mathrm{e}^{2x}, \qquad x \in \mathbb{R}$$

(a) Prove that the composite function gf is

$$\mathrm{gf} : x \rightarrow 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}$$

(4)

(b) In the space provided below, sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

(Total 10 marks)

23. $$f(x) = (x^2 + 1) \ln x, \qquad x > 0.$$

(a) Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e.

(4)

(b) Find the exact value of $\int_1^e f(x)\,dx$

(5)
(Total 9 marks)

24. (a) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)
(Total 13 marks)

25. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}\mathrm{e}^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $\mathrm{f}'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)
(Total 9 marks)

26. (a) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(b) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{8}{(x-1)^3}$.

(6)
(Total 12 marks)

27. The function f is defined by

$$\mathrm{f}: x \mapsto 3 + 2\mathrm{e}^x, \quad x \in \mathbb{R}.$$

(a) Evaluate $\displaystyle\int_0^1 f(x)\,dx$, giving your answer in terms of e.

(3)

The curve C, with equation $y = f(x)$, passes through the y-axis at the point A. The tangent to C at A meets the x-axis at the point $(c, 0)$.

(b) Find the value of c.

(4)

The function g is defined by

$$g: x \mapsto \frac{5x+2}{x+4}, \quad x \in \mathbb{R}, \quad x > -4.$$

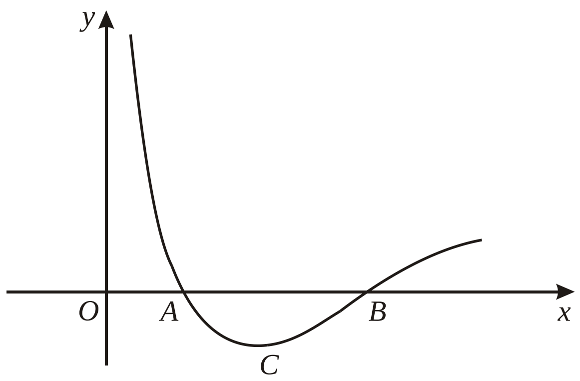
(c) Find an expression for $g^{-1}(x)$.

(3)

(d) Find gf(0).

(2)
(Total 12 marks)

28.



$$f(x) = \frac{1}{2x} - 1 + \ln \frac{x}{2}, x > 0.$$

The diagram above shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points A and B, and has a minimum at the point C.

(a) Show that the x-coordinate of C is $\frac{1}{2}$.

(5)

(b) Find the y-coordinate of C in the form $k \ln 2$, where k is a constant.

(2)

(c) Verify that the x-coordinate of B lies between 4.905 and 4.915.

(2)

(d) Show that the equation $\dfrac{1}{2x} - 1 + \ln \dfrac{x}{2} = 0$ can be rearranged into the form $x = 2e^{\left(1 - \frac{1}{2x}\right)}$.

(2)

The x-coordinate of B is to be found using the iterative formula

$$x_{n+1} = 2e^{\left(1 - \frac{1}{2x_n}\right)}, \quad \text{with } x_0 = 5.$$

(e) Calculate, to 4 decimal places, the values of x_1, x_2 and x_3.

(2)
(Total 13 marks)

29. The curve C with equation $y = k + \ln 2x$, where k is a constant, crosses the x-axis at the point $A\left(\dfrac{1}{2e}, 0\right)$.

(a) Show that $k = 1$.

(2)

(b) Show that an equation of the tangent to C at A is $y = 2ex - 1$.

(4)

(c) Complete the table below, giving your answers to 3 significant figures.

x	1	1.5	2	2.5	3
1 + ln 2x		2.10		2.61	2.79

(2)

(d) Use the trapezium rule, with four equal intervals, to estimate the value of

$$\int_1^3 (1 + \ln 2x) \quad dx$$

.

(4)
(Total 12 marks)

30. $$f(x) = x + \frac{e^x}{5}, \quad x \in \mathbb{R}.$$

(a) Find $f'(x)$.

(2)

The curve C, with equation $y = f(x)$, crosses the y-axis at the point A.

(b) Find an equation for the tangent to C at A.

(3)

(c) Complete the table, giving the values of $\sqrt{\left(x + \frac{e^x}{5}\right)}$ to 2 decimal places.

x	0	0.5	1	1.5	2
$\sqrt{\left(x + \frac{e^x}{5}\right)}$	0.45	0.91			

(2)

(d) Use the trapezium rule, with all the values from your table, to find an approximation for the value of

$$\int_0^2 \sqrt{\left(x + \frac{e^x}{5}\right)} \, dx$$

.

(4)
(Total 11 marks)

31. Use the derivatives of cosec x and cot x to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}\,[\ln(\operatorname{cosec} x + \cot x)] = -\operatorname{cosec} x.$$

(Total 3 marks)

32. Given that $y = \log_a x$, $\quad x > 0$, where a is a positive constant,

(a) (i) express x in terms of a and y,

(1)

(ii) deduce that $\ln x = y \ln a$.

(1)

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x \ln a}$.

(2)

The curve C has equation $y = \log_{10} x$, $x > 0$. The point A on C has x-coordinate 10. Using the result in part (b),

(c) find an equation for the tangent to C at A.

(4)

The tangent to C at A crosses the x-axis at the point B.

(d) Find the exact x-coordinate of B.

(2)
(Total 10 marks)

33. The curve C has equation $y = \dfrac{x}{4 + x^2}$.

(a) Use calculus to find the coordinates of the turning points of C.

(5)

Using the result $\dfrac{d^2y}{dx^2} = \dfrac{2x(x^2 - 12)}{(4 + x^2)^3}$, or otherwise,

(b) determine the nature of each of the turning points.

(3)

(c) Sketch the curve C.

(3)
(Total 11 marks)

34. The curve C has equation $y = f(x)$, where

$$f(x) = 3 \ln x + \frac{1}{x}, \qquad x > 0.$$

The point P is a stationary point on C.

(a) Calculate the x-coordinate of P.

(4)

(b) Show that the y-coordinate of P may be expressed in the form $k - k \ln k$, where k is a constant to be found.

(2)

The point Q on C has x-coordinate 1.

(c) Find an equation for the normal to C at Q.

(4)

The normal to C at Q meets C again at the point R.

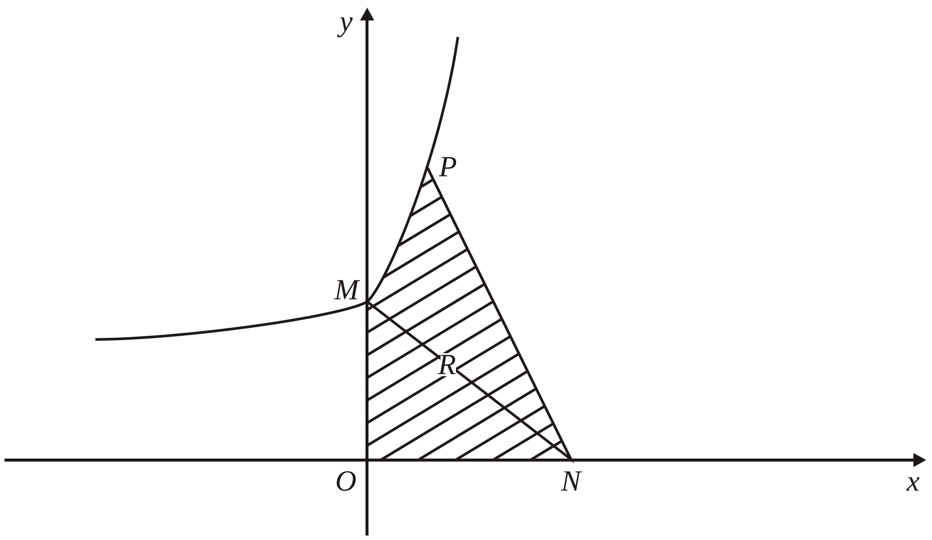
(d) Show that the x-coordinate of R

(i) satisfies the equation $6 \ln x + x + \dfrac{2}{x} - 3 = 0,$

(ii) lies between 0.13 and 0.14.

(4)
(Total 14 marks)

35.



The curve C with equation $y = 2e^x + 5$ meets the y-axis at the point M, as shown in the diagram above.

(a) Find the equation of the normal to C at M in the form $ax + by = c$, where a, b and c are integers.

(4)

This normal to C at M crosses the x-axis at the point $N(n, 0)$.

(b) Show that $n = 14$.

(1)

The point $P(\ln 4, 13)$ lies on C. The finite region R is bounded by C, the axes and the line PN, as shown in the diagram above.

(c) Find the area of R, giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(7)
(Total 12 marks)

36. A curve has equation $7x^2 + 48xy - 7y^2 + 75 = 0$.

A and B are two distinct points on the curve. At each of these points the gradient of the curve is equal to $\frac{2}{11}$.

(a) Use implicit differentiation to show that $x + 2y = 0$ at the points A and B.

(5)

(b) Find the coordinates of the points A and B.

(4)
(Total 9 marks)

37. The curve C has equation $y = 4x^{\frac{3}{2}} - \ln(5x)$, where $x > 0$. The tangent at the point on C where $x = 1$ meets the x-axis at the point A.

Prove that the x-coordinate of A is $\frac{1}{5}\ln(5e)$.

(Total 7 marks)

38. Differentiate with respect to x

(i) $x^3 e^{3x}$,

(3)

(ii) $\dfrac{2x}{\cos x}$,

(3)

(iii) $\tan^2 x$.

(2)

Given that $x = \cos y^2$,

(iv) find $\frac{dy}{dx}$ in terms of y.

(4)
(Total 12 marks)