MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

METHODS FOR ADVANCED MATHEMATICS, C3

Practice Paper C3-B

Additional materials: Answer booklet/paper
Graph paper
List of formulae (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- There is an Insert booklet for use in Question **9.**
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72.**
- **You are reminded of the need for clear presentation in your answers.**

Section A (36 marks)

1 Prove that the product of any three consecutive integers is a multiple of 6. [4]

2 **(i)** Sketch the graph of $y = |2x - 3|$. [2]

(ii) Hence, or otherwise, solve the inequality $|2x - 3| < 5$.
Illustrate your answer on your graph. [2]

3 Differentiate the following functions.

(i) $y = \left(x^2 + 3\right)^5$ [3]

(ii) $y = \dfrac{\sin 2x}{x}$ [3]

4 A curve has equation $y^2 = 5x - 4$.

Find the gradient of the curve at the points where $x = 8$. [5]

5 Given that x and t are related by the formula $x = x_0 e^{-3t}$, show that $t = \ln\left(\dfrac{a}{x}\right)^b$ where a and b are to be determined. [4]

6 **(i)** Find $\int (2x - 3)^7 \, dx$. [3]

(ii) Use the substitution $u = x^2 + 1$, or otherwise, to find $\int_1^2 x\left(x^2 + 1\right)^3 dx$. [5]

7 The functions f, g and h are defined as follows.

$f(x) = 2x$ $\quad$ $g(x) = x^2$ $\quad$ $h(x) = x + 2$

Find each of the following as functions of x.

(i) $f^2(x)$, [1]

(ii) $fgh(x)$, [3]

(iii) $h^{-1}(x)$. [1]

Section B (36 marks)

8 A curve has equation $y = (x+2)\mathrm{e}^{-x}$.

(i) Find the coordinates of the points where the curve cuts the axes. [2]

(ii) Find the coordinates of the stationary point, S, on the curve. [4]

(iii) By evaluating $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at S, determine whether the stationary point is a maximum or a minimum. [3]

(iv) Sketch the curve in the domain $-3 < x < 3$. [1]

(v) Find where the normal to the curve at the point (0, 2) cuts the curve again. [3]

(vi) Find the area of the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$. [5]

9 Answer parts (i) and (iii) on the insert provided.

Fig. 9 shows a sketch graph of $y = \mathrm{f}(x)$.

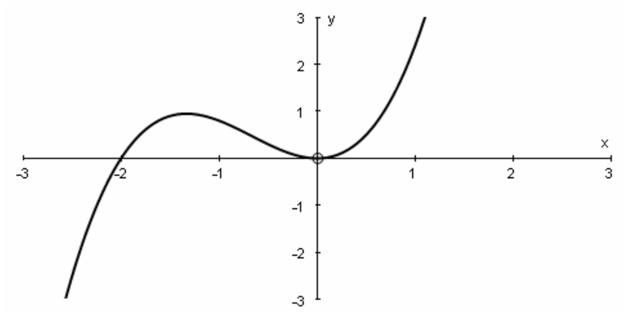

Fig. 9

(i) On the Insert sketch graphs of

(A) $y = 2\mathrm{f}(x)$, (B) $y = \mathrm{f}(-x)$, (C) $y = \mathrm{f}(x-2)$

In each case describe the transformations. [8]

(ii) Explain why the function $y = \mathrm{f}(x)$ does not have an inverse function. [2]

(iii) The function $\mathrm{g}(x)$ is defined as follows:

$$\mathrm{g}(x) = \mathrm{f}(x) \text{ for } x \geq 0$$

On the Insert sketch the graph of $y = \mathrm{g}^{-1}(x)$. [1]

(iv) You are given that $\mathrm{f}(x) = x^2(x+2)$.
Calculate the gradient of the curve $y = \mathrm{f}(x)$ at the point (1, 3).
Deduce the gradient of the function $\mathrm{g}^{-1}(x)$ at the point where $x = 3$. [4]

(v) Show that $\mathrm{g}(x)$ and $\mathrm{g}^{-1}(x)$ cross where $x = -1 + \sqrt{2}$. [3]

NAME:……………………………………………

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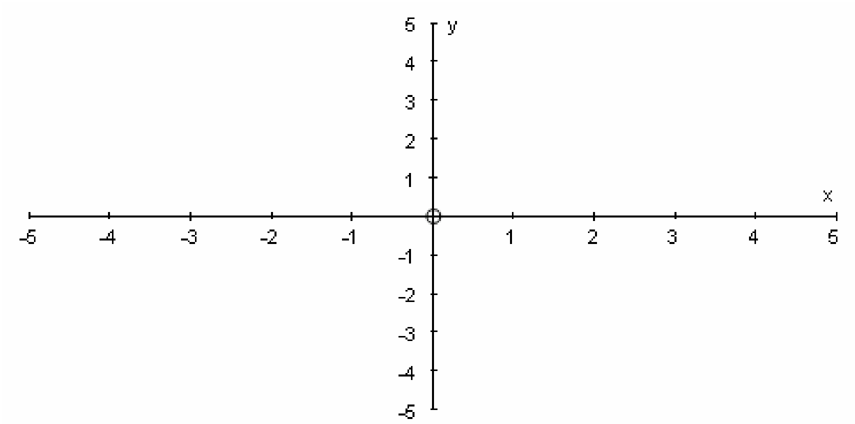
Practice Paper C3-B
INSERT

INSTRUCTIONS

- This insert should be used for question **9.**
- Write your name in the space at the top of this sheet.
- Attach this insert to the rest of your answers.

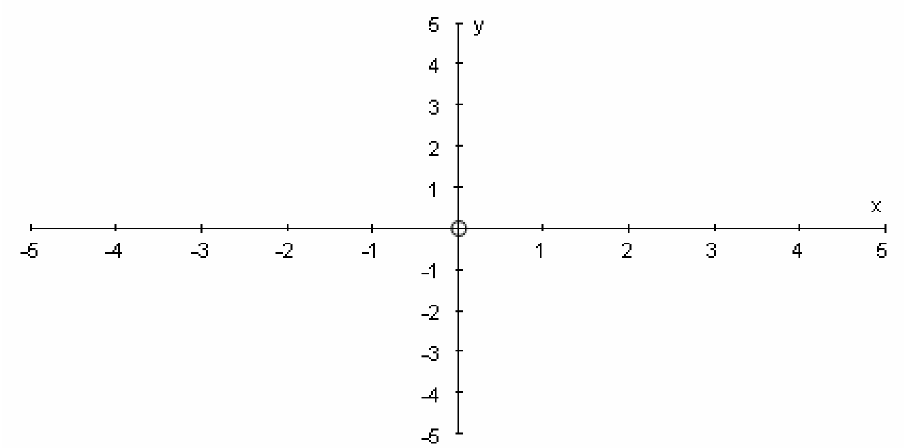
Insert for question 9.

(i) (*A*) On the axes below sketch the graph of $y = 2\mathrm{f}(x)$.
Describe the transformation. [2]



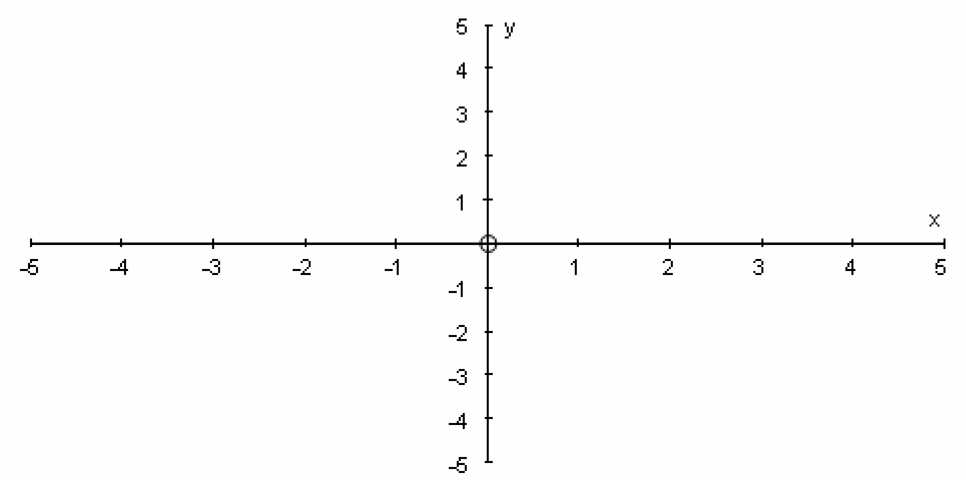
Description:

(i) (*B*) On the axes below sketch the graph of $y = \mathrm{f}(-x)$.
Describe the transformation. [3]



Description:

(i) (*C*) On the axes below sketch the graph of $y = \mathrm{f}(x - 2)$.
Describe the transformation. [3]

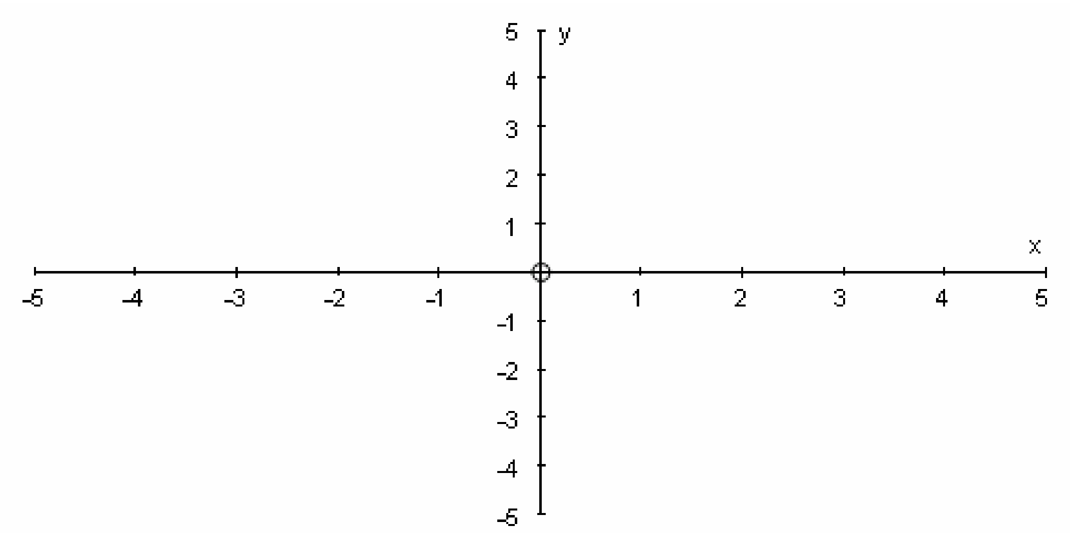


Description:

(iii) The function $\mathrm{g}(x)$ is defined as follows:

$$\mathrm{g}(x) = \mathrm{f}(x) \text{ for } x \geq 0$$

On the axes below sketch the graph of $y = \mathrm{g}^{-1}(x)$. [1]



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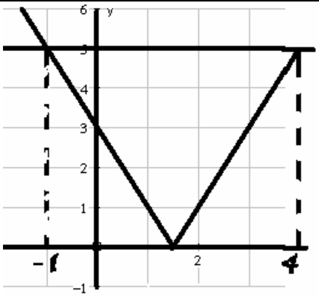
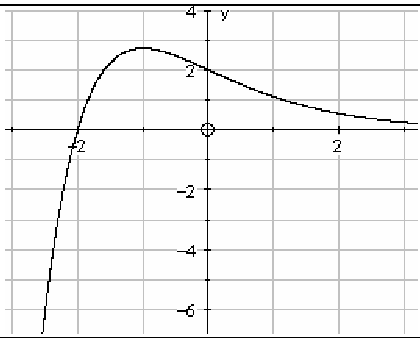
METHODS OF ADVANCED MATHEMATICS, C3

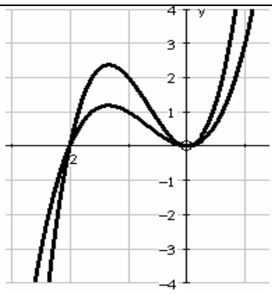
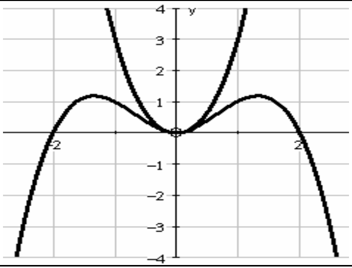
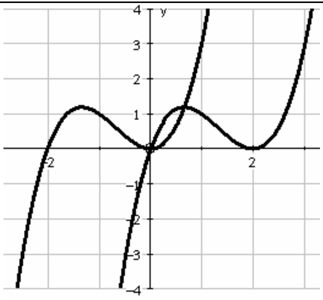
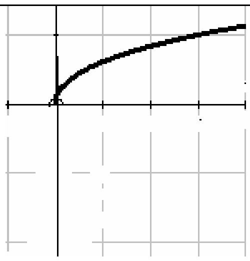
Practice Paper C3-B

MARK SCHEME

Qu		Answer	Mark	Comment
Section A				
1		Call the numbers n, $n + 1$ and $n + 2$ At least one of the numbers is even, and so the product is a multiple of 2. If n is a multiple of 3 then so is the product. If $n = 3k + 1$ then $n + 2$ is a multiple of 3 If $n = 3k + 2$ then $n + 1$ is a multiple of 3. n must have one of the forms $3k$, $3k + 1$ or $3k + 2$. Therefore whichever it is one of the three numbers is a multiple of 3 and so the product is a multiple of 3. Since it is also a multiple of 2 it is a multiple of 6.	B1 M1 M1 E1 **4**	Algebra Divisibility by 2 Divisibility by 3 conclusion
2	**(i)**		B1 B1 **2**	Right part Left part
	(ii)	Line $y = 5$ to be shown on graph. $-1 < x < 4$	M1 A1 **2**	
3	**(i)**	$y = \left(x^2+3\right)^5$ Let $u = x^2+3 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ $y = u^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}u} = 5u^4$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x} = 5u^4 \times 2x = 10x\left(x^2+3\right)^4$	M1 A1 A1 **3**	Chain rule $\frac{\mathrm{d}y}{\mathrm{d}u}$
	(ii)	$y = \frac{\sin 2x}{x}$ Let $u = \sin 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2\cos 2x$ $v = x \Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = 1$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2} = \frac{2x\cos 2x - \sin 2x}{x^2}$	M1 A1 A1 **3**	Quotient rule
4		$y^2 = 5x - 4 \Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5}{2y}$ When $x = 8, y^2 = 36 \Rightarrow y = \pm 6$ $\Rightarrow$ gradients = $\frac{5}{12}$ and $-\frac{5}{12}$	M1 A1 A1 A1 A1 **5**	

5		$x = x_0 e^{-3t} \Rightarrow e^{3t} = \frac{x_0}{x}$ $\Rightarrow 3t = \ln\left(\frac{x_0}{x}\right) \Rightarrow t = \frac{1}{3}\ln\left(\frac{x_0}{x}\right)$ $\Rightarrow t = \ln\left(\frac{x_0}{x}\right)^{\frac{1}{3}}$ i.e. $a = x_0,\ b = \frac{1}{3}$	M1 A1 A1 A1 **4**	Take logs or any equivalent method
6	**(i)**	$\int (2x-3)^7\,dx.$ Let $u = 2x-3,\ \frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}dx$ $= \int \frac{1}{2}u^7\,du = \frac{u^8}{2\times 8} = \frac{1}{16}(2x-3)^8 + c$	M1 A1 A1 **3**	or B3 cao
	(ii)	The substitution $u = x^2+1$ gives $\frac{du}{dx} = 2x$ $\Rightarrow \int_1^2 x(x^2+1)^3dx \quad = \int_2^5 \frac{1}{2}u^3du$ $= \left[\frac{u^4}{8}\right]_2^5$ $= \frac{609}{8}\ (= 76\tfrac{1}{8})$	M1 A1 A1 A1 A1 **5**	Using sub Correct int Correct limits Int Ans
7	**(i)**	$f^2(x) = 4x$	B1 **1**	
	(ii)	$fgh(x) = fg(x+2)$ $= f\,(x+2)^2$ $= 2(x+2)^2$	M1 A1 A1 **3**	correct order of functions
	(iii)	$y = h(x)$ $= x+2$ $\Rightarrow x = y-2$ $h^{-1}(x) = x-2$	B1 **1**	

Section B				
8	**(i)**	$0=(x+2)e^{-x}$ $\Rightarrow x=-2$ so (-2,0) and (0,2)	B1 B1 **2**	
	(ii)	$y=(x+2)e^{-x}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-e^{-x}(x+1)=0\Rightarrow x=-1$ SP is (-1,e)	M1 A1 M1 A1 **4**	Product rule = 0
	(iii)	$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=xe^{-x}$ At (-1,e) this is negative, so SP is a maximum.	M1 A1 A1 **3**	
	(iv)		B1 **1**	
	(v)	At (0,2) gradient is –1 so gradient of normal is 1 Normal is $y = x + 2$. $y=x+2,\ y=(x+2)e^{-x}$ $\Rightarrow 0=(x+2)(1-e^{-x})$ $\Rightarrow x=-2\ (or\ 0)$ New intersection point is (-2,0).	B1 M1 A1 **3**	
	(vi)	Required area is $\int_1^3 (x+2)e^{-x}dx$ $=\left[-e^{-x}(x+2)\right]_1^3+\int_1^3 e^{-x}dx$ $=\left[-e^{-x}(x+2)\right]_1^3+\left[-e^{-x}\right]_1^3$ $=\frac{-6}{e^3}+\frac{4}{e}$	B1 M1 A1 A1 A1 **5**	or equivalent

9	**(i) (A)**		The transformation is a stretch with the x-axis invariant and of scale factor 2.	B1 B1 **2**	Same orientation y values doubled
	(i) (B)		The transformation is a reflection in the y-axis.	B1 B2 **3**	same shape Inversion
	(i) (C)		The transformation is a translation of 2 units parallel to the x-axis, ie $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	B1 B2 **3**	Same shape Moved 2 to the right
	(ii)	There is a set of values of y (for example, y = 1) for which there are three corresponding values of x (so an inverse would be multivalued).		B1 B1 **2**	
	(iii)			B1 **1**	
	(iv)	$\mathrm{f}(x) = x^2(x+2)$ $\Rightarrow \mathrm{f}'(x) = 3x^2 + 4x$ So the gradient at (1,3) is 7. The gradient on the inverse (which is a reflection of the original in $y = x$) is therefore $^{-1}/_{7}$.		M1 A1 M1 A1 **4**	
	(v)	The graph and its reflection must intersect on the axis of reflection, ie y = x, so solve $y = x,\ y = x^2(x+2)$ $\Rightarrow x = x^2(x+2)$ $\Rightarrow 0 = x(x^2 + 2x - 1)$ $\Rightarrow x = 0, -1 \pm \sqrt{2}$ The positive non-zero root is as given.		M1 M1 E1 **3**	