(b) $\sin 2x = \cos x$ $\Rightarrow 2\sin x\cos x = \cos x$ $\Rightarrow 2\sin x\cos x - \cos x = 0$ $\Rightarrow \cos x(2\sin x - 1) = 0$ $\Rightarrow \cos x = 0, x = \pi/2, 3\pi/2$ or $\sin x = 1/2, x = \pi/6, 5\pi/6$	M1 B1 ,B1 B1 , B1 [5]	$\sin 2x = 2\sin x\cos x$ used in the equation. Answers w.w. B1,2,3 or 5. Answers as decimals or in degrees- follow M.S. and –1
(c) let $u = \sin x$, $du = \cos x\, dx$ $\Rightarrow \int \sin^2 x\cos x dx = \int u^2 du$ $= u^3/3 = \frac{1}{3}\sin^3 x + c$ $\int_0^{\pi/2} \sin^2 x\cos x dx = \left[\frac{1}{3}\sin^3 x\right]_0^{\pi/2}$ $= \frac{1}{3}\sin^3\frac{\pi}{2} - \frac{1}{3}\sin^3 0$ $= \frac{1}{3}$	M1 A1 M1 A1 cao [4]	$\int u^2 du$. For I by P, or by inspection, giveB2 for correct result or 0. $\frac{1}{3}\sin^3 x + c$ Condone no c. .Accept $\frac{1}{3}u^3 + c$. substituting correct limits c.a.o. refers to whole of (**c**).
(d) $y = 2^x$ $\Rightarrow \ln y = x\ln 2$ $\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln 2$ **OR** $y = e^{x\ln 2}$ $\Rightarrow \frac{dy}{dx} = y\ln 2$ **OR** $\frac{dy}{dx} = \ln 2\, e^{x\ln 2}$ $= 2^x \ln 2$	B1 B1, B1 B1 cao [4] Total [16]	$x\ln 2$ $\frac{1}{y}\frac{dy}{dx}$, $\ln 2$ **OR** B1 $e^{x\ln 2}$ **OR** B2 cao $\ln 2e^{x\ln 2}$

3 (i) Odd function $f(-x) = -xe^{-(-x)^2/2} = -xe^{-x^2/2} = (-f(x))$	B1 M1 E1 [3]	$f(-x)$ $= -f(x)$ brackets or comment needed to convince re signs
(ii) $y = e^{-x^2/2}$ let $u = -x^2/2$, $du/dx = -2x/2 = -x$ $y = e^u$, $dy/du = e^u$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -x\,e^u = -xe^{-x^2/2}$ $f'(x) = x.(-xe^{-x^2/2}) + 1.\ e^{-x^2/2}$ $= (1 - x^2)\ e^{-x^2/2}$ *	M1 A1 M1 E1 [4]	chain rule s.o.i. product rule (ft) s.o.i. www
(iii) $(1 - x^2)\ e^{-x^2/2} = 0$ $\Rightarrow 1 - x^2 = 0$ $\Rightarrow x^2 = 1$ $\Rightarrow x = 1$ or -1 When $x = 1$, $y = e^{-1/2}$ When $x = -1$, $y = -e^{-1/2}$	M1 A1 A1 A1 [4]	$1 - x^2 = 0$ or first line $=0$ $x = 1$ $y = e^{-1/2}$ $(-1, -e^{-1/2})$ SC A1 for both y-coords decimal only, 0.61 or better
(iv) $A = \int_0^1 xe^{-\frac{1}{2}x^2}\,dx$ let $u = ½\,x^2$, $du/dx = x$ $\Rightarrow du = x\,dx$ When $x = 0$, $u = 0$ When $x = 1$, $u = ½$ $\Rightarrow A = \int_0^{1/2} e^{-u}\,du$ * $= \left[-e^{-u}\right]_0^{1/2}$ $= -e^{-1/2} + 1 = 1 - e^{-1/2}$.	M1 M1 E1 M1 A1 [5]	correct integral (condone missing limits and dx) dealing with dx change of limits shown (convincing recovery needed from no dx) $\left[-e^{-u}\right]$ or equivalent (no decimals)

1(a) $a = 1$ $b = 3$ $c = 2$	B1 B1 B1 [3]	
(b) $y = \dfrac{\ln x}{1+\ln x}$ $\dfrac{dy}{dx} = \dfrac{(1+\ln x)\frac{1}{x} - \ln x.\frac{1}{x}}{(1+\ln x)^2}$ $= \dfrac{1}{x(1+\ln x)^2}$	M1 M1 A1 [3]	$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$ quotient rule or product rule (see below) consistent with their derivatives must cancel terms in numerator, but need not bring down the x
product rule: $\dfrac{dy}{dx} = \ln x(-1)(1+\ln x)^{-2}.\dfrac{1}{x} + \dfrac{1}{x}.(1+\ln x)^{-1}$ $= (1+\ln x)^{-2}.\dfrac{1}{x}(-\ln x + 1 + \ln x)$ $= \dfrac{1}{x(1+\ln x)^2}$	[3]	
(c) $(e^x + e^{-x})^2 = e^{2x} + 2 + e^{-2x}$ $\int (e^x + e^{-x})^2\,dx = \int (e^{2x} + 2 + e^{-2x}\,dx$ $= ½\,e^{2x} + 2x - ½\,e^{-2x} + c$	M1 A1 M1 A1 cao [4]	Correct expansion – condone e^{x^2} simplified – must have 2, allow $(e^x)^2$ and $(e^{-x})^2$ one or other of $\int e^{2x}dx = \dfrac{1}{2}e^{2x}$ or $\int e^{-2x}dx = -\dfrac{1}{2}e^{-2x}$ condone no c
(d) (i) $g(x+2) = a^{x+2} = 3\,a^x$ $\Rightarrow a^x . a^2 = 3\,a^x$ $\Rightarrow a^2 = 3$ $\Rightarrow a = \sqrt{3}$	M1 E1 [2]	$a^{x+2} = 3\,a^x$ or equivalent equation with a particular value of x, e.g. $a^2 = 3$ or $a^3 = 3a$, etc… Allow verifications, but must be exact.
(ii) $(\sqrt{3})^b = 5$ $\Rightarrow \sqrt{3}\,b = 5$ $\Rightarrow b \ln\sqrt{3} = \ln 5$ $\Rightarrow b = \ln 5 / \ln\sqrt{3}$ $\Rightarrow b = 2.93$	M1 M1 A1 [3]	Taking lns and bringing the power down cao – must be 3 s.f.

2(i) $(-1, 0)$	B1 [1]	or $x = -1$, or P = –1. Not (0, -1) or y = -1.
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}.1$ $= \frac{1}{2}(1+x)^{-1/2}(x+2+2x)$ $= \frac{1}{2}(1+x)^{-1/2}(3x+2)$ $= \frac{3x+2}{2\sqrt{1+x}}$ *	M1 M1 E1 [3]	$x.\frac{1}{2}(1+x)^{-1/2}$ $+(1+x)^{1/2}.1$
(iii) $\frac{dy}{dx} = 0$ when $3x + 2 = 0$ $\Rightarrow x = -2/3$ or –0.67 or better $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}} = -\frac{2}{3\sqrt{3}} = -0.385$ Gradient is infinite or undefined at P	M1 B1 cao B1 B1 [4]	$3x + 2 = 0$ soi condone rounding errors, e.g. 0.666 Any correct expression for y. For numerical answers, -0.38 or better, but isw after correct surd expressions.
(iv) Let $u = 1 + x$, $du = dx$, when $x = -1$, $u = 0$; when $x = 0$, $u = 1$ $\Rightarrow \int_{-1}^{0} x\sqrt{1+x}\,dx = \int_0^1 (u-1)u^{1/2}du$ $= \int_0^1 (u^{3/2} - u^{1/2})du$ * $= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \frac{2}{5} - \frac{2}{3} = -\frac{4}{15}$ Area (or –area) between curve and x axis	M1 M1 E1 B1 M1 A1 cao B1 [7]	Changing limits – must show evidence. Substituting $(u-1)u^{1/2}$ for $x\sqrt{(1+x)}$ and $du = dx$ or $dx/du = 1$ or $du/dx = 1$ www $\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}$ or equivalent substituting limits, but must have integrated (not differentiated) must mention x-axis or points O and P, or –1 and 0, ignore negatives.

4(i) P is (½ , 0)	B1 [1]	Allow P = ½ or x = ½ but not (0, ½)
(ii) Let $u = x$ and $v = (1-2x)^{1/2}$ $du/dx = 1$, $dv/dx = ½ .(-2).(1-2x)^{-1/2}$ $= -(1-2x)^{-1/2}$ $\frac{dy}{dx} = -x(1-2x)^{-1/2} + (1-2x)^{1/2}.1$ $= (1-2x)^{-1/2}[-x+1-2x]$ $= \frac{1-3x}{\sqrt{1-2x}}$ *	 B1 B1 M1 E1 (4)	 $½(1-2x)^{-1/2}$ $\times(-2)$ Product rule consistent with their derivatives www
$\frac{dy}{dx} = 0$ when $1-3x=0$ $\Rightarrow\ x = 1/3,\ y = \frac{1}{3}\sqrt{1-\frac{2}{3}} = \frac{1}{3\sqrt{3}}$	M1 A1 B1 (3) [7]	$1-3x=0$ (only) $x = 1/3$ $y = \frac{1}{3\sqrt{3}}$ or equivalent, but must be exact , with 1 – 2/3 simplified – mark final irrational answer
(iii) $A = \int_0^{\frac{1}{2}} x\sqrt{1-2x}\,dx$ let $u = 1-2x$, $du = -2dx$ $\Rightarrow x = (1-u)/2$ when $x = 0$, $u = 1$ when $x = ½$, $u = 0$ $\Rightarrow\ A = \int_1^0 \frac{1-u}{2}.u^{1/2}.(-\frac{1}{2})du$ $= \frac{1}{4}\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}})du$ *	 M1 M1 M1 E1 (4)	 $½(1-u).u^{1/2}$ $\times -½\,du$ $x=0$, $u=1$, $x=½$, $u=0$ [even if wrongly entered into integral] www
$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$ $= \frac{1}{4}(\frac{2}{3}-\frac{2}{5})$ $= \frac{1}{2}\times\frac{2}{15} = \frac{1}{15}$	B1 M1 A1 (3) [7]	$u^{3/2}$ and $u^{1/2}$ correctly integrated substituting (correct) limits (but must have clearly attempted to integrate) cao – must be exact fraction

3(i) P is (0,2) Q is (2,0)	B1 B1 [2]	Not '2' or $y = 2$ Not '2' or $x = 2$ If (2, 0) and (0, 2) without saying which is which, SCB1
(ii) $f(x) = 1 + e^{2x} = y \quad x \leftrightarrow y$ $x = 1 + e^{2y}$ $\Rightarrow x - 1 = e^{2y}$ $\Rightarrow \ln(x - 1) = 2y$ $\Rightarrow y = ½ \ln(x - 1)$ *	M1 M1 E1 [3]	Reasonable attempt to solve for x or y taking lns or $g(x) = ½ \ln(x - 1)$ www
(iii) $f'(x) = 2e^{2x}$ $f'(0) = 2$ $g'(x) = \dfrac{1}{2(x-1)}$ $g'(2) = ½$ $= \dfrac{1}{f'(0)}$ f and g are reflections in $y = x$.	M1 A1 M1 A1 B1 [5]	 $f'(0) = 2$ $g'(2) = ½$
(iii) Area under curve $= \int_0^1 (1+e^{2x})dx$ $= \left[x + \frac{1}{2}e^{2x}\right]_0^1$ $= 1 + ½ e^2 - ½$ $= ½ (1 + e^2)$ Area of rectangle $= 1 \times (1 + e^2)$ So area under curve = ½ area of rectangle	M1 A1 A1 B1 E1 [5]	correct integral and limits correctly integrated allow correct numerical answers allow correct numerical answers but must be exact for final E1

4 (i) $x = 1$	B1 [1]	
(ii) $f'(x) = \frac{(x-1).1 - x.1}{(x-1)^2}$ $= -\frac{1}{(x-1)^2}$. $(x-1)^2 > 0 \Rightarrow f'(x) < 0$ for all x. $\Rightarrow$ gradient is always negative	M1 M1 A1 cao A1 [4]	Correct denominator Correct numerator 'the square is always positive'
(iii) $A = \int_2^3 \frac{x}{x-1}dx$, Let $u = x - 1$, $\Rightarrow du = dx$ When $x = 2$, $u = 1$; when $x = 3$, $u = 2$ $\Rightarrow A = \int_1^2 \frac{u+1}{u}du$ $= \int_1^2 (1 + \frac{1}{u})du$ * $= [u + \ln u]_1^2$ $= 2 + \ln 2 - 1 - \ln 1$ $= 1 + \ln 2$	M1 E1 E1 M1 M1 A1 cao [6]	Correct integral and limits changing limits correct derivation of transformed integral $u + \ln u$ substituting limits into correct integrand www
(iv) $y = \frac{x}{x-1}$ $x \leftrightarrow y$ $x = \frac{y}{y-1}$ $\Rightarrow xy - x = y$ $\Rightarrow xy - y = x$ $\Rightarrow y(x-1) = x$ $\Rightarrow y = \frac{x}{x-1}$ so $f^{-1}(x) = f(x)$ Symmetrical about $y = x$.	M1 M1 A1 cao B1 [4]	Attempt to reverse formula collecting terms *or* Expression for f f(x) M1 Clearing subsidiary denominators M1 Simplifying to x A1

2(i) $a = 1/2$	B1 [1]	
(ii) $\frac{dy}{dx} = \frac{(2x-1)^2.1 - x.2(2x-1).2}{(2x-1)^4}$ $= \frac{(2x-1)(2x-1-4x)}{(2x-1)^4}$ $= -\frac{(2x+1)}{(2x-1)^3}$ *	B1 M1 M1 E1 (4)	derivative of $(2x-1)^2$ is $2(2x-1).2$ numerator consistent with their derivatives denominator correct (or $[(2x-1)^2]^2$) www (not $\frac{-2x+1}{(2x-1)^3}$)
Or $\frac{dy}{dx} = x(-2)(2x-1)^{-3}.2 + (2x-1)^{-2}$ $= (2x-1)^{-3}(-4x+2x-1)$ $= (2x-1)^{-3}(-1-2x)$ $= -\frac{1+2x}{(2x-1)^3}$	B1 M1 M1 E1 (4)	$-4(2x-1)^{-3}$ Product rule consistent with their derivatives Factor of $(2x-1)^{-3}$ or common denominator
$\frac{dy}{dx} = 0$ when $2x + 1 = 0$ $\Rightarrow x = -1/2,$ $y = -1/8$	M1 A1 cao A1 cao (3) [7]	setting *numerator* to 0
(iii) $A = \int_1^2 \frac{x}{(2x-1)^2} dx$ Let $u = 2x - 1$, $du = 2dx$ when $x = 1$, $u = 1$ when $x = 2$, $u = 3$ so $A = \int_1^3 \frac{\frac{1}{2}(u+1)}{u^2}\frac{1}{2}du$ $= \int_1^3 \frac{(u+1)}{4u^2}du$ *	M1 M1 E1 E1 (4)	Correct integral (soi) -condone no dx $u = 2x - 1$ changing limits – must show some working transforming integral (www)
$= \frac{1}{4}\int_1^3 (\frac{1}{u} + \frac{1}{u^2})du$ $= \frac{1}{4}\left[\ln u - \frac{1}{u}\right]_1^3$ $= \frac{1}{4}(\ln 3 - \frac{1}{3} - \ln 1 + 1)$ $= \frac{1}{4}(\ln 3 + \frac{2}{3})$ $= \frac{3\ln 3 + 2}{12}$ *	M1 A1 E1 (3) [7]	splitting fraction correctly integrated www