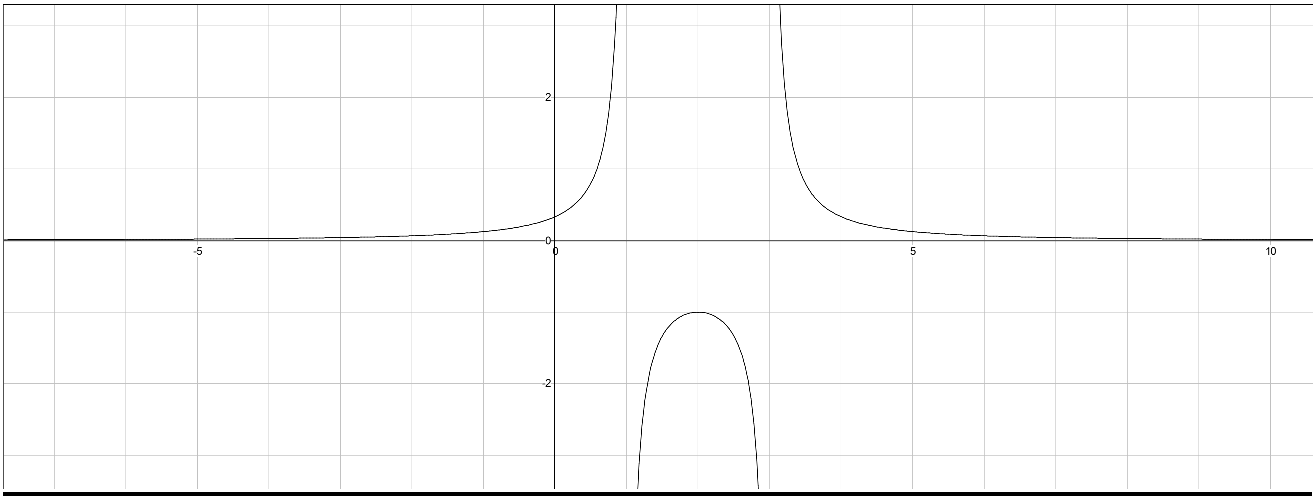
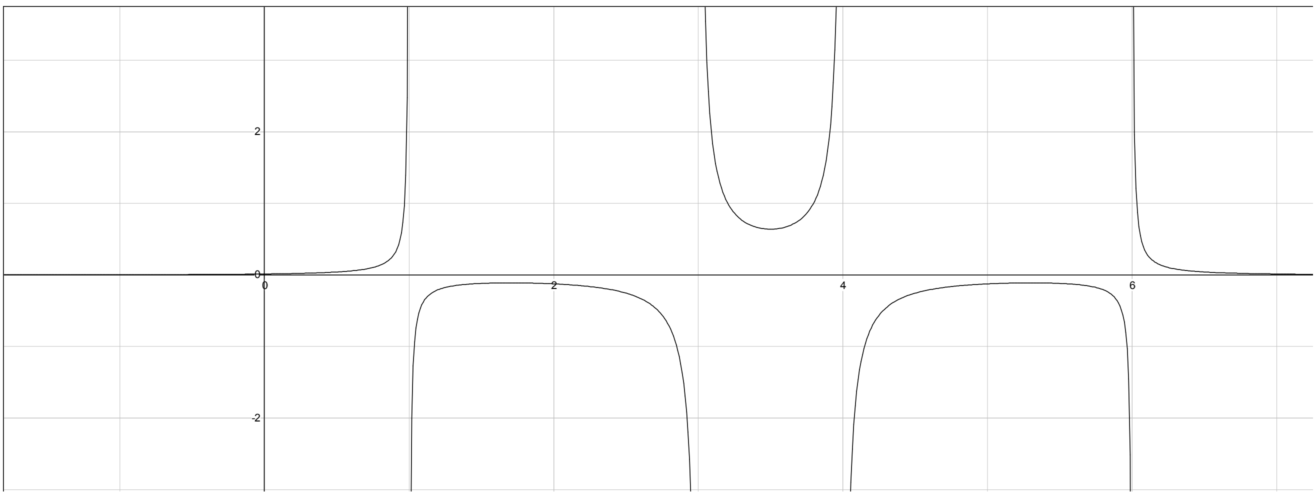
QUESTION 5

Part (i)



This is the graph of $f(x) = \dfrac{1}{(x-2)^2 - 1}$

Part (ii)



This is the graph of $g(x) = \dfrac{1}{((x-2)^2 - 1)((x-5)^2 - 1)}$

i.e $b = 5, a = 2, b > a + 2$

It has asymptotes $x = 1, x = 3, x = 4, x = 6, y = 0$

$$g'(x) = -\frac{\dfrac{d\left[((x-2)^2-1)((x-5)^2-1)^2\right]}{dx}}{(((x-2)^2-1)((x-5)^2-1))^2}$$

$$= -\frac{\left[2(x-5)((x-2)^2-1)+2(x-2)((x-5)^2-1)\right]}{(((x-2)^2-1)((x-5)^2-1))^2}$$

$$= -\frac{2\left[(x-5)(x-3)(x-1)+(x-2)(x-4)(x-6)\right]}{(((x-2)^2-1)((x-5)^2-1))^2}$$

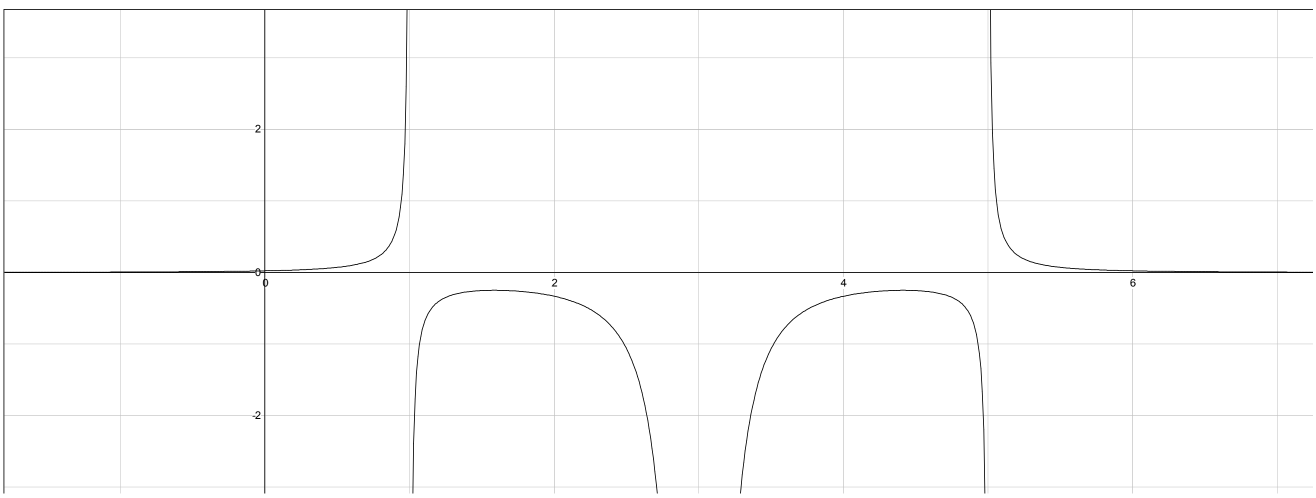
Note at this point it can be seen that one solution is $\dfrac{dy}{dx} = 0$ when $x = 3.5$

$$= -\frac{2\left[\left(x^3-9x^2+23x-15\right)+\left(x^3-12x^2+44x-48\right)\right]}{(((x-2)^2-1)((x-5)^2-1))^2}$$

$$= -\frac{2\left[2x^3-21x^2+67x-63\right]}{(((x-2)^2-1)((x-5)^2-1))^2}$$

$$= -\frac{2\left[(2x-7)\left(x^2-7x+9\right)\right]}{(((x-2)^2-1)((x-5)^2-1))^2}$$ (Using the note above)

Therefore $\dfrac{dy}{dx} = 0$ when $x = 3.5$ or $x^2-7x+9=0 \Rightarrow x = \dfrac{7\pm\sqrt{13}}{2}$



This is the graph of $g(x) = \dfrac{1}{((x-2)^2-1)((x-4)^2-1)}$

i.e $b=4, a=2, b=a+2$

It has asymptotes $x=1, x=3, x=5, y=0$

$$g'(x) = -\frac{\frac{d\left[((x-2)^2-1)((x-4)^2-1)^2\right]}{dx}}{(((x-2)^2-1)((x-4)^2-1))^2}$$

$$= -\frac{\left[2(x-4)((x-2)^2-1)+2(x-2)((x-4)^2-1)\right]}{(((x-2)^2-1)((x-4)^2-1))^2}$$

$$= -\frac{2\left[(x-4)(x-3)(x-1)+(x-2)(x-5)(x-3)\right]}{(((x-2)^2-1)((x-4)^2-1))^2}$$

$$= -\frac{2(x-3)\left[(x-4)(x-1)+(x-2)(x-5)\right]}{(x-3)(x-1)(x-3)(x-5)}$$

$$= -\frac{4\left[x^2-6x+7\right]}{(x-1)(x-3)(x-5)}$$

Therefore $\frac{dy}{dx}=0$ when $x^2-6x+7=0 \Rightarrow x=3\pm\sqrt{2}$