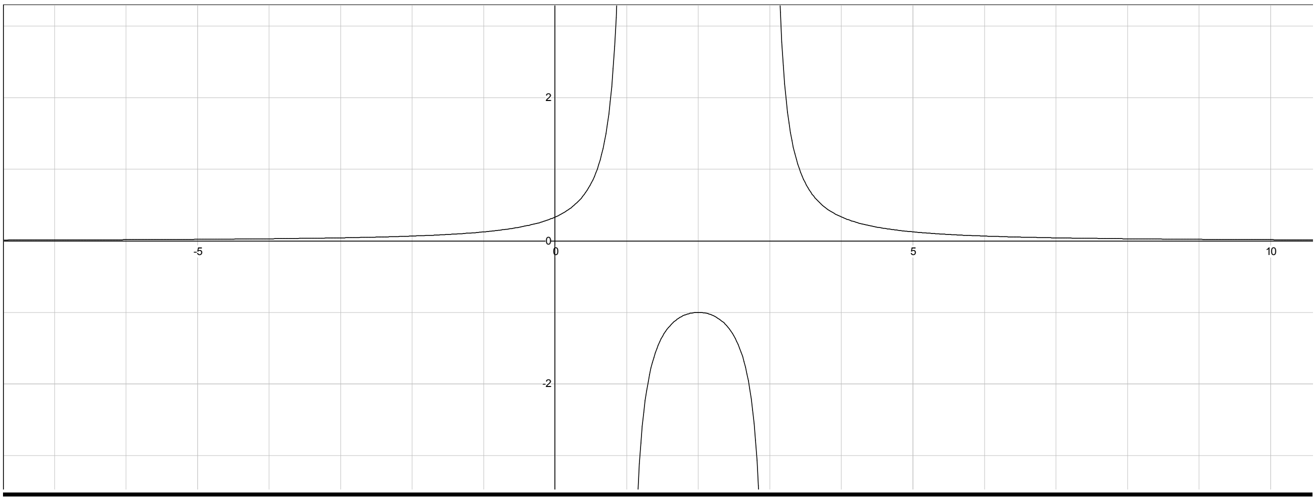
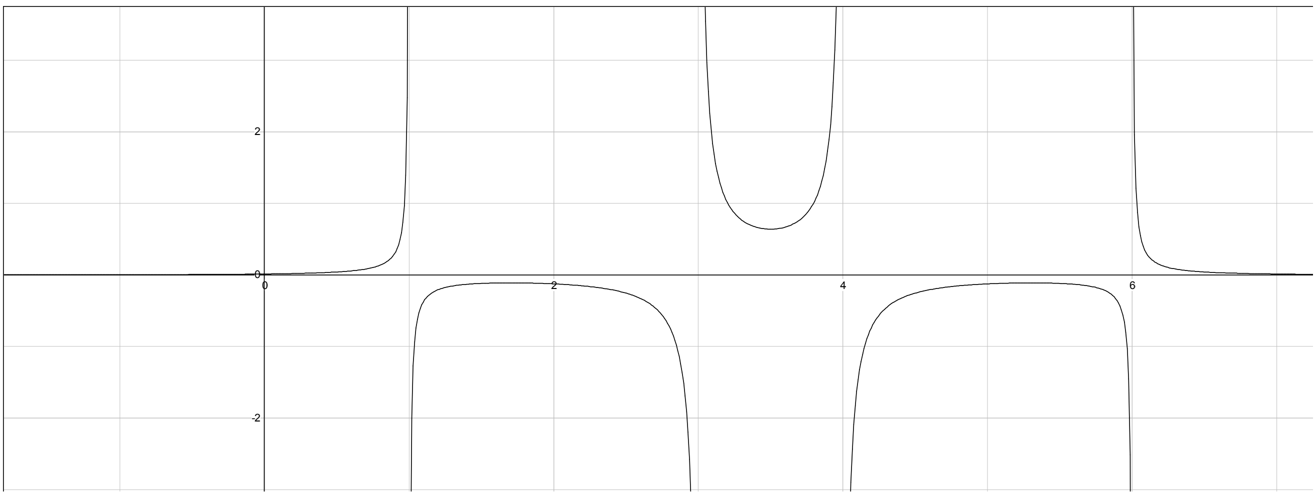
QUESTION 5

Part (i)



This is the graph of $f(x)=\dfrac{1}{(x-2)^2-1}$

Part (ii)



This is the graph of $g(x)=\dfrac{1}{((x-2)^2-1)((x-5)^2-1)}$

i.e $b=5, a=2, b>a+2$

It has asymptotes $x=1, x=3, x=4, x=6, y=0$

Which translated back would be $x = a - 1, x = a + 1, x = b - 1, x = b + 1, y = 0$

$$g'(x) = -\frac{\dfrac{d\left[((x-a)^2 - 1)((x-b)^2 - 1)^2\right]}{dx}}{(((x-a)^2 - 1)((x-b)^2 - 1))^2}$$

$$= -\frac{\left[2(x-b)((x-a)^2 - 1) + 2(x-a)((x-b)^2 - 1)\right]}{(((x-a)^2 - 1)((x-b)^2 - 1))^2}$$

$$= -\frac{2\left[(x-b)(x-a-1)(x-a+1) + (x-a)(x-b-1)(x-b+1)\right]}{(((x-a)^2 - 1)((x-b)^2 - 1))^2}$$

$$= -\frac{2\left[\left(x^3 - (b+2a)x^2 + \left(a^2 - 1 + 2ab\right)x - a\left(b^2 - 1\right)\right) + \left(x^3 - (a+2b)x^2 + \left(b^2 - 1 + 2ab\right)x - b\left(a^2 - 1\right)\right)\right]}{(((x-2)^2 - 1)((x-5)^2 - 1))^2}$$

$$= -\frac{\left[2x^3 - 3(a+b)x^2 + \left(a^2 + b^2 + 4ab - 2\right)x - \left(b\left(a^2 - 1\right) + a\left(b^2 - 1\right)\right)\right]}{(((x-a)^2 - 1)((x-b)^2 - 1))^2}$$

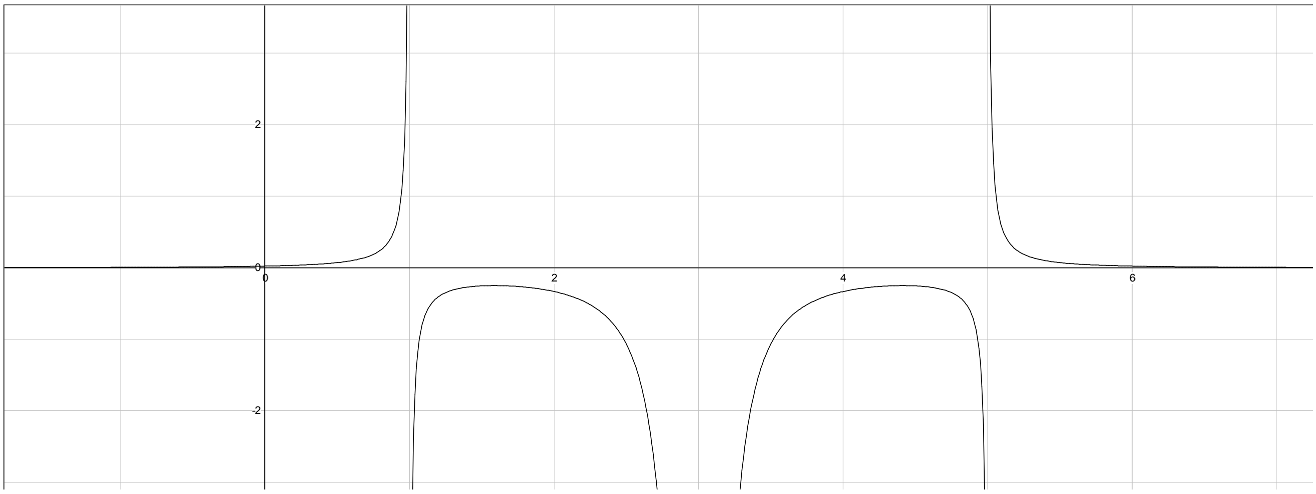
<u>Note</u> at this point it can be seen from the graph that one solution is:-

$$\frac{dy}{dx} = 0 \text{ when } x = \frac{1}{2}\left[(a+1) + (b-1)\right] = \frac{1}{2}(a+b)$$

$$= -\frac{\left[\left(2x - (a+b)\right)\left(x^2 - (a+b)x + (ab-1)\right)\right]}{(((x-a)^2 - 1)((x-b)^2 - 1))^2}$$

Therefore $\dfrac{dy}{dx} = 0$ when $x = \dfrac{1}{2}(a+b)$ or

$$x = \frac{(a+b) \pm \sqrt{(a+b)^2 - 4(ab-1)}}{2} = \frac{(a+b) \pm \sqrt{(a-b)^2 + 4}}{2}$$



This is the graph of $g\left(x\right)=\dfrac{1}{((x-2)^2-1)((x-4)^2-1)}$

i.e $b=4,a=2,b=a+2$

It has asymptotes $x=1,x=3,x=5,y=0$

Which translated back would be $x=a-1,x=a+1=b-1,x=b+1,y=0$

$$g'\left(x\right)=-\frac{2\left[\left(x-b\right)\left(x-a-1\right)\left(x-a+1\right)+\left(x-a\right)\left(x-b-1\right)\left(x-b+1\right)\right]}{(((x-a)^2-1)((x-b)^2-1))^2}$$

(as before)

This time since $b=a+2\Rightarrow b-1=a+1$ we have :-

$$x-\left(b-1\right)=x-\left(a+1\right)\Rightarrow x-b+1=x-a-1$$

So $g'\left(x\right)=-\dfrac{2\left(x-a-1\right)\left[\left(x-b\right)\left(x-a+1\right)+\left(x-a\right)\left(x-b-1\right)\right]}{(((x-a)^2-1)((x-b)^2-1))^2}$

$$=-\frac{2\left(x-a-1\right)\left[\left(x-b\right)\left(x-a+1\right)+\left(x-a\right)\left(x-b-1\right)\right]}{\left(x-a-1\right)\left(x-a+1\right)((x-b)^2-1)^2}$$

$$= -\frac{2\left[(x-b)(x-a+1)+(x-a)(x-b-1)\right]}{(x-a+1)((x-b)^2-1)^2}$$

$$= -\frac{2\left[\left(x^2-(a+b)x+ab-b\right)+\left(x^2-(a+b)x+ab+a\right)\right]}{(x-a+1)((x-b)^2-1)^2}$$

$$= -\frac{2\left[2x^2-2(a+b)x+2ab-(b-a)\right]}{(x-a+1)((x-b)^2-1)^2}$$

Therefore $\frac{dy}{dx}=0$ when $2x^2-2(a+b)x+2ab-(b-a)=0$

$$\Rightarrow x=\frac{2(a+b)\pm\sqrt{4(a+b)^2-4\times2\times(2ab+a-b)}}{4}$$

$$\Rightarrow x=\frac{(a+b)\pm\sqrt{(b-a)^2+2(b-a)}}{2}$$