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Mark Scheme (Results)

January 2013

GCE Further Pure Mathematics FP1 (6667/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- Unless indicated in the mark scheme a correct answer with no working should gain full marks for that part of the question.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but incorrect answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

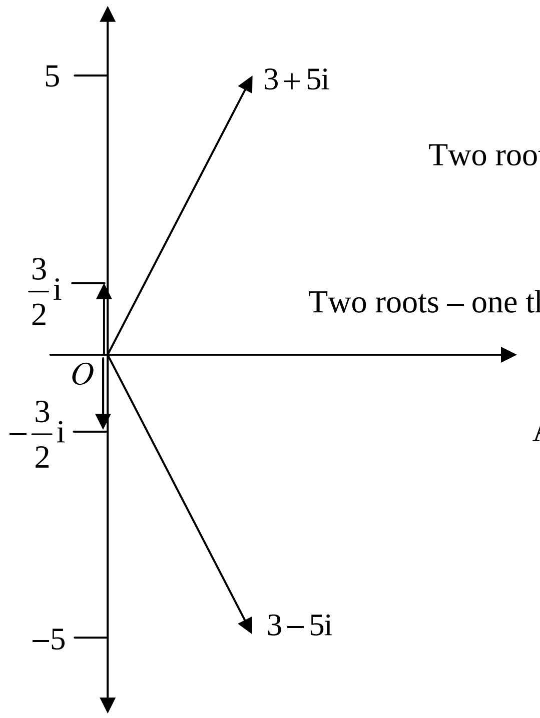
Jan 2013
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme	Marks
1.	$\sum_{r=1}^{n} 3(4r^2 - 4r + 1) = 12\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + \sum_{r=1}^{n} 3$	M1
	$= \frac{12}{6}n(n+1)(2n+1) - \frac{12}{2}n(n+1), \quad +3n$	A1, B1
	$= n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	M1
	$= n\left[4n^2 - 1\right] = n(2n+1)(2n-1)$	A1 cso
		[5]
Notes:	Induction is not acceptable here First M for expanding given expression to give a 3 term quadratic and attempt to substitute. First A for first two terms correct or equivalent. B for $+3n$ appearing Second M for factorising by n Final A for completely correct solution	

Question Number	Scheme	Marks
2.	(a) $\dfrac{50}{3+4i}=\dfrac{50(3-4i)}{(3+4i)(3-4i)}=\dfrac{50(3-4i)}{25}=6-8i$	M1 A1cao **(2)**
	(b) $z^2=\left(6-8i\right)^2=36-64-96\text{i}=-28-96\text{i}$	M1 A1 **(2)**
	(c) $\lvert z\rvert=\sqrt{6^2+\left(-8\right)^2}=10$	M1 A1ft **(2)**
	(d) $\tan\alpha=\dfrac{-96}{-28}$	M1
	so $\alpha=-106.3^{\text{o}}$ or 253.7^{o}	A1cao **(2)** **[8]**
	<u>Alternatives</u> (c) $\lvert z\rvert=\dfrac{50}{\lvert 3+4\text{i}\rvert}=10$	M1 A1
	(d) arg (3+4i) = 53.13.. so $\arg\left(\dfrac{50}{3+4\text{i}}\right)^2=-2\times 53.13\ldots$ = -106.3	M1 A1
Notes:	(a) M for $\times\dfrac{3-4i}{3-4i}$ (accept use of -3+4i) and attempt to expand using $i^2=-1$, A for 6-8i only (b) M for attempting to expand their z^2 using $i^2=-1$, A for -28-96i only. If using original z then must attempt to multiply top and bottom by conjugate and use $i^2=-1$. (c) M for $\sqrt{a^2+b^2}$, A for 'their 10' (d) M for use of tan or $\tan^{-1}$ and values from their z^2 either way up ignoring signs. Radians score A0.	

Question Number	Scheme	Marks
3.	(a) $f'(x) = x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$	M1 A1 **(2)**
	(b) f(5) = - 0.0807	B1
	$f'(5) = 0.4025$	M1
	$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 5 - \dfrac{-0.0807}{0.4025}$	M1
	=5.2(0)	A1 **(4)** **[6]**
Notes	The B and M marks are implied by a correct answer only with no working or by $\frac{5}{9}(10\sqrt{5}-13)$ (a) M for at least one of $\pm ax^{-\frac{1}{2}}$ or $\pm bx^{-\frac{3}{2}}$, A for correct (equivalent) answer only (b) B for awrt -0.0807, first M for attempting their f'(5), M for correct formula and attempt to substitute, A for awrt 5.20, but accept 5.2	

Question Number	Scheme	Marks
4.	(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 **(1)**
	(b) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 **(1)**
	(c) **R = QP**	B1 **(1)**
	(d) $R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 cao **(2)**
	(e) Reflection in the *y* axis	B1 B1 **(2)** **[7]**
Notes	(a) and (b) Signs must be clear for B marks. (c) Accept **QP** or their 2x2 matrices in the correct order only for B1. (d) M for their **QP** where answer involves ±1 and 0 in a 2x2 matrix, A for correct answer only. (e) First B for Reflection, Second B for '*y* axis' or 'x=0'. Must be single transformation. Ignore any superfluous information.	

Question Number	Scheme	Marks
5.	(a) $4x^2+9=0 \quad \Rightarrow \quad x=k\mathrm{i}, \quad x=\pm\frac{3}{2}\mathrm{i}$ or equivalent	M1, A1
	Solving 3-term quadratic by formula or completion of the square	M1
	$x=\frac{6\pm\sqrt{36-136}}{2}$ or $(x-3)^2-9+34=0$	
	$=3+5\mathrm{i}$ and $3-5\mathrm{i}$	A1 A1ft **(5)**
	(b)	
	 	
	Two roots on imaginary axis	B1ft
	Two roots – one the conjugate of the other	B1ft
	Accept points or vectors	**(2)** **[7]**
Notes	(a) Final A follow through conjugate of their first root. (b) First B award only for first pair imaginary, Second B award only if second pair complex. Complex numbers labelled , scales or coordinates or vectors required for B marks.	

Question Number	Scheme	Marks
6.	(a) Determinant: $2 - 3a = 0$ and solve for $a =$	M1
	So $a = \frac{2}{3}$ or equivalent	A1 **(2)**
	(b) Determinant: $(1\times 2)-(3\times -1)=5 \quad (\Delta)$	
	$Y^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \quad \left[= \begin{pmatrix} 0.4 & 0.2 \\ -0.6 & 0.2 \end{pmatrix} \right]$	M1A1 **(2)**
	(c) $\frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1-\lambda \\ 7\lambda-2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2-2\lambda+7\lambda-2 \\ -3+3\lambda+7\lambda-2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda-1 \end{pmatrix}$	M1depM1A1 A1 **(4)** **[8]**
	<u>Alternative method for (c)</u>	
	$\begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-\lambda \\ 7\lambda-2 \end{pmatrix}$ so $x - y = 1 - \lambda$ and $3x + 2y = 7\lambda - 2$	M1M1
	Solve to give $x = \lambda$ and $y = 2\lambda - 1$	A1A1
Notes	(b) M for $\frac{1}{\text{their det}}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}$	
	(c) First M for their $\mathbf{Y}^{-1}\mathbf{B}$ in correct order with $\mathbf{B}$ written as a 2x1 matrix**,** second M dependent on first for attempt at multiplying their matrices resulting in a 2x1 matrix, first A for λ, second A for $2\lambda - 1$	
	Alternative for (c) First M to obtain two linear equations in x, y, λ Second M for attempting to solve for x or y in terms of λ	

Question Number	Scheme	Marks
7.	(a) $y=\frac{25}{x}$ so $\frac{dy}{dx}=-25x^{-2}$	M1
	$\frac{dy}{dx}=-\frac{25}{(5p)^2}=-\frac{1}{p^2}$	A1
	$y-\frac{5}{p}=-\frac{1}{p^2}(x-5p) \Rightarrow \quad p^2y+x=10p \qquad (*)$	M1 A1 **(4)**
	(b) $q^2y+x=10q$ only	B1 **(1)**
	(c) $(p^2-q^2)y=10(p-q)$ so $y=\frac{10(p-q)}{(p^2-q^2)}=\frac{10}{p+q}$	M1 A1cso
	$x=10p-p^2\frac{10}{p+q}=\frac{10pq}{p+q}$	M1 A1 cso **(4)**
	(d) Line *PQ* has gradient $\frac{\frac{5}{p}-\frac{5}{q}}{5p-5q}\left(=-\frac{1}{pq}\right)$	M1 A1
	ON has gradient $\frac{\frac{10}{p+q}}{\frac{10pq}{p+q}}\left(=\frac{1}{pq}\right)$ or $\frac{-1}{\frac{-1}{pq}}(=pq)$ could be as unsimplified equivalents seen anywhere	B1
	As these lines are perpendicular $\frac{1}{pq}\times-\frac{1}{pq}=-1$ so $p^2q^2=1$	
	OR for *ON* $y-y_1=m(x-x_1)$ with gradient (equivalent to) pq and sub in points *O* AND *N* to give $p^2q^2=1$ **OR** for *PQ* $y-y_1=m(x-x_1)$ with gradient (equivalent to) -pq and sub in points *P* AND *Q* to give $p^2q^2=1$. NB –pq used as gradient of *PQ* implies first M1A1	M1 A1 **(5)** **[14]**

Question Number	Scheme	Marks
	Alternatives for first M1 A1 in part (a) $x\frac{dy}{dx}+y=0 \Rightarrow \frac{dy}{dx}=-\frac{y}{x}$ So at P gradient = $\frac{-\frac{5}{p}}{5p}=-\frac{1}{p^2}$ Or $x=5t,\ y=\frac{5}{t}\ \Rightarrow\ \frac{dx}{dt}=5,\ \frac{dy}{dt}=-\frac{5}{t^2}$ so $\frac{dy}{dx}=$ $\frac{-\frac{5}{t^2}}{5}=-\frac{1}{t^2}$ so at P gradient = $-\frac{1}{p^2}$	M1 A1 M1 A1
Notes	(a) First M for attempt at explicit, implicit or parametric differentiation not using p or q as an initial parameter, first A for $\frac{-1}{p^2}$ or equivalent. Quoting gradient award first M0A0. Second M for using $y-y_1=m(x-x_1)$ and attempt to substitute or $y=mx+c$ and attempt to find c; gradient in terms of p only and using $\left(5p,\frac{5}{p}\right)$, second A for correct solution only. (c) First M for eliminating x and reaching $y=\mathrm{f}(p,q)$, second M for eliminating y and reaching $x=\mathrm{f}(p,q)$, both As for given answers. Minimum amount of working given in the main scheme above for 4/4, but do not award accuracy if any errors are made. (d) First M for use of $\frac{y_2-y_1}{x_2-x_1}$ and substituting, first A for $\frac{-1}{pq}$ or unsimplified equivalent . Second M for their product of gradients=-1 (or equating equivalent gradients of *ON* or equating equivalent gradients of *PQ*), second A for correct answer only.	

Question Number	Scheme	Marks
8.	(a) If $n=1$, $\sum_{r=1}^{n} r(r+3)=1\times 4=4$ and $\frac{1}{3}n(n+1)(n+5)=\frac{1}{3}\times 1\times 2\times 6=4$**,**	B1
	(so true for n = 1. Assume true for n = k)	
	So $\sum_{r=1}^{k+1} r(r+3)=\frac{1}{3}k(k+1)(k+5)+(k+1)(k+4)$	M1
	$= \frac{1}{3}(k+1)\left[k(k+5)+3(k+4)\right] = \frac{1}{3}(k+1)\left[k^2+8k+12\right]$	A1
	$= \frac{1}{3}(k+1)(k+2)(k+6)$ which implies is true for $n=k+1$	dA1
	As result is true for $n=1$ this implies true for all positive integers and so result is true by induction	dM1A1cso
		(6)
	(b) $u_1=1^2(1-1)+1=1$	B1
	(so true for $n=1$. Assume true for n = k)	
	$u_{k+1}=k^2(k-1)+1+k(3k+1)$	M1,
	$=k(k^2-k+3k+1)+1=k(k+1)^2+1$ which implies is true for n = k + 1	A1
	As result is true for n = 1 this implies true for all positive integers and so result is true by induction	M1A1cso **(5)**
		[11]
Notes	(a) First B for LHS=4 and RHS =4 First M for attempt to use $\sum_{1}^{k} r(r+3)+u_{k+1}$ First A for $\frac{1}{3}(k+1)$, $\frac{1}{3}(k+2)$ or $\frac{1}{3}(k+6)$ as a factor before the final line Second A dependent on first for $\frac{1}{3}(k+1)(k+2)(k+6)$ with no errors seen Second M dependent on first M and for any 3 of 'true for n=1' 'assume true for n=k' 'true for n=k+1', 'true for all n' (or 'true for all positive integers') seen anywhere Third A for correct solution only with all statements and no errors	

	(b) First B for both some working and 1. First M for $u_{k+1} = u_k + k(3k+1)$ and attempt to substitute for u_k First A for $k(k+1)^2 + 1$ with some correct intermediate working and no errors seen Second M dependent on first M and for any 3 of 'true for n=1' 'assume true for n=k' 'true for n=k+1', 'true for all n' (or 'true for all positive integers') seen anywhere Second A for correct solution only with all statements and no errors	

Question Number	Scheme	Marks
9.	(a) $y = 6x^{\frac{1}{2}}$ so $\frac{dy}{dx} = 3x^{-\frac{1}{2}}$	M1
	Gradient when x = 4 is $\frac{3}{2}$ and gradient of normal is $-\frac{2}{3}$	M1 A1
	So equation of normal is $(y-12) = -\frac{2}{3}(x-4)$ (or $3y + 2x = 44$)	M1 A1
		(5)
	(b) S is at point (9,0)	B1
	N is at (22,0), found by substituting y=0 into their part (a)	B1ft
	Both B marks can be implied or on diagram.	
	So area is $\frac{1}{2}\times 12\times(22-9) = 78$	M1 A1 cao
		(4)
		[9]
	Alternatives:	
	First M1 for $ky\frac{dy}{dx} = 36$ **or** for	
	$x = 9t^2, y = 18t \rightarrow \frac{dx}{dt} = 18t, \frac{dy}{dt} = 18 \rightarrow \frac{dy}{dx} = \frac{1}{t}$	
Notes	(a) First M for $\frac{\mathrm{d}y}{\mathrm{d}x} = ax^{-\frac{1}{2}}$, Second M for substituting x=4 (or y=12 or t=2/3 if alternative used) into their gradient and applying negative reciprocal. First A for $-\frac{2}{3}$ Third M for $y - y_1 = m(x - x_1)$ or $y = mx + c$ and attempt to substitute a changed gradient AND (4,12) Second A for $3y + 2x = 44$ or any equivalent equation (b) M for Area= $\frac{1}{2}$ base x height and attempt to substitute including their numerical ‘(22-9)’ or equivalent complete method to find area of triangle *PSN*.	

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